INFLUENCE OF LOW-DENSITY PLASMA ON GYROTRON OPERATION

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A non-linear theory of gyrotron with low-density plasma filling is developed. The self-consistent set of the averaged nonlinear equations for the slowly varying electromagnetic fields and the momentum of electrons of a beam is obtained. The influence of a low-density plasma on the gyrotron operation is numerically examined.

1. Introduction

Among high-power microwave (HPM) devices of vacuum electronics, gyrotrons are the most developed and prospective. Their applications in such advanced technological fields as high-power radars, plasma heating and diagnostics, high-power accelerators, material processing, and so forth demonstrate the substantial progress. At present, the output power of microwave radiation has reached a level of some GW in pulse regimes and some MW in continuous and quasi-continuous regimes. To achieve such high output powers, high-current electron beams should be used. However, the effects from the self-fields of such beams become essential, which results in the deterioration of the beam quality. One of the simplest ways to solve this problem is to use a background plasma which often appears inevitably in the cavities of high-power gyrotrons [1]. It is able to compensate the self-fields of high-current electron beams [1].

On the other hand, there exist now many types of plasma-based microwave tubes [2]. The enhancement of the output power and efficiency due to the presence of plasma in them has been demonstrated experimentally. These positive effects have been reached not only due to the compensation of the self-fields of electron beams, but also due to changing the electrodynamic characteristics of their cavities. It should be pointed out that the presence of plasma in the cavities of electronic devices causes some negative effects like a broadening of the spectrum of output radiation, the instability of generation, and the bombarding of the cathode by plasma ions. However, these negative effects take place only when the density of plasma is rather high (the plasma frequency is comparable with the operational frequency) or when the operational modes are the purely plasma modes, i.e. the modes which do not exist without plasma. Thus, the use of the electromagnetic modes which are slightly modified by a low-density plasma is more preferable.

The gyrotron cavity is a piece of the waveguide with a slowly varying cross section. The method of description of the nonlinear beam-wave interaction in plasma, which was used in [3,4], is not applied to this case. The analysis of waveguides with a slowly varying cross section is introduced in many papers (see, for example, [5, 6]). But such a description does not take into account a low-density plasma and the finite Larmor radius of the beam electrons which can be comparable with the transverse wavelength.

The main result of the present work is the creation of a non-linear theory of plasma-filled gyrotrons which includes the effects of the finite Larmor radius of the beam electrons and the slowly varying waveguide radius of the gyrotron cavity. The self-consistent set of averaged nonlinear equations for the slowly varying electromagnetic fields and the momentum of the beam electrons are obtained. In a case where the variation of the field in the radial direction over the distance of the order of the Larmor radius is weak, the set of equations is transformed to the equations which are well known from the traditional gyrotron theory.

The conditions when the traditional approach is applicable [7, 8] are obtained as well.

Analyzing the obtained set of non-linear equations, it is shown that the RF power yield of a plasma gyrotron can be higher than that in the vacuum case for some values of beam currents.

The present description is valid when the condition of the normal Doppler resonance is satisfied.

2. Equations for a Gyrotron without Plasma

The electromagnetic fields of one TE mode in a waveguide are given by the expressions

$$B_z = b\,(z,t)\,J_m\,(k_\perp r)\exp\,(-i\omega t + im\theta) \tag{1}$$

and

$$(B_\theta, E_r) = \left\{ i\frac{m}{rk_\perp^2}\frac{\partial}{\partial z}, -\frac{\omega m}{crk_\perp^2} \right\} b\,(z,t) \times$$
$$\times J_m\,(k_\perp r)\exp\,(-i\omega t + im\theta)\,, \tag{2}$$

$$(B_r, E_\theta) = \left\{ (l/k_\perp)\frac{\partial}{\partial z}, -i\omega/ck_\perp \right\} b\,(z,t) \times$$
$$\times J'_m\,(k_\perp r)\exp\,(-i\omega t + im\theta)\,, \tag{3}$$

where $b\,(z,t)$ is the complex amplitude of the mode, m is the azimuthal number, $J_m(x)$ and $J'_m(x) = \frac{d}{dx}J_m(x)$ are, respectively, the Bessel function and its derivative. In many papers (see, for example, [9]), the authors use the membrane function $\Psi_s = -i\,(c/\omega)\,J_m\,(k_\perp, r)$ which satisfies the equation $\left(\Delta_\perp + k_\perp^2\right)\Psi_s = 0$ with the boundary conditions $\frac{\partial\Psi_s}{\partial n}\Big|_s = \frac{\partial\Psi_s}{\partial r}\Big|_{r=0} = 0$. The components of the electromagnetic field of TE mode (1)—(3) also satisfy the same conditions as the membrane function.

