# DUALITY-SYMMETRIC GRAVITY AND SUPERGRAVITY: TESTING THE PST APPROACH

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Drawing an analogy between the gravity dynamical equation of motion and that of the Maxwell electrodynamics with an electric source, we outline a way of the appearance of a field dual to a graviton one. We propose a dimensional reduction ansatz for the strength of this field which reproduces the correct duality relations between the fields arising in the dimensional reduction of a $D$-dimensional gravity action to that in the 1D dimensions. Modifying the Pasti—Sorokin—Tonin (PST) approach, we construct a new term entering the action of $D = 11$ duality-symmetric gravity and, by using the proposed ansatz, we confirm the relevance of such a term to reproduce the correct duality-symmetric structure of the reduced theory. We end up by extending the results to the bosonic sector of $D = 11$ supergravity.

## 1. Introduction

The recent progress in studying the hidden symmetry group of Superstring/M-theory [1—16] reveals the necessity of taking seriously into account a "graviton dual" field. The dynamics of such a field has been intensively studied in the literature in the linearized limit [17, 2, 7, 18, 19] together with studying the actions that lead, in the same approximation, to the equation of motion for the "graviton dual" field. The attempts to extend the linear theory to a non-linear one were faced with the troubles that were summarized in the "no-go" theorems [18] which forbid the action for the interaction theory solely in terms of the "graviton dual" field. This situation is very similar to that of constructing the eleven-dimensional supergravity with a six-index photon field [20, 21], where the simple arguments [22] lead to the conclusion on the impossibility of constructing the gauge sector of the theory solely in terms of a six-index photon field. However, a way to go beyond these arguments is to consider a duality-symmetric theory of $D = 11$ supergravity with the original three-index photon field and with the "dual-to-three index photon" field that enter the action on equal footing [23] (see also [24] for an early attempt). Here, we will follow the same way and will closely inspect a possibility to construct a non-linear theory of gravity and supergravity with the "graviton dual" field standing on a point of the duality-symmetric formulation. The arguments in favor of such a consideration are as follows. First, we recall the close connection between $D = 11$ supergravity and type IIA theory that follows from $D =11$ supergravity by the dimensional reduction. The importance of having the "graviton dual" field in $D = 11$ supergravity for $D = 10$ type IIA theory has been pointed out as in the purely algebraic aspect [2], as well as in the context of searching for the complete duality-symmetric version of $D = 11$ supergravity [25] involving the "graviton dual" field. As well as recovering the correct algebra for a non-linear realization of type IIA supergravity requires to have a generator corresponding to the graviton dual partner in its eleven-dimensional counterpart [7], this dual field is required for recovering the complete duality-symmetric formulation of type IIA supergravity in a straightforward way [26]. Secondly, since the duality-symmetric version of type IIA supergravity was realized on the ground of PST technique [27], it is naturally to expect the PST-like structure of additional terms in the $D = 11$ supergravity action which encode the duality relations between the graviton field and its dual partner. As we have known the structure of the duality-symmetric type IIA supergravity [26], we can therefore use this advantage for deducing an ansatz for the dimensional reduction of the "graviton dual" field. Finally, we can try to apply the ansatz for constructing the additional terms in the $D = 11$ supergravity action which will reproduce, after reduction, the correct duality-symmetric structure of $D = 10$ type IIA supergravity. To this end, we take into account the observations that have been previously made under construction of duality-symmetric theories for different sub-sectors of maximal higher-dimensional supergravities [23, 28, 29, 26].

It has to be emphasized that, as a first step towards completing our task, we will find, without the appeal to a method of constructing the action, a convenient representation of the gravity equation of motion in a way that allows us to present the latter as the Bianchi identity for a dual field. It turns out to be convenient, for this purpose, to write down the gravity equation of motion in a form which is similar to the Maxwell theory dynamics with a source. As soon as such a representation is recovered, it suggests a way of extracting the dual field and to apply the PST formalism to construct the action, from which the duality relations between the graviton field and its dual partner will follow as equations of motion.

For the sake of simplicity, we will start our quest of the complete duality-symmetric formulation of $D = 11$ supergravity with the pure gravity case and extend, hereafter, the results to the bosonic sector of $D = 11$ supergravity. The scheme we propose does not depend on the space-time dimension $D \geq 4$, where gravity has dynamical degrees of freedom. Moreover, since the gravity is the key ingredient of supergravity, the results can be applied for all theories of supergravity in diverse dimensions, though, as we have mentioned above, we will end up with the space-time dimension $D = 11$ having in mind mostly the applying to this case.

