LONG-RANGE EFFECTIVE POTENTIALS OF GRAIN INTERACTIONS IN PLASMAS

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Analytical estimates of the effective potential of a dust particle embedded into a plasma are performed with regard for its charging by plasma current. The plasma dynamics is described within the drift-diffusion approximation. It is shown that the effective potential in the case under consideration manifests the Coulomb-like asymptotic behaviour. The approximate expressions for the electric grain charge and plasma particle distributions around a grain are derived and compared with the earlier obtained numerical solutions of the problem.

Studying the effective potentials of a grain, whose charge is maintained by plasma currents, still remains one of the important issues of dusty plasma theory. This problem has been intensively studied in many papers within the framework of various models (see, for example, [1—14] and references cited therein). The variety of important details such as the dependence of the effective potentials on the plasma dynamics, mechanisms of plasma regeneration, charging processes, the presence of collisions of plasma particles with neutrals, the existence of ionic bound states, etc. has been discovered. In particular, it has been shown that, in the case of the fixed grain charge with no fluxes through the grain surface, the effective potential has the Yukawa-type asymptotics and can be described by the Debye potential with the effective charge [2, 3]. In such a case, the potential under consideration does not depend on the details of the plasma dynamics. If the grain charge is maintained by the plasma currents, the potential is considerably dependent on the charging kinetics (in the case of collisionless plasma at large distances, the potential is inversely proportional to the square of distance [5—7], while the asymptotics manifests the Coulomb-like behaviour in the case of strongly collisional dynamics [14]). Moreover, it has been established that, in the latter case, the potential asymptotics is dependent on the distribution of the sources of plasma regeneration [14, 15].

However, it is necessary to point out that the major part of the results mentioned above was obtained using the numerical studies of the appropriate models. The problem is that, due to a strong coupling between plasma particles and grains, the methods of linear analysis cannot be applied. Nevertheless, as was shown in [15], linearized equations make it possible to describe the asymptotic behaviour of the potential. Since the analytical relations for the effective potentials can be very useful for various applications in dusty plasma theory, it looks reasonable to find approximate analytical solutions of the problem of grain screening, which describe not only the qualitative asymptotic behaviour, but give also the quantitative estimates for the grain charge, plasma particle distributions around a grain, and effective potential. In the present paper, we propose such approximate solution for the case of a dust particle embedded in the weakly ionized plasma assuming that the grain absorbs all encountered electrons and ions. We also assume that the plasma sources, which compensate the plasma particle absorption by the grain, are located at the boundary of the system, i.e. at large distances from the grain.

Let us consider a grain located at the origin of the coordinate system. The grain is surrounded by the weakly ionized plasma. Due to the absorption of electrons and ions by the grain, the fluxes of plasma particles $\vec{\Gamma}_\sigma$ toward the grain arise (subscript σ labels plasma particle species, $\sigma = e$ – electron, $\sigma = i$ – ion). In the stationary case, these fluxes satisfy the equation

$$\mathrm{div}\vec{\Gamma}_\sigma = 0. \qquad (1)$$

In view of the spherical symmetry of the problem, $\vec{\Gamma}_\sigma = (\Gamma_{\sigma r}, 0, 0)$, one has

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\Gamma_{\sigma r}\right) = 0, \qquad (2)$$

or

$$\Gamma_{\sigma r} = \frac{B_\sigma}{r^2}, \qquad (3)$$

where B_σ is the constant to be determined using boundary conditions.

We assume that the plasma particle motion is governed by the drift-diffusive equations. Within such an approximation, the plasma particle fluxes can be written as

$$\Gamma_{\sigma r}(r) = -\frac{e_\sigma n_\sigma(r)}{m_\sigma \nu_\sigma}\nabla\Phi(r) - D_\sigma \nabla n_\sigma(r), \quad (4)$$

where $D_\sigma = T_\sigma/(m_\sigma \nu_\sigma)$ is the diffusion coefficient, ν_σ is the collision frequency, $e_i = e, e_e = -e$. The rest of notation is conventional.