In the case of TE polarization, Maxwell equations yield the equation for B_z:

$$\left(-\frac{\partial^2}{\partial z^2} + k_\perp^2 - \frac{\omega^2}{c^2}\right) B_z = \frac{4\pi}{c}\left[\frac{1}{r}\frac{d}{dr}\,(rJ_\theta) - \frac{1}{r}imJ_r\right]. \tag{4}$$

By applying the operator $\hat{Q} = \int\limits_0^{r_w} rdr \int\limits_0^{2\pi} d\theta J_m(k_{ms}r)\times$ $\times\exp(-im\theta)$ to both sides of this equation, we obtain

$$b\pi r_w^2 \left\{ J_m^2\,(k_{ms}r_w)\left[1 - m^2/\,(k_{ms}r_w)^2\right]\right\} \times$$
$$\times\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2} - k_{ms}^2\right) =$$
$$= -\,(4\pi i/\omega b^*)\,k_{ms}^2 \int\limits_0^{r_w} rdr \int\limits_0^{2\pi} d\theta \left(\vec{J}\vec{E}^*\right), \tag{5}$$

where $\vec{J}$ is electron current density.

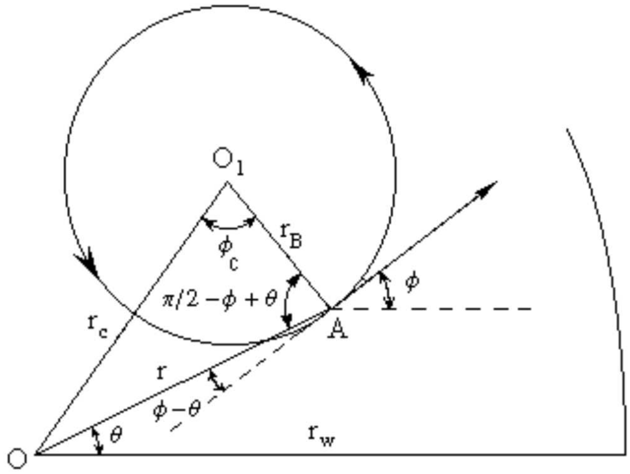

Fig. 1. Coordinate system of a moving electron

The boundary conditions $\frac{\partial\Psi_s}{\partial n}\Big|_s = \frac{\partial\Psi_s}{\partial r}\Big|_{r=0} =0$ are equivalent to the requirement of nullifying the component E_θ at the waveguide boundary. This condition determines the transverse wavenumber $k_\perp = k_{ms} = x_{ms}/r_w$, where x_{ms} is the s-root of the equation $\frac{d}{dx}J_m\,(x) = 0$.

Usually, the electron beam in the gyrotron cavity has form of a thin cylindrical layer. So one can consider that all electrons of the beam are placed at the same radius r_c.

In the calculations of the interaction between electrons and the field of TE mode, we need to pass to the coordinate system moving with the electron which follows its helical orbit. The axis of such an orbit is parallel to the axis of the waveguide. This system of coordinates is shown in Fig. 1. The components of the field in the new system of coordinates can be written as a linear combination of fields (1)—(3) (see, for example, [10]):

$$b_z = B_z,$$

$$b_\Phi = B_\theta sin(\Phi - \theta) + B_r cos\,(\Phi - \theta)\,,$$

$$b_R = -B_\theta cos\,(\Phi - \theta) + B_r in\,(\Phi - \theta)\,,$$

$$e_R = -E_\theta cos\,(\Phi - \theta) + E_r sin\,(\Phi - \theta)\,,$$

$$e_\Phi = E_\theta cos\,(\Phi - \theta) + E_r sin\,(\Phi - \theta)\,. \tag{6}$$

Components of the RF field on the electron orbit of radius R_{BA} can be written as

$$(b_z, b_\Phi, e_R) = \sum_q J_q(k_{ms} r_c) J_{m+q}(k_{ms} r_B) \times$$

$$\times \exp(-i\omega t + im\Phi + iq\Phi_c + i\pi m/2) \times$$

$$\times \left(b, \frac{im}{rk_{ms}^2}\frac{\partial b}{\partial z}, -\omega\frac{m+q}{k_{ms}cr}b\right), \qquad (7)$$

$$(b_R, e_\Phi) = \sum_q J_q(k_{ms} r_c) J'_{m+q}(k_{ms} r_B) \times$$

$$\times \exp(-i\omega t + ik_z z + im\Phi + iq\Phi_c + i\pi m/2) \times$$

$$\times \left[(1/k_{ms})\frac{\partial b}{\partial z}, -\frac{i\omega}{k_{ms}c}b\right]. \qquad (8)$$

In this case, $\Phi_c = \Phi_{c0} + \omega_B t$, $\Phi = \Phi_0 + \omega_B t$, where Φ_{c0} and Φ_0 are slowly varying phases which contain the information about the initial positions of particles, and $z = z_0 + v_z t$. In the case where the wave–particle resonance takes place and $\omega \approx n\omega_B$, (where $\omega_B = \frac{eB}{m_e c}$ is the electron cyclotron frequency) the terms in the infinite sums in (7) and (8), for which the equation $m + q = n$ is valid, are much larger than others. Therefore, we can keep only them in expressions (7) and (8):

$$(b_z, b_\Phi, e_R) = J_{n-m}(k_{ms} r_c) J_n(a) \exp(2i\pi\zeta) \times$$

$$\times \left[b, \frac{im}{rk_{ms}^2}\frac{\partial b}{\partial z}, -\omega\frac{m+q}{k_{ms}cr}b\right], \qquad (9)$$

$$(b_R, e_\Phi) = J_{n-m}(k_{ms} r_c) J'_n(a) \exp(2i\pi\zeta) \times$$

$$\times \left[\frac{1}{k_{ms}}\frac{\partial b}{\partial z}, -\frac{i\omega}{k_{ms}c}b\right]. \qquad (10)$$

Here, $\zeta = \frac{1}{2\pi}\left(-\omega t + n\omega_B t + m\Phi_0 + (n-m)\Phi_{c0} + \frac{\pi m}{2}\right)$, $a = k_{ms} r_B$, and $r_B = v_\Phi/\omega_B$ is the Larmor radius of an electron.