## 2. Kaluza—Klein Ansatz for the Graviton Dual Field

Let us introduce both the graviton field described by a vielbein $\hat{e}^{\hat{a}[1]}$, whose index $\hat{a}$ runs from zero to $D-1$ and takes the value in the tangent space with the Lorentz group $\mathrm{SO}(1, D-1)$, and the field $\hat{A}^{\hat{a}[D-3]}$ dual to the vielbein. The way, in which the latter appears, is as follows. The first-order action for the pure gravity is

$$S_{\mathrm{EH}} = \int\limits_{\mathcal{M}^D} \hat{R}^{\hat{a}\hat{b}} \wedge \hat{\Sigma}_{\hat{a}\hat{b}}, \tag{1}$$

where $\hat{R}^{\hat{a}\hat{b}} = d\hat{\omega}^{\hat{a}\hat{b}} - \hat{\omega}^{\hat{a}}{}_{\hat{c}} \wedge \hat{\omega}^{\hat{c}\hat{b}}$ is the curvature 2-form and

$$\hat{\Sigma}_{\hat{a}_1\ldots\hat{a}_n} = \frac{1}{(D-n)!}\epsilon_{\hat{a}_1\ldots\hat{a}_D}\hat{e}^{\hat{a}_{n+1}} \wedge \ldots \wedge \hat{e}^{\hat{a}_D} \tag{2}$$

is a $(D-n)$-form constructed out of vielbeins $\hat{e}^a$. The equations of motion following from the action are

$$\hat{\Sigma}_{\hat{a}\hat{b}\hat{c}} \wedge \hat{R}^{\hat{b}\hat{c}} = 0, \tag{3}$$

$$\hat{\Sigma}_{\hat{a}\hat{b}\hat{c}} \wedge \hat{T}^{\hat{c}} = 0, \tag{4}$$

where we have introduced the torsion 2-form $\hat{T}^{\hat{a}} = d\hat{e}^{\hat{a}} - \hat{e}^{\hat{b}} \wedge \hat{\omega}_{\hat{b}}{}^{\hat{a}}$. The latter equation sets the torsion to zero and is the algebraic relation expressing the connection $\hat{\omega}^{\hat{a}\hat{b}}$ through the vielbeins and their derivatives. In the sequel, we will be mostly concentrated on the dynamical equation of motion.

It is easy to check that the Einstein equation (3) admits the following representation:

$$d(\hat{\omega}^{\hat{b}\hat{c}} \wedge \hat{\Sigma}_{\hat{a}\hat{b}\hat{c}}) = \hat{\omega}^{\hat{b}\hat{c}} \wedge d\hat{\Sigma}_{\hat{a}\hat{b}\hat{c}} + (-)^{D-3}\, \hat{\omega}^{\hat{b}}{}_{\hat{d}} \wedge \hat{\omega}^{\hat{d}\hat{c}} \wedge \hat{\Sigma}_{\hat{a}\hat{b}\hat{c}}. \tag{5}$$

One can rewrite this equation in the form

$$d\hat{*}\hat{f}^{[2]}_{\hat{a}} = \hat{*}\hat{J}^{[1]}_{\hat{a}} \tag{6}$$

with $\hat{*}\hat{f}^{[2]}_{\hat{a}} \equiv \hat{\omega}^{\hat{b}\hat{c}} \wedge \hat{\Sigma}_{\hat{a}\hat{b}\hat{c}}$, that looks very much like the equation of motion for Maxwell electrodynamics with an electric source. This analogy has been pointing out in the literature for a long time (cf., e.g., [30, 31]). However, the difference between the electrodynamics equation of motion and (6) is apparent. The former contains a "bare" potential $A^{[1]}$ whereas the latter does not, since the r.h.s. and the l.h.s. of (6) depend on vielbeins and the connection. The only vielbein is the true dynamical field, hence one should resolve the connection through vielbeins using the torsion free constraint (4). The answer is

$$\hat{\omega}_{\hat{m}}{}^{\hat{a}\hat{b}} = \frac{1}{2}(\hat{e}^{\hat{n}\hat{a}}\partial_{[\hat{m}}\hat{e}^{\hat{b}}_{\hat{n}]} - \hat{e}^{\hat{n}\hat{b}}\partial_{[\hat{m}}\hat{e}^{\hat{a}}_{\hat{n}]} - \hat{e}_{\hat{m}\hat{c}}\hat{e}^{\hat{n}\hat{a}}\hat{e}^{\hat{s}\hat{b}}\partial_{[\hat{n}}\hat{e}^{\hat{c}}_{\hat{s}]}), \tag{7}$$