Combining Eqs. (3) and (4), we have the equation for plasma particle densities

$$\frac{dn_\sigma(r)}{dr} + \frac{e_\sigma}{T_\sigma} n_\sigma(r)\frac{d\Phi(r)}{dr} = -\frac{B_\sigma}{D_\sigma r^2}. \quad (5)$$

With regard for the boundary condition

$$n_\sigma(\infty) = n_0, \quad (6)$$

which means that plasma particle densities are fixed at a large distance from the grain, the solution of Eq. (5) can be written as

$$n_\sigma(r) = e^{-e_\sigma\Phi(r)/T_\sigma}\left(n_0 + \frac{B_\sigma}{D_\sigma}\int\limits_r^\infty \frac{dr'}{r'^2} e^{e_\sigma\Phi(r')/T_\sigma}\right). \quad (7)$$

Since the grain absorbs plasma particles contacting its surface, the following boundary condition at the grain surface can be used:

$$n_\sigma(a) = 0. \quad (8)$$

Here, a is the grain radius. Equations (7) and (8) yield

$$B_\sigma = -\frac{D_\sigma n_0}{\int\limits_a^\infty \frac{dr'}{r'^2} e^{e_\sigma\Phi(r')/T_\sigma}}, \quad (9)$$

$$n_\sigma(r) = n_0 e^{-e_\sigma\Phi(r)/T_\sigma}\frac{\int\limits_a^r \frac{dr'}{r'^2} e^{e_\sigma\Phi(r')/T_\sigma}}{\int\limits_a^\infty \frac{dr'}{r'^2} e^{e_\sigma\Phi(r')/T_\sigma}}. \quad (10)$$

The potential $\Phi(r)$ is governed by the Poisson equation

$$\triangle\Phi(r) = -4\pi \sum_{\sigma=e,i} e_\sigma n_\sigma(r). \quad (11)$$

As is seen, Eqs. (10) and (11) generate a nonlinear equation for the effective potential. However, since the plasma particle densities are considerably reduced in the vicinity of the grain due to the electron and ion absorption, it is possible to expect the weakening of the nonlinear effects, as it is observed in the case of collisionless plasma [7], and thus to make analytical estimates using a linear approximation. A linearized version of Eq. (11) looks as

$$\triangle\Phi - k_D^2\Phi\left(1 - \frac{a}{r}\right) - k_D^2 a\int\limits_r^\infty dr'\frac{\Phi(r')}{r'^2} =$$

$$= -\frac{k_D^2 a^2}{r}\int\limits_a^\infty dr'\frac{\Phi(r')}{r'^2}, \quad (12)$$

where $k_D^2 = \sum\limits_{\sigma=e,i} k_{D\sigma}^2$, $k_{D\sigma}^2 = 4\pi e_\sigma^2 n_0/T_\sigma$.

It is possible to show that, with the accuracy up to the main contribution of the terms associated with the plasma particle absorption, Eq. (12) can be reduced to

$$\triangle\Phi - k_D^2\Phi = -\frac{k_D^2 a^2}{r}\int\limits_a^\infty dr'\frac{\Phi(r')}{r'^2}. \quad (13)$$

The solution of this equation consists of two parts

$$\Phi(r) = \frac{C}{r}e^{-k_D r} + \frac{a^2}{r}\int\limits_a^\infty dr'\frac{\Phi(r')}{r'^2}, \quad (14)$$

the first of which describes the screened Debye potential, while the second one is of the Coulomb-like form.

Equation (14) can be rewritten as

$$\Phi(r) = \frac{C}{r}\left(e^{-k_D r} + 2a^2\int\limits_a^\infty dr'\frac{e^{-k_D r'}}{r'^3}\right). \quad (15)$$

Using the boundary condition

$$\left.\frac{d\Phi}{dr}\right|_{r=a} = -\frac{q}{a^2}, \quad (16)$$

where q is the stationary grain charge to be determined from the condition of the zero value of the electric current through the grain surface, we obtain

$$\Phi(r) = \frac{q}{r}\frac{e^{-k_D r} + C_1}{e^{-k_D a}(k_D a + 1) + C_1}, \quad (17)$$

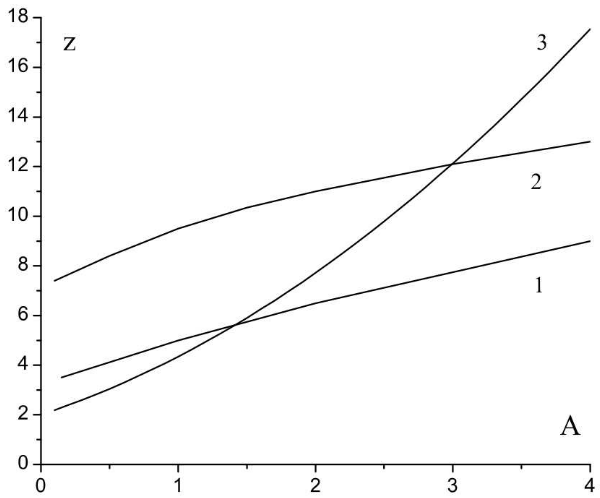


Fig. 1. Dependences of the normalized grain charge z on $A = ak_D$ at $t = T_i/T_e = 1$, $d = D_i/D_e = 0.001$: *1* — the collisionless OML theory (based on the approach presented in [7]), *2* — results of numerical studies [14], *3* — the curve described by Eq. (23)