Now, using the obtained expressions for the electric field components (9) and (10), Eq. (5) can be written in the form

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2}{c^2} - k_{ms}^2\right) b = -4ik_{ms}^2 \times$$

$$\times \left[\omega r_w^2 b^* J_m^2(x_{ms})\left(1 - \frac{m^2}{x_{ms}^2}\right)\right]^{-1} \times$$

$$\times \int_0^{r_w} rdr \int_0^{2\pi} d\theta \vec{J}\vec{E}^* = -(4N_{b0}e\omega_B/c) \times$$

$$\times \left[r_w^2 J_m^2(x_{ms})\left(1 - \frac{m^2}{x_{ms}^2}\right)\right]^{-1} J_{n-m}(k_{ms} r_c) \times$$

$$\times \frac{1}{N}\sum_{j=1}^{N} a_j J'_n(a_j) \exp(-2i\pi\zeta_j),$$

where $b = |b|\exp(i\varphi)$ is the complex amplitude of the field.

The equations of motion of the beam electrons can be written as

$$v_\Phi \frac{d\Phi_c}{dt} = \frac{e}{m_e}\left[e_R - \frac{v_z}{c}b_\Phi + \frac{v_\Phi}{c}b_z\right] =$$

$$= v_\Phi \frac{e}{mc\omega_B} J_{n-m}(k_{ms} r_c) J_n(a) \exp(2i\pi\zeta) \times$$

$$\times \left[\left(\omega_B - \omega\frac{n}{a^2}\right) b - i\frac{nv_z}{a^2}\frac{\partial b}{\partial z}\right], \qquad (11)$$

$$\frac{dv_\Phi}{dt} = -\frac{e}{m_e}\left[e_\Phi + \frac{v_z}{c}b_R\right] = i\frac{e}{mck_{ms}} \times$$

$$\times J_{n-m}(k_{ms} r_c) J'_n(a) \exp(2i\pi\zeta) \times$$

$$\times \left[\omega \cdot b + iv_z\frac{\partial b}{\partial z}\right], \qquad (12)$$

$$\frac{dv_z}{dt} = -\frac{e}{m_e}\left[-\frac{v_\Phi}{c}b_R\right] = \frac{ev_\Phi}{mck_{ms}}\frac{\partial b}{\partial z} \times$$

$$\times J_{n-m}(k_{ms} r_c) J'_n(a) \exp(2i\pi\zeta). \qquad (13)$$

On the other hand, taking into account the z-dependence, the motion equations can be rewritten as

$$2\pi\frac{d\zeta}{dz} = \frac{n\omega_{B0} - \omega}{v_z} + \frac{n\omega_{B0}}{2v_z}\left[\beta_{\perp 0}^2 - \beta_\perp^2 - \right.$$

$$\left. - 2\beta_{z0}(v_z - v_{z0})/c\right] + n\frac{e}{m_e c v_z} J_{n-m}(k_{ms} r_c) \times$$

$$\times J_n(a) \exp(2i\pi\zeta)\left[\left(1 - \frac{n^2}{a^2}\right) b - i\frac{n^2 v_z}{\omega a^2}\frac{\partial b}{\partial z}\right], \qquad (14)$$

$$\frac{da}{dz} = in\frac{e\omega}{m_e c\omega_B v_z} J_{n-m}(k_{ms} r_c) J'_n(a) \times$$

$$\times \exp(2i\pi\zeta)\left[b + i\frac{v_z}{\omega}\frac{\partial b}{\partial z}\right], \qquad (15)$$

$$\frac{d}{dz}\left(\frac{1}{v_z}\right) = -\frac{e\omega_B}{k_{ms}^2 v_z^3 m_e c}\frac{\partial b}{\partial z}J_{n-m}\left(k_{ms}r_c\right)a\times$$

$$\times J_n'\left(a\right)\exp\left(2i\pi\zeta\right), \qquad (16)$$

where $\omega_{B0} = \frac{eB}{m_e}\left(z=0\right) = \frac{eB}{m_{e0}c}\left(1-\beta_{\perp 0}^2-\beta_{z0}^2\right)^{1/2}$.