where the indices from the second middle of the Latin alphabet are the curved ones. Substituting this expression into the l.h.s. of (5), we arrive at

$$d(\hat{\omega}^{\hat{b}\hat{c}} \wedge \hat{\Sigma}_{\hat{a}\hat{b}\hat{c}}) = \frac{1}{\alpha_D} d(\hat{*}d\hat{e}_{\hat{a}}) + d\hat{S}^{[D-2]}_{\hat{a}}, \tag{8}$$

where we have introduced the form $\hat{S}^{[D-2]}_{\hat{a}}$, which is a function of vielbeins and their derivatives, and the numerical coefficient $\alpha_D$ which takes $\pm 1$ in dependence on the space-time signature setting and the Hodge star definition. It is worth mentioning that the r.h.s. of Eq. (6) is nothing but the Landau—Lifshits pseudotensor which is a conserved "current" and therefore admits the representation

$$\hat\omega^{\hat b\hat c}\wedge d\hat\Sigma_{\hat a\hat b\hat c}+(-)^{D-3}\,\hat\omega^{\hat b}{}_{\hat d}\wedge\hat\omega^{\hat d\hat c}\wedge\hat\Sigma_{\hat a\hat b\hat c}:=\hat{*}\hat J^{[1]}_{\hat a}=d\hat{*}\hat G^{[2]}_{\hat a}. \quad (9)$$

Taking into account (8) and (9), Eq. (5) can be rewritten as

$$d(\hat{*}d\hat e_{\hat a})=\hat{*}\hat{\tilde J}^{[1]}_{\hat a} \quad (10)$$

with

$$\hat{*}\hat{\tilde J}^{[1]}_{\hat a}=d\hat{*}\hat{\tilde G}^{[2]}_{\hat a}, \quad (11)$$

where we have denoted $\hat{\tilde G}^{[2]}_{\hat a}=\alpha_D(\hat G^{[2]}_{\hat a}-\hat{*}\hat S^{[D-2]}_{\hat a})$. Notice that $\hat{\tilde G}^{[2]}_{\hat a}$ is also a function of vielbeins and their derivatives.

One more comment is needed before proceeding further. The Einstein equation (3) is generally covariant while its representations (5), (6), and (10) are recorded w.r.t. the usual (non-covariant) derivatives that could lead to the conclusion on their non-covariance. This puzzle can be resolved through the observation (cf., for instance, [33]) that the r.h.s. of Eqs. (5), (6), and (10) is not a true tensor, but a pseudotensor. Hence, the non-covariance of the l.h.s. of the above-mentioned equations is compensated by the pseudotensor character of the r.h.s. leaving nevertheless Eqs. (5), (6), and (10) to be covariant. Another way to see that is, for instance, to notice the relation

$$d(\hat{*}d\hat e_{\hat a}-\hat{*}\hat{\tilde G}^{[2]}_{\hat a})=\alpha_D(-)^{D-3}\Sigma_{\hat a\hat b\hat c}\wedge\hat R^{\hat b\hat c}.$$

Now we have the almost complete analogy between gravity and the Maxwell theory with an electric source in the sense of representing the former through the only true potentials. The field strength dual to $d\hat e_{\hat a}$ is

$$\hat F^{[D-2]}_{\hat a}=d\hat A^{[D-3]}_{\hat a}+\hat{*}\hat{\tilde G}^{[2]}_{\hat a}\equiv\hat{\mathbb F}^{[D-2]}_{\hat a}+\hat{*}\hat{\tilde G}^{[2]}_{\hat a}, \quad (12)$$

and the "graviton dual" field has appeared.

Since a part of the field dual to the graviton will become dual fields to a dilaton and a Kaluza—Klein vector field after the dimensional reduction from $D$ to $D-1$, we will use this fact to deduce the reduction ansatz for the field strength of $\hat A^{\hat a[D-3]}$ that will reproduce the correct structure of the duality-symmetric gravity in $D-1$ space-time dimensions. After establishing the ansatz, we will try to apply it for lifting the known $D-1$-dimensional action up to the $D$-dimensional space-time.

