ON THE POSSIBLE JUMP OF T_λ IN NANOFILMS OF He-II

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It is suggested that the microscopic vortex rings (MVRs) induce the λ-transition in helium-II and define substantially the value of T_λ. For very thin films of He-II with thickness d less than the size of the smallest MVR, the rings do not fit in and do not exist in such films. As a result, a jump-like peculiarity for superfluid films of He-II should exist in the curve $T_\lambda(d)$ at d approximately equal to the size of the smallest MVR, $d \approx (6 \pm 3)$ Å. The absence of a similar peculiarity will be an evidence for that MVRs do not influence the values of T_λ and do not play any key role in the λ-transition. The currently available experimental data are insufficiently complete and precise for revealing the predicted peculiarity.

1. Introduction

The microscopic nature of the λ-transition in He-II is still not quite clear. It is safe to say that the λ-transition is caused by the destruction of the off-diagonal long-range order (ODLRO) and is accompanied by the exhaustion of the condensate which probably has composite nature. The viewpoint, according to which MVRs play an important role in the λ-transition, is also popular enough [1—6]. The latter idea was proposed about 50 years ago [1], but a role of MVRs in the λ-transition is not clear until now. Here, as microscopic rings, we understand the vortex rings with radius $R \leq 10$ Å and with quantized circulation $\kappa = \hbar/m$ [7].

For He-II films, the value of T_λ is known to decrease with the decrease of the thickness of a film, mainly as a consequence of the finite-size (FS) scaling [8, 9]:

$$T_\lambda(d) - T_\lambda(\infty) \sim d^{-1/\Theta}. \quad (1)$$

In the present work, d is the thickness of the superfluid layer of the film. It is observed that $\Theta \approx \nu = 0.67$ (ν is the theoretical value) at $d \gtrsim 2 \times 10^3$ Å, but $\Theta \approx 0.54$ [8, 10, 11] at $d \simeq 20 \div 2000$ Å. The difference of the value $\Theta \approx 0.54$ from the theoretical value is probably caused [12] by the fractal geometry of the substrates. In consequence of the FS-scaling, the λ-peak in the curve of the dependence of the specific heat on temperature, $C(T)$, becomes finite and is smoothed out [13, 8, 9, 14]. So, in He-II films, we observe the FS-modified λ-transition (FS λ-transition).

For thick films with $d \gtrsim 20$ Å, the dependence $T_\lambda(d)$ is caused by FS-scaling [8]. At $d \lesssim 20$ Å(for Nuclepore [8]), the deviation from the scaling law (3) is observed (see Fig. 3 in [8]), which indicates [8] the appearance of the contribution to the specific heat from Kosterlitz—Thouless (KT) vortices. With decrease in d, the contribution of KT-vortices to the specific heat increases. For the films on such substrates as Vicor [15]—[17] and Nuclepore [8, 10], the broad peak (BP) on the curve $C(T)$ corresponds to the FS λ-transition, though KT-vortices give also some (not crucial) contribution to BP at $d \lesssim 20$ Å. With decrease in d, BP decreases and becomes "smeared out"[10, 13], which indicates a key role of bulk quasiparticles to BP. We denote the temperature of the maximum of such BP as $T_\lambda(d)$ (the temperature of the FS λ-transition).

But for some substrates (Millipore, Anopore [18], $d \approx 1 \div 3$ Å), BP with different properties is observed on the curve $C(T)$. This BP decreases (in comparison with the background) with increase in d, at $d \geq 2$ Å, which signifies that such BP is an effect of mainly two-dimensional quasiparticles [18]. According to [18, 19], this BP is caused by the dissociation of the small pairs of KT-vortices. We denote the temperature of the maximum of such BP as T_m^*.