When the arguments of Bessel functions are small, Eqs. (11) and (12) yield the equations of motion of an electron in the well-known form [5—8] using the new dimensionless variable

$$\xi_G = \frac{\beta_{\perp 0}^2\omega_{B0}z}{2v_z}. \qquad (17)$$

After this, the equations of motion of an electron (11) and (12) transform to

$$\frac{d\vec{A}}{d\xi_G} + i\left[\Delta + |A|^2 - 1\right]A = -iF^*A^{*n-1}, \qquad (18)$$

where $A = \frac{\tilde{a}}{a_0}$, $\tilde{a} = a\exp\left\{\frac{2\pi i\zeta}{n}\right\}$,

$$\Delta = \frac{2\left(\omega - n\omega_{B0}\right)}{n\omega_{B0}\beta_{\perp 0}^2}, \qquad (19)$$

$$F = 4\frac{n^2}{2^n n!}\frac{\left[a_0^{n-2}eb\exp\left(i\varphi\right)J_{n-m}\left(k_{ms}r_c\right)\right]}{cm\beta_{\perp 0}^2\omega_{B0}}. \qquad (20)$$

The condition of smallness of the arguments of Bessel functions is satisfied if the wave amplitude does not vary significantly over the wavelength in the longitudinal direction, i.e.

$$|b| \gg \left|\frac{v_z}{\omega}\frac{\partial b}{\partial z}\right|.$$

The second term on the left-hand side of Eq. (15) takes into account changing the masses of electrons as a result of increasing or decreasing the transverse electron velocity v_Φ:

$$\frac{-\omega + n\omega_B}{n\omega_{B0}} = \left[\frac{-\omega + n\omega_{B0}}{n\omega_{B0}} + \frac{\beta_{\perp 0}^2 - \beta_\perp^2}{2}\right], \qquad (21)$$

where $\beta_\perp = \frac{v_\Phi}{c}$.

In the small Larmor radius approximation, the wave field equation (11) can be rewritten in the following way:

$$\frac{d^2F}{d\xi_G^2} + \gamma_G^2 F = -I_V\frac{1}{N}\sum_{j=1}^{N}\left(\frac{a_j}{a_{j0}}\right)^n\times$$

$$\times\exp\left(-2i\pi\zeta_j\right) = -I_V\left\langle A^{*n}\right\rangle. \qquad (22)$$

If the temporal evolution of the wave amplitude is taken into account, Eq. (22) should be changed to the form [11, 12]

$$\frac{\partial^2 F}{\partial\xi_G^2} + i\frac{\partial F}{\partial\tau_G} + \gamma_G^2 F = -I_V\frac{1}{N}\sum_{j=1}^{N}\left(\frac{a_j}{a_{j0}}\right)^n\times$$

$$\times\exp\left(-2i\pi\zeta_j\right) = -I_v\left\langle A^{*n}\right\rangle, \qquad (23)$$

where $\tau_G = \frac{\beta_{\perp 0}^4 t\omega_{B0}}{8n\beta_{z0}^2}$,

and I_V characterizes the beam current and is represented by the expression

$$I_V = \frac{64eI_0n^3}{m_{e0}c^3}\frac{\beta_{z0}}{\beta_{\perp 0}^{2(4-n)}}\left(\frac{n^n}{2^n n!}\right)^2\frac{J_{n-m}^2\left(k_{ms}r_c\right)}{J_m^2\left(x_{ms}\right)\left(x_{ms}^2-m^2\right)}. \qquad (24)$$

where $I_0 = N_{b0}ev_{z0}$ is the initial beam current, and $\gamma_G^2 = 4v_z^2\frac{\omega^2/c^2-k_{ms}^2}{\left(\beta_{\perp 0}^2\omega_{B0}\right)^2}$.

If we make the substitution $F^* \to F$, the set of equations (18) and (22) will be similar to that which was used in many papers (see, e.g., [14]). The difference in expressions is caused only by choosing the time factor of all perturbations as $\exp\left(-i\omega t\right)$ instead of $\exp\left(i\omega t\right)$, which have been used in the above-mentioned papers.

In the case where the Larmor radius of electrons of a beam is not small, the set of equations which describes the stationary wave amplification in the longitudinal direction can be written as

$$\left(\frac{d^2}{d\xi_V^2} + \gamma_V^2\right)E_V = -\frac{1}{N}\sum_{j=1}^{N}a_jJ_n'\left(a_j\right)\exp\left(-2i\pi\zeta_j\right), \qquad (25)$$

$$2\pi\frac{d\zeta}{d\xi_V} = \sigma\left[\Delta_V + \frac{a_0^2-a^2}{2\varepsilon n^2} + \beta_{z0}^2\frac{(\sigma-1)}{\sigma\varepsilon}\right] +$$

$$+nJ_n\left(a\right)\exp\left(2i\pi\zeta\right)\times$$

$$\times\left[\left(1-\frac{n^2}{a^2}\right)\sigma E_V - i\varepsilon\frac{n^2}{a^2}\frac{\partial^2 E_V}{\partial\xi_V}\right], \qquad (26)$$

$$\frac{da}{d\xi_V} = inJ_n'\left(a\right)\exp\left(2i\pi\zeta\right)\left[\sigma E_V + i\varepsilon\frac{\partial E_V}{\partial\xi_V}\right], \qquad (27)$$

$$\frac{d\sigma}{d\xi_V} = -R_V\sigma^3\frac{\partial E_V}{\partial\xi_V}aJ_n'\left(a\right)\exp\left(2i\pi\zeta\right), \qquad (28)$$

where $\xi_V = z\rho_V$, $E_V = \frac{e}{m_{e0}c\rho_V v_{z0}} J_{n-m}(k_{ms}r_c)$, $\sigma = \frac{v_{z0}}{v_z}$, $R_V = \rho_V \omega_B / k_{ms}^2 v_{z0}$, $\varepsilon = \frac{\rho_v v_{z0}}{\omega}$, $\Delta_V = \frac{n\omega_{B0}-\omega}{\rho_V v_{z0}}$, $\gamma_V^2 = \frac{1}{\rho_V^2}\left[\frac{\omega^2}{c^2} - k_{ms}^2\right]$, $\rho_V^3 = \frac{4\pi N_{b0} e^2 \omega_B}{m_e c^3 \beta_{z0}} \left[r_w^2 J_m^2(x_{ms})\left(1 - \frac{m^2}{x_{ms}'^2}\right)\right]^{-1} J_{n-m}^2(k_{ms}r_c)$, and $\omega_{B0} = \omega_B \ (z=0)$.