Splitting the tangent space index as $\hat a=(a,z)$, $a=0,\dots,D-2$, we choose the standard Kaluza—Klein ansatz for the vielbein

$$\hat e^{a[1]}=e^{\alpha\phi}e^{a[1]},\qquad \hat e^{z[1]}=e^{\beta\phi}(dz+A^{[1]}), \quad (13)$$

where $\phi$ denotes the dilaton field, $A^{[1]}$ stands for the Kaluza—Klein vector field, and $z$ is the direction of the reduction. All the fields on the r.h.s. of Eq. (13) are independent on the reduction coordinate. The parameters $\alpha$ and $\beta$ are related to fixing the space-time dimensions $D$ and to each other via [34]

$$\alpha^2=\frac{1}{2(D-2)(D-3)},\qquad \beta=-(D-3)\alpha, \quad (14)$$

that corresponds to the Einstein frame after the reduction.

The vielbeins' dimensional reduction ansatz falls into a general class of ansätze which determine the reduction of a (Lorentz-valued) $n$-form

$$\hat\Omega^{(a,z)[n]}=\Omega^{(a,z)[n]}+\omega^{(a,z)[n-1]}\wedge(dz+A^{[1]}). \quad (15)$$

To specify the ansatz, one has to determine the forms $\Omega^{(a,z)[n]}$, $\omega^{(a,z)[n-1]}$ for each case under consideration. The other relation which will be under the usage in what follows is

$$\hat{*}\,\hat\Omega^{(a,z)[n]}=a_{(n)}e^{-2(n-1)\alpha\phi}*\Omega^{(a,z)[n]}\wedge(dz+A^{[1]})+$$
$$+a_{(n-1)}e^{2(D-n-1)\alpha\phi}*\omega^{(a,z)[n-1]}. \quad (16)$$

The numerical coefficients $a_{(n)}$, $a_{(n-1)}$ taking the values $\pm1$ encode the information on the space-time signature choice and the definition of the Hodge operator. To distinguish $D$ and $(D-1)$-dimensional Hodge stars, we have equipped the former with a hat.

Let us now turn to the analysis of the gravity equation of motion (10). The equation reads, at least for the trivial topology setting, as

$$\hat{*}d\hat e^{\hat a[1]}=d\hat A^{\hat a[D-3]}+\hat{*}\hat{\tilde G}^{\hat a[2]}=\hat{\mathbb F}^{\hat a[D-2]}+\hat{*}\hat{\tilde G}^{\hat a[2]}. \quad (17)$$

Splitting the tangent space index $\hat a$, one arrives at the relations

$$\hat{*}d\hat e^{a[1]}=d\hat A^{a[D-3]}+\hat{*}\hat{\tilde G}^{a[2]}=\hat{\mathbb F}^{a[D-2]}+\hat{*}\hat{\tilde G}^{a[2]}, \quad (18)$$

$$\hat{*}d\hat e^{z[1]}=d\hat A^{z[D-3]}+\hat{*}\hat{\tilde G}^{z[2]}=\hat{\mathbb F}^{z[D-2]}+\hat{*}\hat{\tilde G}^{z[2]}. \quad (19)$$

Since the duality relation for the dilaton and the Kaluza—Klein vector field is encoded into the latter equation, let us begin our analysis with (19).

Taking into account (13), (15), and (16), one arrives at the relation

$$a_{(2)}e^{(\beta-2\alpha)\phi}*dA^{[1]}\wedge(dz+A^{[1]})-$$

$$-a_{(1)}e^{2(D-3)\alpha\phi+\beta\phi}\beta * d\phi =$$

$$= \mathbb{F}^{z[D-2]} + \mathbb{F}^{z[D-3]} \wedge (dz + A^{[1]})+$$

$$+a_{(2)}e^{-2\alpha\phi} * G^{z[2]} \wedge (dz + A^{[1]}) + a_{(1)}e^{2(D-3)\alpha\phi} * g^{z[1]} \quad (20)$$

which contains two independent parts

$$a_{(2)}e^{(\beta-2\alpha)\phi} * dA^{[1]} = \mathbb{F}^{z[D-3]} + a_{(2)}e^{-2\alpha\phi} * G^{z[2]}, \quad (21)$$

and

$$-a_{(1)}e^{2(D-3)\alpha\phi+\beta\phi}\beta * d\phi = \mathbb{F}^{z[D-2]}+$$

$$+a_{(1)}e^{2(D-3)\alpha\phi} * g^{z[1]}. \quad (22)$$

Here, we have denoted $\hat{\mathbb{F}}^{z[D-2]} \equiv d\hat{A}^{z[D-3]} = \mathbb{F}^{z[D-2]} + \mathbb{F}^{z[D-3]} \wedge (dz + A^{[1]})$.