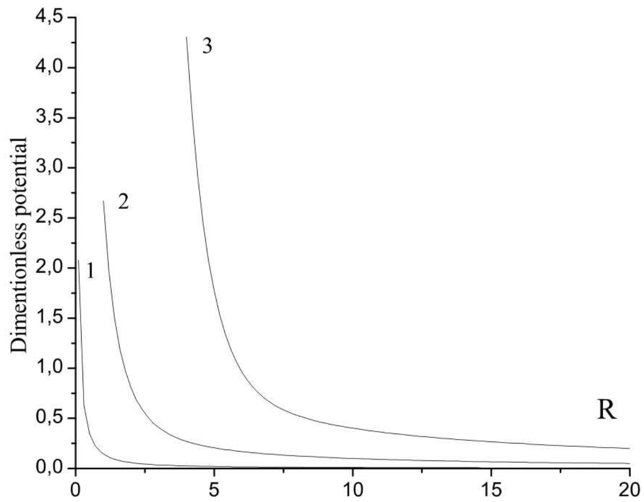


Fig. 2. Dimensionless potential $e_e\Phi/T_e$ vs distance $R = rk_D$ at $t = T_i/T_e = 1$, $d = D_i/D_e = 0.001$ for different values of the normalized grain radius $A = ak_D$: *1* — $A = 0.1$; *2* — $A = 1$; *3* — $A = 4$

which is in agreement with the estimates presented in [15]. Here,

$$C_1 = 2a^2 \int\limits_a^\infty dr \frac{e^{-k_D r}}{r^3} = e^{-A}(1-A) + \mathrm{Ei}(1,A)A^2, \quad (18)$$

$A = ak_D$, and $\mathrm{Ei}(n,x)$ is the integral exponent, $\mathrm{Ei}(n,x) = \int\limits_1^\infty dt\ t^{-n} e^{-xt}$. In the system with no plasma particle absorption by a grain ($C_1 = 0$), Eq. (17) recovers the well-known Derjagin—Landau—Verwey—Overbeek potential [16].

At $k_D r \gg 1$, Eq. (17) reduces to the Coulomb potential with the effective charge

$$\tilde{q} = q \frac{C_1}{e^{-k_D a}(k_D a + 1) + C_1}. \quad (19)$$

Obviously, this effect is related to the plasma currents associated with the plasma particle absorption by the grain. As was mentioned above, the total electric current through the grain surface is equal to zero in the stationary state, which requires

$$D_i n_0 \int\limits_a^\infty \frac{dr}{r^2} e^{e_e \Phi(r)/T_e} = D_e n_0 \int\limits_a^\infty \frac{dr}{r^2} e^{e_i \Phi(r)/T_i}. \quad (20)$$

In the linear approximation, this gives

$$\int\limits_a^\infty \frac{dr}{r^2} \Phi(r) = \frac{D_i - D_e}{a(D_e e_i/T_i - D_i e_e/T_e)}. \quad (21)$$

The substitution of Eq. (17) in (21) leads to the following formula for the grain charge:

$$q = \frac{e^{-k_D a}(k_D a + 1) + C_1}{C_1} \frac{a(D_i - D_e)}{D_e e_i/T_i - D_i e_e/T_e}. \quad (22)$$

The dimensionless grain charge $z = qe_e/(T_e a)$ is appropriately given by

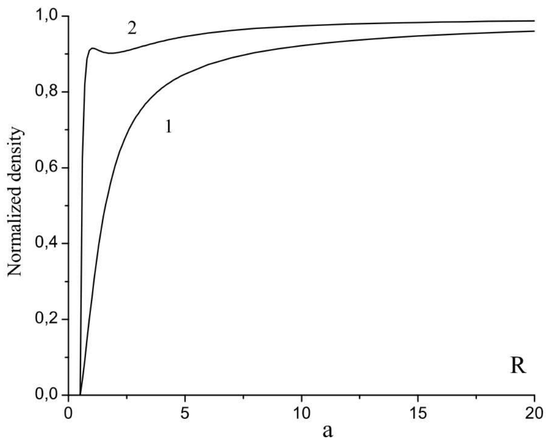
$$z = f(A) \frac{1-d}{1/t + d}, \quad (23)$$

where $f(A) = (2e^{-A} + \mathrm{Ei}(1,A)A^2)/C_1$; $d = D_i/D_e$, $t = T_i/T_e$.