3. Equations for a Plasma-filled Gyrotron

In a plasma-filled waveguide in the presence of an axial magnetic field, TM and TE modes are coupled, by forming hybrid modes. If the plasma density is rather small, and the longitudinal component of the wave vector is much less than the transverse one, the hybrid modes practically are similar to slightly coupled TM and TE modes of a vacuum waveguide [13]. Usually, this condition holds in the high-power gyrotrons for fusion applications.

In this case, there are appear two small parameters: the first one is $(k_z/k_\perp)$, and the second is $g = \left(\omega^2 \varepsilon_{xy}/c^2 k_\perp^2\right) = \frac{\omega^2 \varepsilon_2}{c^2 k_\perp^2}$, where $\vec{k} = \left(k_z, \vec{k}_\perp\right)$ is the wave vector, and $\varepsilon_{xy} = \varepsilon_2$ is the transverse component of the dielectric permittivity tensor of cold plasma [14]. Using these two small parameters, one can modify the well-known gyrotron equations [5,6] to new equations which take into account the influence of a low-density plasma.

Consider the excitation of TE mode in a waveguide with the plasma density which is rather small. So the electron plasma frequency ω_{pe} is much less than the n-harmonic electron cyclotron frequency $n\omega_B$ and the operational frequency ω:

$$\omega_{pe} = \left(\frac{4\pi e^2 n_{0e}}{m_e}\right)^{1/2} \ll n\Omega_B, \omega. \quad (29)$$

If inequality (29) is satisfied, the parameter g is also small ($g \ll 1$). So it is possible to neglect the terms which are proportional to g^2, $\left(k_z^2/k_\perp^2\right)$, or $g\left(k_z/k_\perp\right)$. The effects of collision between the particles in plasma are also neglected ($\nu \ll \omega_{pe}, \Omega_B, \omega$,where ν is the collision frequency).

The interaction between a wave and the beam of electrons with current density $\vec{J}$ in a magnetized plasma is described by the Maxwell equations

$$\left[\nabla, \vec{E}\right] = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}, \quad \left[\nabla, \vec{H}\right] = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c}\vec{J}.$$

Consider a homogeneous plasma waveguide in the absence of the beam of electrons. For a perturbation proportional to $\exp\left\{-i\left(\omega t - k_z z - \vec{r}_\perp \vec{k}_\perp\right)\right\}$, one can write a dispersion relation which is describes the dependence of the frequency on the wave vector $\omega\left(\vec{k}\right)$

$$D\left(\omega, \vec{k}\right) \equiv \frac{k_z^2 + k_\perp^2 - \omega^2 \varepsilon_1/c^2}{k_z^2 - \omega^2 \varepsilon_1/c^2} = 0. \quad (30)$$

Here, the non-zero components of the permittivity tensor of a cold plasma are given by the expressions

$$\varepsilon_{11} = \varepsilon_{22} = \varepsilon_1 = 1 - \frac{\omega_{pe}^2}{\omega^2 - \Omega_B^2},$$

$$\varepsilon_{12} = -\varepsilon_{21} = i\varepsilon_2 = \frac{i\omega_{pe}^2 \Omega_B}{\omega\left(\omega^2 - \Omega_B^2\right)},$$

$$\varepsilon_{33} = \varepsilon_3 = 1 - \frac{\omega_{pe}^2}{\omega^2},$$

$$\Omega_B = \frac{eB_0}{m_{e0}c} = \omega_{B0}\left(1 - \beta_{\perp 0}^2 - \beta_{z0}^2\right)^{-1/2}.$$

The gyrofrequency of electrons in a beam ω_{B0} is lower than that of electrons in a plasma Ω_B, because the masses of the beam electrons higher than the masses of the plasma electrons due to the relativistic effect.

In a plasma-filled gyrotron, two modes appear instead of one TE mode. One of them has the frequency which is higher than the gyrofrequency of the plasma electrons Ω_B. Modes of the other type have a frequency which is smaller than the gyrofrequency of the plasma electrons.