The contribution of KT-vortices to the specific heat, C_{VKT}, depends strongly on d and on the substrate potential and can differ by several orders of magnitude for various substrates and values of d. Actually, C_{VKT} is proportional to the concentration of KT-vortices, N_{VKT}, and $N_{VKT} \sim a^{-3}$ [22], where a is the core radius of KT-vortices. According to [20, 21], $a \approx \left(\hbar^2 d/2mU_0 n^{(2)}\right)^{1/2}$, where $U_0 = \int d\mathbf{r} U(r)$, and $U(r)$ includes the potential of the substrate. So we have, roughly,

$$C_{VKT} \sim N_{VKT} \sim \left(n^{(2)} U_0/d\right)^{3/2}.$$

The value of a is minimal at $d \gtrsim 1$ a.l. (atomic layer, 3.6 Å) [21]. Therefore, it should be expected that $C_{VKT} \sim a^{-3}$ is highest at $d \simeq 1$ a.l.

Thus, for various substrates and values of d, the following versions of BP on the curve $C(T)$ could be realized:
(i) a single BP caused by the dissociation of small pairs of KT-vortices [19]; this is typical of thin films with $d \lesssim 1$ a.l. [18];
(ii) a single BP which is an FS-rounded λ-peak; this case is observed for such substrates as Vicor [15—17] and Nuclepore [10, 8] at $d \gtrsim 1$ a.l.;
(iii) two different BPs at a given $d \simeq 1$ a.l., one being caused by KT-vortices and the other one being the FS-rounded λ-peak (this version was not observed untill now).

Moreover, as known [15, 23], a narrow peak is observed on the curve $C(T)$ at $T \approx T_{\rm KT}$. This peak corresponds to the Kosterlitz–Thouless transition [22, 24] which is caused by the dissociation of big pairs of the KT-vortices [22, 25]. The relation $T_{\rm KT} < T_m^*, T_\lambda$ holds at arbitrary d.

It should be emphasized that, in this work, we will interested, first of all, in the curve $T_\lambda(d)$. A peculiarity similar to the predicted below, but weaker, can also exist on the curves $T_m^*(d)$ and $T_{\rm KT}(d)$ (see below).

2. On the Possibility of a Jump on the Curve $T_\lambda(d)$

We suggest that the ensemble of MVRs induces the λ-transition in the bulk He-II and determines the value of T_λ (whatever the mechanism is). Let us consider the properties of thin superfluid films of He-II. We would like to call attention to the dependence of the value of T_λ on the thickness d of the superfluid layer. Clearly, in very thin films of He-II with the thickness d less than the size d_0 of the smallest MVR, the rings do not fit in and, therefore, do not exist in such films. We suggest that, in the films with $d > d_0$, the FS λ-transition is caused mainly by vortex rings, but the rings do not already exist in the films with $d \approx d_0$. So, for the system to undergo the λ-transition for $d < d_0$, it is necessary that the number of remaining quasiparticles be larger than the number that would be required if MVRs exist in the system. This means that, at $d \approx d_0$, the temperature of the λ-transition should increase by a jump. Consequently, a peculiarity should exist on the curve $T_\lambda(d)$ at $d \approx d_0$ similar to that shown in Fig. 1. The jump is somewhat "smeared out" because of the finite-size scaling, a possible heterogeneity of the substrates, and since some of the rings have a size larger than d_0 [26].

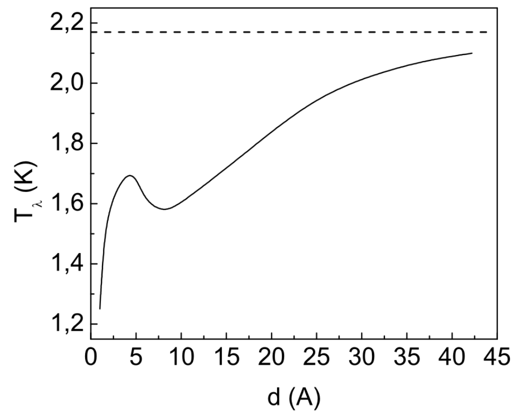

Fig. 1. Proposed $T_\lambda(d)$ curve for He-II films, with an anomaly at $d \approx d_0 \simeq 6$ Å. T_λ for films is the temperature of the maximum of the broad peak on the curve of heat capacity $C(T)$, and d is the thickness of the superfluid layer of a film. The values of T_λ at $d < 2.5$ Å and $d > 10$ Å correspond approximately to the crosses and squares in Fig. 2, respectively; the dotted line is T_λ for bulk He-II