Since, by construction [cf. (8), (10), and (11)], $\hat{\hat{G}}^{z[2]}$ depends on veilbeins and their derivatives, $G^{z[2]}$ and $g^{z[1]}$ are completely determined by the reduction ansatz (13). Hence, our aim is to deduce the reduction ansatz for $\hat{\mathbb{F}}^{z[D-2]}$, i.e. to determine $\mathbb{F}^{z[D-2]}$ and $\mathbb{F}^{z[D-3]}$, which will lead to the correct duality relations between the fields.

The ansatz we propose for $\mathbb{F}^{z[D-3]}$ has the following form:

$$\mathbb{F}^{z[D-3]} = -a_{(2)}e^{-\beta\phi}dA^{[D-4]} - a_{(2)}e^{-2\alpha\phi} * G^{z[2]}. \quad (23)$$

It is easy to see that the substitution of this ansatz into (21) leads to the correct duality relation between the Kaluza—Klein vector field and its dual partner

$$e^{2(\beta-\alpha)\phi} * dA^{[1]} = -dA^{[D-4]}, \quad (24)$$

which can be extracted from the Kaluza—Klein vector field equation of motion after dimensional reduction of pure gravity from $D$ to $D-1$.

The other part forming the ansatz for $\hat{\mathbb{F}}^{z[D-2]}$ is

$$\mathbb{F}^{z[D-2]} = a_{(1)}e^{-\beta\phi}(dA^{[D-3]}-$$

$$-(D-2)\alpha dA^{[D-4]} \wedge A^{[1]})+$$

$$+a_{(1)}e^{2(D-3)\alpha\phi+\beta\phi}(1-\beta) * d\phi - a_{(1)}e^{2(D-3)\alpha\phi} * g^{z[1]}. \quad (25)$$

Substituting the latter into (22), one can find the correct duality relation between the dilaton and its dual partner:

$$*d\phi = -dA^{[D-3]} + (D-2)\alpha\, dA^{[D-4]} \wedge A^{[1]}. \quad (26)$$

Dealing with Eq. (18) is quite a bit complicated. By use of (13), (15), and (16), one obtains

$$a_{(2)}e^{-2\alpha\phi} * d(e^{\alpha\phi}e^a) \wedge (dz + A^{[1]}) = \mathbb{F}^{a[D-2]}+$$

$$+\mathbb{F}^{a[D-3]} \wedge (dz + A^{[1]}) + a_{(1)}e^{2(D-3)\alpha\phi} * g^{a[1]}+$$

$$+a_{(2)}e^{-2\alpha\phi} * G^{a[2]} \wedge (dz + A^{[1]}), \quad (27)$$

from which it immediately follows

$$\mathbb{F}^{a[D-2]} = -a_{(1)}e^{2(D-3)\alpha\phi} * g^{a[1]}. \quad (28)$$

The other equation that is contained into (27) is

$$a_{(2)}e^{-\alpha\phi} * (de^a - \alpha\, d\phi \wedge e^a) = \mathbb{F}^{a[D-3]}+$$

$$+a_{(2)}e^{-2\alpha\phi} * G^{a[2]}. \quad (29)$$

To find the correct ansatz for $\mathbb{F}^{a[D-3]}$, let us introduce a new form $\mathfrak{G}^{a[2]}$. This form is defined by

$$d * \mathfrak{G}^{a[2]} = *\mathfrak{J}^{a[1]} \quad (30)$$

with

$$*\mathfrak{J}^{[1]}_a = \omega^{bc} \wedge d\Sigma_{abc} + (-)^{D-4}\, \omega^b{}_d \wedge \omega^{dc} \wedge \Sigma_{abc}-$$

$$-dS^{[D-3]}_a + M^{[D-2]}_a. \quad (31)$$

Here, $\omega^{ab}$ is the $(D-1)$-dimensional connection one-form, $\Sigma_{abc}$ stands for the lower-dimensional analog of (2), $S^{[D-3]}_a$ is the $(D-1)$-dimensional counterpart of the form $\hat{S}^{[D-2]}_{\hat{a}}$ introduced in (8) that appears after resolving the lower-dimensional torsion free constraint, and $M^{[D-2]}_a$ is the energy-momentum tensor of the dilaton and of the Kaluza—Klein vector field which is a closed form. Therefore,

$$M^{[D-2]}_a = dk^{[D-3]}_a. \quad (32)$$

Taking the ansatz to be

$$\mathbb{F}^{a[D-3]} = -a_{(2)}e^{-\alpha\phi}[dA^{a[D-4]} + *\mathfrak{G}^{a[2]}+$$