The wave energy on the unit length of a waveguide can be written in this case as

$$W_E = \frac{1}{8\pi}\frac{\partial \omega D\left(\omega, \vec{k}\right)}{\partial \omega} \left|b(z,t)\right|^2 \pi r_w^2 \times$$

$$\times \left[J_m'^2(x_{ms}') + J_m^2(x_{ms}')\left(1 - \frac{m^2}{x_{ms}'^2}\right)\right]. \quad (31)$$

In a plasma-filled waveguide, the group velocity of RF mode increases with the axial component of the wave vector. In this case, the group velocity of the low-frequency mode decreases. We have

$$v_g = -\frac{\partial \omega D\left(\omega, \vec{k}\right)}{\partial k_z} / \frac{\partial \omega D\left(\omega, \vec{k}\right)}{\partial \omega} = \frac{k_z c^2}{\omega G}, \quad (32)$$

where

$$G = \frac{\left(\omega^2 - \Omega_B^2\right)^2 + \omega_{pe}^2 \Omega_B^2}{\left(\omega^2 - \Omega_B^2\right)^2}. \quad (33)$$

The electromagnetic fields of TE mode in the waveguide are given by the expressions

$$B_z = b(z,t)\, J_m(k_\perp r)\exp(-i\omega t + im\theta,) \quad (34)$$

$$(B_\theta, E_r) = \left[\frac{m}{r}J_m(k_\perp r) + \frac{\omega^2\varepsilon_2}{c^2k_\perp^2}k_\perp J_m'(k_\perp r)\right]\times$$
$$\times\exp(-i\omega t + ik_z z + im\theta)\left(\frac{i}{k_\perp^2}\frac{\partial b}{\partial z}, -\frac{\omega}{ck_\perp^2}b\right), \quad (35)$$

$$(B_r, E_\theta) = \left(J_m'(k_\perp r) + \frac{\omega^2\varepsilon_2}{c^2k_\perp^3}\frac{m}{r}J_m(k_\perp r)\right)\times$$
$$\times\exp(-i\omega t + ik_z z + im\theta)\left(\frac{1}{k_\perp}\frac{\partial b}{\partial z}, -i\frac{\omega}{ck_\perp}b\right). \quad (36)$$

The transverse component of the wave vector $k_\perp$ can be found from the equation

$$J_m'(x_{ms}') + \frac{\omega^2\varepsilon_2 r_w^2}{c^2x_{ms}'^2}\frac{m}{x_{ms}'}J_m(x_{ms}') = 0. \quad (37)$$

This equation means that the tangential component of the electric field of a wave is equal to zero on the walls of the waveguide. Thus,

$$k_\perp = k_{ms}' = \frac{x_{ms}'}{r_w}, \quad (38)$$

where x_{ms}' is the s-th root of Eq. (37).

The components of the field in the new system of coordinates which is associated with a helical orbit of an electron can be written in such a way:

$$b_z' = b_z,$$
$$b_\Phi' = b_\Phi + igb_R,$$
$$b_R' = b_R - igb_\Phi,$$
$$e_R' = e_R - ige_\Phi,$$
$$e_\Phi' = e_\Phi + ige_R. \quad (39)$$

The field components are determined by expressions (9) and (10). So the equation for the field can be written as

$$\left(\frac{d^2}{dz^2} + \frac{\omega^2\varepsilon_1}{c^2} - k_{ms}'^2\right)b\pi r_w^2\left\{J_m'^2(k_{ms}'r_w) + \right.$$
$$\left. + J_m^2(k_{ms}'r_w)\left[1 - \frac{m^2}{(k_{ms}'r_w)^2}\right]\right\} =$$
$$= -\frac{4\pi i}{\omega b^*}k_{ms}'^2\int_0^{r_w} rdr\int_0^{2\pi}d\theta\left(\vec{J}\vec{E}^*\right). \quad (40)$$

By substituting expressions (37) into (42), we obtain

$$\left(\frac{\partial^2}{\partial z^2} + \frac{\omega^2\varepsilon_1}{c^2} - k_{ms}'^2\right)b = -\frac{4N_{b0}e\omega_B k_{ms}'^2}{c}\times$$
$$\times\left[x_{ms}'^2J_m'^2(x_{ms}') + J_m^2(x_{ms}')\left(x_{ms}^2 - m^2\right)\right]^{-1}\times$$
$$\times J_{n-m}(k_{ms}r_c)\frac{1}{N}\sum_{j=1}^N\left[a_jJ_n'(a_j) + \right.$$
$$\left. + g\frac{n}{a_j}J_n(a_j)\right]\exp(-2i\pi\zeta_j). \quad (40a)$$

The equations of motion of electrons can be written as

$$2\pi\frac{d\zeta}{dz} = \frac{n\omega_{B0} - \omega}{v_z} + \frac{n\omega_{B0}}{2v_z}\left[\beta_{\perp0}^2 - \beta_\perp^2 - \right.$$
$$\left. -2\beta_{z0}(v_z - v_{z0})/c\right] + n\frac{e}{m_ecv_z}J_{n-m}(k_{ms}'r_c)\times$$
$$\times\exp(2i\pi\zeta)\left\{\left[\left(1 - \frac{n^2}{a^2}\right)J_n(a) - g\frac{n}{a}J_n'(a)\right]\times\right.$$
$$\left.\times b - \frac{in^2v_z}{\omega a^2}\left[J_n(a) + g\frac{a}{n}J_n'(a)\right]\frac{\partial b}{\partial z}\right\}, \quad (41)$$

$$\frac{da}{dz} = in\frac{e\omega}{m_ec\omega_Bv_z}\left[J_n'(a) + g\frac{n}{a}J_n(a)\right]\times$$
$$\times J_{n-m}(k_{ms}r_c)\exp(2i\pi\zeta)\left[b + i\frac{v_z}{\omega}\frac{\partial b}{\partial z}\right], \quad (42)$$

$$\frac{d}{dz}\left(\frac{1}{v_z}\right) = -\frac{e\omega_B}{k_{ms}^2v_z^3m_ec}\frac{\partial b}{\partial z}J_{n-m}(k_{ms}r_c)\,a\times$$
$$\times\left[J_n'(a) + g\frac{n}{a}J_n(a)\right]\exp(2i\pi\zeta). \quad (43)$$