Experimentally, it is known [27—29] that vortex rings with very small core radius $a \approx 0.8 \div 1.5$ Å may exist in He-II, and the smallest radius of MVRs detected in the experiment is $R \gtrsim 5$ Å[28]. Such small size of MVRs is caused by the fact that MVRs are vortices in the probability field [30] and are not in an ordinary classical fluid.

Note that the MVRs observed in all experiments of which we are aware emerged as a result of some external influence. The observation of the predicted anomaly would be the first experimental evidence of the existence of an ensemble of MVRs as *thermal* excitations in He-II.

The smallest ring looks like a torus with a very small hole. So it is clear that the size d_0 of such a ring should be around two to three core diameters:

$$d_0 \approx 4a \div 6a \approx 3 \div 9 \text{ Å}. \tag{2}$$

According to the approximate model [30] describing the circular vortex ring as a solution of the Gross–Pitaevskii equation, $d_0 \approx 4a$, where a is the core radius of large rings.

Note that the core of an MVR can be circular, as well as elliptic in form, as a result of some interactions. Also MVRs can be "lying" in parallel to the substrate. The "lying" ring, like each vortex ring, should move parallel to its axis [7, 26] with the velocity $v \sim 1/R$. Such rings cannot be stable. Therefore, they should not be numerous, cannot give any substantial contribution to the heat capacity, and should not influence the value of T_λ. Elliptic rings must be numerous in the thin films, and they give, probably, an essential contribution to the heat capacity of the ensemble of MVRs. However, since each ring has a core, both the elliptic and "lying" rings have nonzero "height" d_0. For "lying" rings, the "height" should be $d_0 \approx 3a$ (the core plus the minimal layer of the fluid rotating around the core). For an elliptic ring, we should take into consideration another core: $d_0 \approx 5a$. Elliptic rings are taken into account in (2).

Let us estimate the value of the possible jump of T_λ for He-II films. We assume that the λ-transition in the bulk He-II is accompanied by the complete exhaustion of the one-particle condensate. According to the calculation [31], the fraction of the one-particle condensate is $n_0 = 0.078$ at $T = 0$, and $n_0 = 0.058$ at $T = T_\lambda = 2.17$ K. That is, the condensate does not vanish completely at $T = T_\lambda$, although $n_0(T_\lambda) \approx 0$ in the experiment. It is suggested in [31] that the one-particle condensate in He-II is exhausted completely $[n_0(T_\lambda) = 0]$ because of vortex rings. The rings were not taken into account in the calculation [31], and the decrease in n_0 at $T \to T_\lambda$ was due to rotons: the number of atoms pulled out of the condensate was directly proportional to the number of rotons. The concentration of free rotons is known [20]:

$$n_r = 0.051 \cdot e^{-\Delta/kT} \left(\frac{q_r}{1.925 \ \text{Å}^{-1}} \right)^2 \sqrt{\frac{\mu T}{0.14 m_4}} \quad \text{Å}^{-3}. \tag{3}$$

In order that rotons provide $n_0 = 0$, it is necessary that their number be four times greater than that at $T = 2.17$ K; in this case, we have $n_0 = 0$ instead of $n_0 = 0.058$. For this, temperature $T \approx 3.12$ K is required according to (3). Thus, if the calculation of n_0 in [31] is correct, the value of T_λ would be higher, than the observed 2.17 K, by $\delta T_\lambda = 0.95$ K in the absence of vortex rings in He-II. Some authors obtained $n_0(2.17 \text{ K}) \leq 0.02$ without taking MVRs into account. For $n_0(2.17 \text{ K}) = 0.02$, the temperature $T = 2.17 + 0.15$ K is required in order that rotons provide $n_0 = 0$. Such estimates give the value of the possible jump of T_λ for thin He-II films: $\delta T_\lambda = 0.1 \div 1$ K. The stronger the influence of MVRs on T_λ, the larger is the magnitude of the jump. Even if the λ-transition is not accompanied by the exhaustion of the one-particle condensate, but MVRs influence the value of $T_\lambda(d > d_0)$ in some other way, we will still observe some jump of T_λ in He-II thin films at $d \approx d_0$.