$$+\alpha * (d\phi \wedge e^a) + e^{-\alpha\phi} * G^{a[2]}], \quad (33)$$

one can obtain the duality relation (cf. (17))

$$*de^a = -(dA^{a[D-4]} + *\mathfrak{G}^{a[2]}) \quad (34)$$

that reproduces the correct equation of motion for the graviton field

$$d(*de^a) = - * \mathfrak{J}^{a[1]}. \quad (35)$$

To summarize, the ansatz we proposed so far is

$$\hat{\mathbb{F}}^{a[D-2]} = -a_{(1)}e^{2(D-3)\alpha\phi} * g^{a[1]} - a_{(2)}e^{-\alpha\phi}[dA^{a[D-4]}+$$

$$+ * \mathfrak{G}^{a[2]} + \alpha * (d\phi \wedge e^a) + e^{-\alpha\phi} * G^{a[2]}] \wedge (dz + A^{[1]}), \quad (36)$$

$$\hat{\mathbb{F}}^{z[D-2]} = a_{(1)}e^{-\beta\phi}(dA^{[D-3]}-$$

$$-(D-2)\alpha\phi dA^{[D-4]}\wedge A^{[1]})+$$

$$+a_{(1)}e^{2(D-3)\alpha\phi+\beta\phi}(1-\beta)*d\phi-a_{(1)}e^{2(D-3)\alpha\phi}*g^{z[1]}-$$

$$-a_{(2)}e^{-\beta\phi}(dA^{[D-4]}+e^{-2\alpha\phi+\beta\phi}*G^{z[2]})\wedge(dz+A^{[1]}).\quad(37)$$

Let us close this part of the paper with some remarks on the proposed ansatz structure. First, the exponential factors of the leading expressions in (23), (25), and (33) are the same as in (13) but are opposite in sign. This is the indication of duality. Secondly, there are the uncommon quantities entering the ansatz such as that of coming from the reduction of $\hat{\tilde{G}}^{\hat{a}[2]}$. We are forced to include such quantities into the ansatz, since we cannot overcome the technical problem of evaluating the explicit expression for $\hat{\tilde{G}}^{\hat{a}[2]}$ which is defined by (11). The same concerns to the $\mathfrak{G}^{a[2]}$. However, the ansatz we proposed does not depend on the explicit structure of the above-mentioned expressions and is universal, though requires the very well knowledge of the duality relations in the lower-dimensional theory after reduction.

## 3. Duality-symmetric Action for Gravity and Complete Duality-symmetric Action for *D* = 11 Supergravity

Having established the reduction ansatz, let us turn to the more sophisticated problem of constructing the action for the duality-symmetric gravity. Since that requires more tuning, we are fixing the space-time dimensions $D$ to be eleven, and will follow the notation of [26]. In this notation, after the dimensional reduction, we should reproduce, in particular, the duality relations (24), (26) from the action, and they now have the following form:

$$\mathcal{F}^{[8]}=dA^{[7]}+e^{-\frac{3}{2}\phi}*dA^{[1]}=0,\quad(38)$$

$$\mathcal{F}^{[9]}=(dA^{[8]}-\frac{3}{4}dA^{[7]}\wedge A^{[1]})+*d\phi=0.\quad(39)$$

Here, we have fixed the parameters $\alpha$ and $\beta$ to be

$$\alpha=+\frac{1}{12},\qquad\beta=-\frac{2}{3}.\quad(40)$$

The relevant part of the ten-dimensional duality-symmetric action, from which these relations come, is (cf. [26])

$$S_{\rm d.s}=\frac{1}{2}\int\limits_{\mathcal{M}^{10}}\sum_{n=1}^{2}(F^{[10-n]}\wedge F^{[n]}+$$

$$+i_v\mathcal{F}^{[10-n]}\wedge v\wedge F^{[n]}+v\wedge F^{[10-n]}\wedge i_v\mathcal{F}^{[n]}).\quad(41)$$

Our aim is to demonstrate that action (41) follows after the dimensional reduction from the part of the gravity duality-symmetric action

$$S_{\rm PST}=\int\limits_{\mathcal{M}^{11}}\frac{1}{2}\hat{v}\wedge\hat{\mathcal{F}}^{\hat{a}[9]}\wedge i_{\hat{v}}\hat{\mathcal{F}}^{\hat{b}[2]}\eta_{\hat{a}\hat{b}},\quad(42)$$

where we have introduced, following to the PST approach [27], a scalar field $a(\hat{x})$ that enters the action in a non-polynomial way through the one-form

$$\hat{v}=\frac{da(\hat{x})}{\sqrt{-\partial_{\hat{m}}a\;\hat{g}^{\hat{m}\hat{n}}\;\partial_{\hat{n}}a}},\quad(43)$$