The set of equations which describe the stationary wave amplification in the longitudinal direction can be determined in case where the Larmor radius of electrons of a beam is not small as

$$\left(\frac{d^2}{d\xi_P^2} + \gamma_P^2\right)E_P = -\frac{1}{N}\sum_{j=1}^N a_j\left[J_n'(a_j) + \right.$$
$$\left. + g\frac{n}{a}J_n(a_j)\right]\exp(-2i\pi\zeta_j), \quad (44)$$

$$2\pi\frac{d\zeta}{d\xi_P}=\sigma\left[\Delta_P+\frac{a_0^2-a^2}{2\varepsilon' n^2}+\beta_{z0}^2\frac{(\sigma-1)}{\sigma\varepsilon'}\right]+$$
$$+\exp(2i\pi\zeta)\,n\left\{\left[\left(1-\frac{n^2}{a^2}\right)J_n(a)-\right.\right.$$
$$\left.-g\frac{n}{a}J_n'(a)\right]\sigma E_P-\frac{i\varepsilon' n^2}{a^2}\left[J_n(a)+\right.$$
$$\left.\left.+g\frac{a}{n}J_n'(a)\right]\frac{\partial E_P}{\partial\xi_P^2}\right\}, \qquad (45)$$

$$\frac{da}{d\xi_P}=in\left[J_n'(a)+g\frac{n}{a}J_n(a)\right]\times$$
$$\times\exp(2i\pi\zeta)\left[\sigma E_P+i\varepsilon'\frac{\partial E_P}{\partial\xi_P}\right], \qquad (46)$$

$$\frac{d\sigma}{d\xi_P}=-R_P\sigma^3\frac{\partial E_P}{\partial\xi_P}a\left[J_n'(a)+g\frac{n}{a}J_n(a)\right]\exp(2i\pi\zeta), \qquad (47)$$

where

$$\xi_P=z\rho_P,\quad E_P=\frac{eJ_{n-m}(k_{ms}'r_c)}{m_{e0}c\rho_P v_{z0}},\quad \sigma=\frac{v_{z0}}{v_z},$$

$$R_P=\frac{\rho_P\omega_B}{k_{ms}'^2 v_{z0}},\quad \varepsilon'=\frac{\rho_P v_{z0}}{\omega},\quad \Delta_P=\frac{n\omega_{B0}-\omega}{\rho_P v_{z0}},$$

$$\gamma_P^2=\frac{1}{\rho_P^2}\left[\frac{\varepsilon_1\omega^2}{c^2}-k_{ms}'^2\right],$$

and

$$\rho_P^3=\frac{4N_{b0}e^2\omega_B k_{ms}'^2}{m_e c^3\beta_{z0}}\left[x_{ms}'^2J_m'^2(x_{ms}')+\right.$$
$$\left.+J_m^2(x_{ms}')\left(x_{ms}'^2-m^2\right)\right]^{-1}J_{n-m}^2(k_{ms}'r_c).$$

If we take into account the temporal evolution of the wave amplitude, Eq. (45) should be transformed into

$$\left(\frac{\partial^2}{\partial\xi_P^2}+i\frac{\partial}{\partial\tau_P}+\gamma_P^2\right)E_P=$$
$$=-\frac{1}{N}\sum_{j=1}^{N}a_jJ_n'(a_j)\exp(-2i\pi\zeta_j), \qquad (48)$$

where $\tau_P=\rho_P^2c^2/2\omega G$.

With refard for the new dimensionless variable $\xi_G=\frac{\beta_\perp^2\omega_{B0}z}{2v_z}$ and the smallness of the arguments of Bessel functions, Eqs. (45) and (46) yield the equation of motion of an electron which is similar to (18). Then the equations of motion of electrons (45) and (46) are transformed to

$$\frac{dA}{d\xi_G}+i\left[\Delta+|A|^2-1\right]A=-i(1+g)F^*A^{*n-1}, \qquad (49)$$

where

$$F=4\frac{n^2}{2^n n!}\frac{a_0^{n-2}eb\exp(i\varphi)J_{n-m}(k_{ms}'r_c)}{cm_{e0}\beta_{\perp0}^2\omega_{B0}}. \qquad (50)$$

The condition of smallness of the arguments of Bessel functions is satisfied if the RF field varies weakly over the wavelength in the longitudinal direction, i.e. $|b|\gg\left|\frac{v_z}{\omega}\frac{\partial b}{\partial z}\right|$.