It is difficult to make the exact calculation of the λ-transition in thin helium films, taking into account all kinds of quasiparticles, even if we would have an exact solution for MVRs (only a solution in the mean field approximation is known [30]). But our simple estimates are sufficient for the prediction of a smoothed-out jump on the curve $T_\lambda(d)$ and for the approximate demonstration of the form and location of the anomaly. Using the estimates for d_0 and δT_λ and also taking into consideration the smoothing of the jump, we show approximately the proposed anomaly in the $T_\lambda(d)$ curve in Fig. 1.

As the simple estimates showed, one could expect a similar, but small, anomaly also on the curves $T^*_m(d)$, $T_{\text{KT}}(d)$ (see Introduction) at $d \approx d_0$. Since the rings disappear from the He-II film at $d < d_0$, the value of ρ_s should grow at $d \approx d_0$ compared to ρ_s at d just larger than d_0. So far as $T_{\text{KT}} \sim d\rho_s$, a bump-like peculiarity similar to the anomaly on the curve $T_\lambda(d)$ should also exist on the curve $T_{\text{KT}}(d)$ at $d \approx d_0$. However, the value of $T_{\text{KT}}(d_0)$ is appreciably smaller than $T_\lambda(d_0)$, but the number of MVRs N_{vr} and their contribution to ρ_s are proportional to $\exp(-E_0/kT)$ (E_0 is the energy of the smallest MVR; the interaction between rings may also be important and must be included in E_0). So, the contribution of MVRs to ρ_s should be 3—10 times smaller at $T_{\text{KT}}(d_0)$ than that at $T_\lambda(d_0)$. These simple estimates show that a jump on $T_{\text{KT}}(d)$ must be roughly three to ten times weaker (or even may be negligible if E_0 is high enough) than a jump on $T_\lambda(d)$. The experimental data [32] give no clear evidence for the anomaly on $T_{\text{KT}}(d)$ at $d < 7$ Å.

Since $T^*_m > T_{\text{KT}}$, we can expect that the peculiarity on the curve $T^*_m(d)$ will be larger than that on the curve $T_{\text{KT}}(d)$. But the T^*_m-peak itself on the curve $C(T)$ should be very weak relative to the background at $d \approx d_0 \simeq 2$ a.l. [18] and thus may be hardly discernible. Thus, one should look for an anomaly, first of all, on the curve $T_\lambda(d)$.

Thus, taking into account our estimates and also the theory and experiment for vortex rings, one can see that an ensemble of microscopic vortex rings, in which the smallest MVR have size about $d_0 \simeq 6$ Å, should exist in He-II. If the λ-transition in the bulk He-II is induced by MVRs, then the anomaly should exist on the curve $T_\lambda(d)$ at $d \approx d_0 \simeq (6\pm3)$ Å. We have drawn this anomaly approximately in Fig. 1.

The experimental data on the dependence $T_\lambda(d)$ for thin films of He-II are shown in Fig. 2. The crosses in Fig. 2 are obtained using the data of Fig. 1 from [15]. According to [15], the Brewer's curve (triangles) should be shifted to the left (circles in Fig. 2); in this case, the data by Finotello *et al.* [15] (crosses) well agrees with that obtained by Brewer and his colleagues [16, 17]. As a whole, as can be seen from Fig. 2, the data of different