Since the diffusion coefficient for electrons is usually much larger than that for ions, one has

$$z \simeq t f(A), \quad (24)$$

i.e. the dimensionless grain charge is proportional to the ion to electron temperature ratio. The dependence of z on the dimensionless grain radius is presented in Fig. 1. As is seen from the comparison with the appropriate numerical results, the obtained estimates give quantitative predictions of the same order as those obtained on the basis of the numerical solution of the problem. Unfortunately, the qualitative behavior of the analytical results is not in so good agreement with the numerical studies.



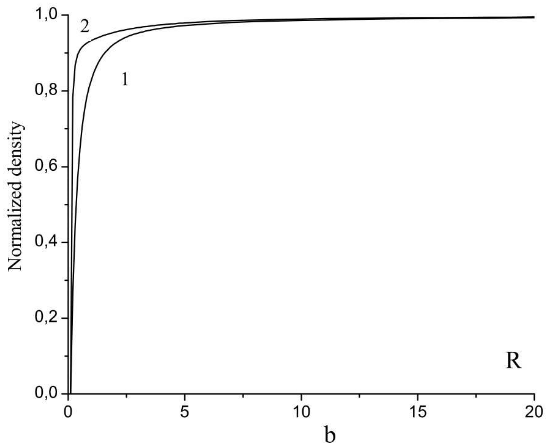


Fig. 3. Normalized density distribution of electrons n_e/n_{e0} (*1*) and ions n_i/n_{i0} (*2*), at $t = 1$, $d = 0.001$. a — $A = 0.5$, b — $A = 0.1$

Notice that, in spite of the long-range asymptotics of the potential, the induced charge density decreases exponentially.

$$\rho(r) = \sum_\sigma e_\sigma n_\sigma(r) = n_0 e \frac{\left(1 - \frac{D_i}{D_e}\right)\left(1 + \frac{T_i}{T_e}\right)}{\left(1 + \frac{D_i T_i}{D_e T_e}\right)} \frac{a}{C_1} \frac{e^{-k_D r}}{r}. \quad (25)$$

Equations (10), (17), and (22) give the approximate solution of the problem under consideration. The typical behaviour of the dimensionless potential $e_e\Phi/T_e$ and plasma particle densities as functions of the dimensionless distance $k_D r$ are presented in Figs. 2—4. These dependences are in qualitative agreement with the numerical results, and thus the obtained solution can be used for analytical estimates of the effective potentials in the appropriate cases. In particular, relations (19), (22) can be used to find the effective charge for a Coulomb-like potential.

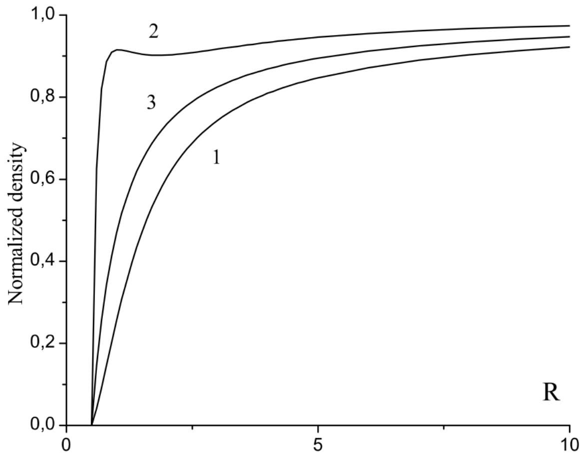


Fig. 4. Plasma particle density distribution for different values of $t = T_i/T_e$. Curves *1, 2* are all the same as in Fig. 3 (a), curve *3* describes the normalized electron density distribution n_e/n_{e0} at $t = 0.1$; the normalized ion density distribution n_i/n_{i0} at $t = 0.1$ coincides with *2*

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ДАЛЕКОДІЙНІ ЕФЕКТИВНІ ПОТЕНЦІАЛИ ВЗАЄМОДІЇ ПОРОШИНКИ У ПЛАЗМІ

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Р е з ю м е

Проведено аналітичні дослідження ефективного потенціалу порошинки у плазмі з урахуванням її заряджання плазмовими струмами. Динаміка плазми описується у дрейфово-дифузійному наближенні. Показано, що у розглянутому випадку ефективний потенціал має кулонівську асимптотику. Отримано наближені вирази для електричного заряду порошинки і розподілу плазмових частинок навколо неї. Проведено порівняння з отриманими раніше числовими розв'язками задачі.