$\eta_{\hat{a}\hat{b}}$ is the Minkowski metric tensor, and

$$\hat{\mathcal{F}}^{\hat{a}[2]}=d\hat{e}^{\hat{a}}-\hat{*}(d\hat{A}^{\hat{a}[8]}+\hat{*}\hat{\tilde{G}}^{\hat{a}[2]}),\quad(44)$$

$$\hat{\mathcal{F}}^{\hat{a}[9]}=-\hat{*}\hat{\mathcal{F}}^{\hat{a}[2]}.\quad(45)$$

Note that, though these generalized field strengths are constructed out of the non-covariant quantities, they enter the generalized field strengths in the covariant combinations. Therefore, $\mathcal{F}^{a[2]}$ and $\mathcal{F}^{a[9]}$ are the covariant objects.

Splitting the indices $\hat{a}$ and $\hat{b}$, we can rewrite (42) as

$$S_{\rm PST}=\int\limits_{\mathcal{M}^{11}}[\frac{1}{2}\hat{v}\wedge\hat{\mathcal{F}}^{a[9]}\wedge i_{\hat{v}}\hat{\mathcal{F}}^{b[2]}\eta_{ab}$$

$$-\frac{1}{2}\hat{v}\wedge\hat{\mathcal{F}}^{z[9]}\wedge i_{\hat{v}}\hat{\mathcal{F}}^{z[2]}].\quad(46)$$

By use of ansätze (36) and (37) (in our notation $a_{(1)}=a_{(2)}=-1$), one gets

$$\hat{\mathcal{F}}^{a[2]}=e^{\frac{1}{12}\phi}[de^a+*(dA^{a[7]}+*\mathfrak{G}^{a[2]})]\equiv e^{\frac{1}{12}\phi}\mathcal{F}^{a[2]},\quad(47)$$

$$\hat{\mathcal{F}}^{a[9]}=e^{-\frac{1}{12}\phi}\mathcal{F}^{a[8]}\wedge(dz+A^{[1]}),\quad\mathcal{F}^{a[8]}=*\mathcal{F}^{a[2]},\quad(48)$$

$$\hat{\mathcal{F}}^{z[9]}=-e^{\frac{2}{3}\phi}(\mathcal{F}^{[9]}-\mathcal{F}^{[8]}\wedge(dz+A^{[1]})),\quad(49)$$

$$\hat{\mathcal{F}}^{z[2]}=e^{-\frac{2}{3}\phi}(\mathcal{F}^{[2]}-\mathcal{F}^{[1]}\wedge(dz+A^{[1]})),\quad\mathcal{F}^{[9]}=*\mathcal{F}^{[1]},$$

$$\mathcal{F}^{[8]}=e^{-\frac{3}{2}\phi}*\mathcal{F}^{[2]},\quad(50)$$

where $\mathcal{F}^{[8]}$, $\mathcal{F}^{[9]}$ have been defined in (38), (39) (they are not equal to zero, of course; the latter follows from the action as an equation of motion). Supplying the stuff by the ansatz for $\hat{v}$ [26]

$$\hat{v}=e^{\frac{1}{12}\phi}v,\quad i_v(dz+A^{[1]})=0,$$

$$v = \frac{da(x)}{\sqrt{-\partial_m a \; g^{mn} \; \partial_n a}}, \quad v \neq v(z), \tag{51}$$

and using the standard rules of dimensional reduction (see, e.g., [35] and references therein, or Appendices in [26]), one arrives after integration over $(dz + A^{[1]})$ at

$$S_{\rm PST} = -\frac{1}{2} \int\limits_{\mathcal{M}^{10}} [v \wedge \mathcal{F}^{a[8]} \wedge i_v \mathcal{F}^{b[2]} \eta_{ab} -$$

$$-v \wedge \mathcal{F}^{[8]} \wedge i_v \mathcal{F}^{[2]} - v \wedge \mathcal{F}^{[9]} \wedge i_v \mathcal{F}^{[1]}]. \tag{52}$$