In the small Larmor radius approximation, the field equation (44) can be rewritten as

$$\frac{d^2F}{d\xi_G^2}+\gamma_G'^2F=-I_P\frac{1}{N}(1+g)\times$$
$$\times\sum_{j=1}^{N}\left(\frac{a_j}{a_{j0}}\right)^n\exp(-2i\pi\zeta_j)=-I_P(1+g)\left\langle\vec{A}^{*n}\right\rangle. \qquad (51)$$

If we take into account the temporal evolution of the wave amplitude, Eq. (51) should be changed into

$$\frac{d^2F}{d\xi_G^2}+i\frac{\partial F}{\partial\tau_G'}+\gamma_G'^2F=-I_P\frac{1}{N}(1+g)\times$$
$$\times\sum_{j=1}^{N}\left(\frac{a_j}{a_{j0}}\right)^n\exp(-2i\pi\zeta_j)=-I_P(1+g)\left\langle\vec{A}^{*n}\right\rangle, \qquad (52)$$

where

$$I_P=\frac{64eI_0n^3}{m_{e0}c^3}\frac{\beta_{z0}}{\beta_{\perp0}^{2(4-n)}}\left(\frac{n^n}{2^n n!}\right)^2\times$$
$$\times\frac{J_{n-m}^2(k_{ms}'r_c)}{J_m'^2(x_{ms}')x_{ms}'^2+J_m^2(x_{ms}')(x_{ms}'^2-m^2)}, \qquad (53)$$

$I_0=N_{b0}ev_{z0}$ is the initial beam current, and $\gamma_G'^2=4v_z^2\frac{\varepsilon_1\omega^2/c^2-k_{ms}^2}{\left(\beta_{\perp0}^2\omega_{B0}\right)^2}$.

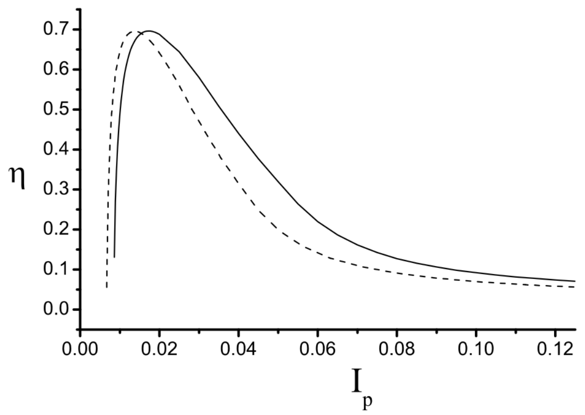

Fig. 2. Dependence of the transverse electron efficiency ($\eta_\perp$) on the normalized current (I_p). The dotted and solid lines correspond, respectively, to the vacuum case and the plasma-filled case ($\omega_{\rm pl}/\omega_{\rm cr}$=0.1)

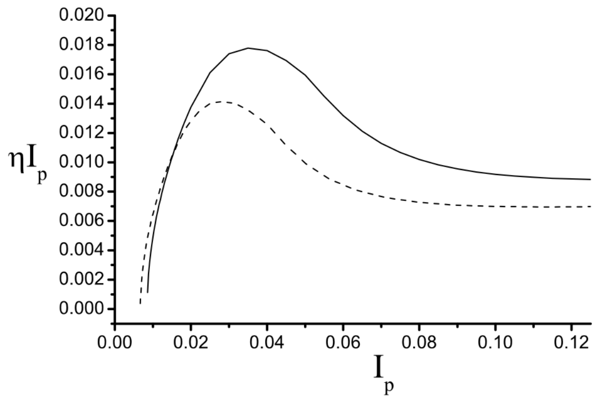

Fig. 3. Output power ($I_p \cdot \eta_\perp$) versus the normalized current (I_p) in the vacuum case (the dotted line) and in the plasma-filled case (the solid line) ($\omega_{\rm pl}/\omega_{\rm cr}$=0.1)

4. Numerical Results and Conclusion

Equations (49) and (52) have been solved numerically in order to find the longitudinal profile of the TE field in a waveguide, trajectories of electrons in the phase space, and the electron efficiency of a gyrotron.

In Fig. 2, the transverse electron efficiency $\eta_\perp$ as a function of the current strength for empty and plasma-filled waveguides is shown. The numerical simulations were made for the following parameters: $\beta_z = 0.24$, $\beta_\perp = 0.336$, $\frac{\omega_H}{\omega_{\rm cr}} = 1.062$, $z_{\rm out} = 12$, $\Delta = 0.6$, and $\frac{\omega_{\rm pl}}{\omega_{\rm cr}} = 0.1$. The dependence of the output power ($I_p \cdot \eta_\perp$) on the current is show in Fig. 3 for the same values of the parameters. Note that, for such parameters, $g \approx -0.078$.

As we can see, the maximal efficiency is reached for the current $I_p \approx 0.025$. For this and greater currents, the output power in the plasma-filled case is significantly larger than that in an empty gyrotron. In the case of the presence of a plasma, the plot of the efficiency versus the current is shifted to larger values of the current, and the maximal efficiency is reached for a larger current. So the filling with a plasma gives us the possibility to increase the output power for currents which correspond to the maximal efficiency or are larger.

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ВПЛИВ ПЛАЗМИ МАЛОЇ ЩІЛЬНОСТІ
НА РОБОТУ ГІРОТРОНА

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Р е з ю м е

Розроблено нелінійну теорію гіротронів з плазмовим наповненням малої щільності. Отримано самоузгоджену систему усереднених рівнянь для повільно змінного електромагнітного поля та імпульсу електронів пучка. Чисельно вивчено вплив плазми малої щільності на роботу гіротрона.