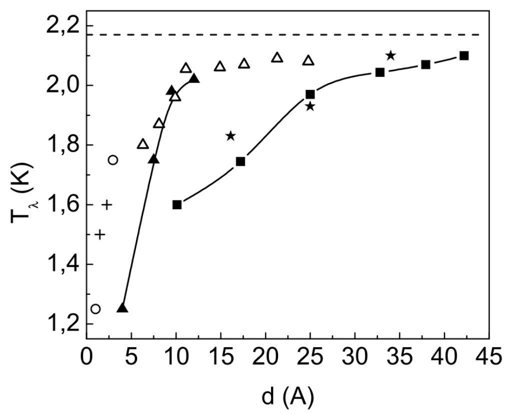

Fig. 2. Experimental $T_\lambda(d)$ curve for He-II films on the substrates of jeweller's rouge [13] (stars), Vycor [16, 17] (full triangles), N_2-plated Vycor [17] (open triangles), 2000 Å Nuclepore [10] (squares), and Vycor [15] (crosses); the circles mark $T_\lambda = 1.25$ K and $T_\lambda = 1.75$ K from [16, 17], with d defined more precisely according to [15]; the dotted line is T_λ for bulk He-II

works do not fully agree with each other, and there is a large dispersion of the experimental points. The main causes for this disagreement are the imprecise measurement of the film thickness and the difference in the substrates. Using these data, we cannot determine the existence of the predicted peculiarity on the curve $T_\lambda(d)$.

The more precise measurements of the dependence $T_\lambda(d)$ are necessary for several substrates, for d in the interval between 1 and 20 Åwith small step $\triangle d \leq 1$ Å. The Vycor glass, Nuclepore, and, perhaps, Mylar and some other substrates can be used in this case (see Introduction). These must be substrates on which the He^4 films are superfluid, and the KT-effect is observed very well. The ordered substrates with strong attraction, such as graphite substrates, are not suitable. The precise measurements of $T^*_m(d)$ and $T_{\rm KT}(d)$ for $d = 1 \div 20$ Å can also be interesting, especially for substrates of the type of [18], for which the observation of two different broad peaks on the curve $C(T)$ is possible (see version III in Introduction).

If the predicted anomaly will be detected, one should investigate its dependence on pressure. Since the radius of the MVR core increases with pressure, $a(p) \sim \rho(p)$ [33, 34], the size of the smallest MVR should increase proportionally, and, therefore, the anomaly in Fig. 1 must accordingly shift to the right, towards larger values of d.

A discovery of such an anomaly could stimulate the further theoretical and experimental investigations of vortex rings and the λ-transition in He-II. As known, the understanding of the physical nature of the λ-transition and the knowledge of the properties of MVRs did not advance noticeably for the last 40 years (after the experiments of [27—29]).

The presence of the anomaly will indicate that MVRs play an important role in the bulk λ-transition and substantially influence the value of T_λ, whereas the absence of the anomaly will be an evidence for that MVRs do not influence the value of T_λ and do not play any key role in the λ-transition in He-II. In any case, we would obtain information on the nature of the λ-transition in He-II. Therefore, the exact measurement of the dependence $T_\lambda(d)$ for He-II films, in our opinion, is of great interest.

The idea of this work is developed in more details in [26].

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ПРО МОЖЛИВІСТЬ СТРИБКА T_λ В НАНОПЛІВКАХ He-II

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Р е з ю м е

Обговорюється λ-перехід у наноплівках гелію-II. Припускається, що ансамбль мікроскопічних вихрових кілець (МВК) викликає λ-перехід в гелії-II, зокрема в значній мірі визначає значення T_λ. В дуже тонких плівках He-II, в яких товщина d надплинного шару менша за розмір найменшого МВК, вихрові кільця не вміщуються, і в таких плівках ансамбль МВК відсутній. Внаслідок цього для плівок He-II на кривій залежності $T_\lambda(d)$ повинна бути особливість у формі згладженого стрибка при d, приблизно рівному розміру найменшого МВК, $d \approx (6 \pm 3)$ Å. Відсутність подібної особливості означатиме, що вихрові кільця не впливають на значення T_λ і не відіграють суттєвої ролі у λ-переході. Існуючі експериментальні дані не достатньо повні та точні для виявлення передбачуваної особливості.