After the standard manipulations, one can convince yourself that the reduction of the complete duality-symmetric $D = 11$ gravity action $S = S_{\rm EH} + S_{\rm PST}$ results in

$$S = -\int\limits_{\mathcal{M}^{10}} \left( R^{ab} \wedge \Sigma_{ab} + \frac{1}{2} v \wedge \mathcal{F}^{a[8]} \wedge i_v \mathcal{F}^{b[2]} \eta_{ab} \right) +$$

$$+\frac{1}{2} \int\limits_{\mathcal{M}^{10}} \sum_{n=1}^{2} (F^{[10-n]} \wedge F^{[n]} + i_v \mathcal{F}^{[10-n]} \wedge v \wedge F^{[n]} +$$

$$+v \wedge F^{[10-n]} \wedge i_v \mathcal{F}^{[n]}). \tag{53}$$

It is easy to recognize the duality-symmetric structure of the $D = 10$ gravity action, and the second line is precisely action (41) we are looking for. Therefore, we have established the relevance of the proposed $S_{\rm PST}$ term in the action for the duality-symmetric gravity.

One may wonder why, having a new term entering the action, we did not modify $\hat{\tilde{G}}^{\hat{a}[2]}$, though this term is not a topological term and therefore contributes to the energy-momentum tensor. The reason is the same as for another application of the PST approach. The PST terms do not spoil the dynamics of an original theory, since their contribution to the equations of motion is a combination of the duality relations which is zero on-shell. The latter is guaranteed by a special symmetry, one of the two PST symmetries [27]. That is, for instance, why we do not need to take into account the contribution of two last terms of (53) to $\mathfrak{G}^{a[2]}$. Even leaving the issue of establishing the PST symmetries out of consideration, one could notice the indication of their presence in the eleven-dimensional theory, since the special symmetries of two last terms of action (53) are the bits of the corresponding eleven-dimensional symmetries.

To give more rigorous arguments in favor of the discussion above, let us consider a variation of the action

$$S = S_{\rm EH} + S_{\rm PST} \tag{54}$$

with $S_{\rm EH}$ of (1) and $S_{\rm PST}$ of (42). The variation of the Einstein—Hilbert term results in

$$\delta S_{\rm EH} = \int\limits_{\mathcal{M}^{11}} \delta \hat{e}^{\hat{a}} \wedge d\hat{\mathcal{F}}^{\hat{b}[9]} \eta_{\hat{a}\hat{b}}. \tag{55}$$

What concerns the PST term, it is convenient to split its variation into the part standard for such an approach, where we will effectively treat the vielbeins on the same footing as another gauge field, i.e. as fields completely independent of the metric, and that varying the metric:

$$\delta S_{\rm PST} = \int\limits_{\mathcal{M}^{11}} \delta_0 \mathcal{L}_{\rm PST} + \delta_* \mathcal{L}_{\rm PST}. \tag{56}$$

The standard variation ends up with

$$\delta_0 S_{\rm PST} = \int\limits_{\mathcal{M}^{11}} \left( \delta \hat{A}^{\hat{a}[8]} + \frac{\delta a}{\sqrt{-(\partial a)^2}} i_{\hat{v}} \hat{\mathcal{F}}^{\hat{a}[8]} \right) \times$$

$$\times \eta_{\hat{a}\hat{b}} \wedge d(\hat{v} \wedge i_{\hat{v}} \hat{\mathcal{F}}^{\hat{b}[2]}) +$$

$$+ \int\limits_{\mathcal{M}^{11}} \left( \delta \hat{e}^{\hat{a}} + \frac{\delta a}{\sqrt{-(\partial a)^2}} i_{\hat{v}} \hat{\mathcal{F}}^{\hat{a}[2]} \right) \times$$

$$\times \eta_{\hat{a}\hat{b}} \wedge d(\hat{v} \wedge i_{\hat{v}} \hat{\mathcal{F}}^{\hat{b}[9]}) -$$

$$- \int\limits_{\mathcal{M}^{11}} \left( \delta \hat{e}^{\hat{a}} \eta_{\hat{a}\hat{b}} \wedge d\hat{\mathcal{F}}^{\hat{b}[9]} + \delta [\hat{*} \hat{\tilde{G}}^{\hat{a}[2]}] \eta_{\hat{a}\hat{b}} \wedge \hat{v} \wedge i_{\hat{v}} \hat{\mathcal{F}}^{\hat{b}[2]} \right). \tag{57}$$

To read off the other part of the variation, we will proceed as in the case of extracting the energy-momentum tensor. In terms of differential forms, this variation is

$$\delta_* (\Omega^{[n]} \wedge * \Omega^{[n]}) = (\delta_* \Omega^{[n]}) \wedge * \Omega^{[n]} + \Omega^{[n]} \wedge \delta_* (* \Omega^{[n]}) =$$

$$= \frac{1}{(n-1)!} \delta e^{a_n} \wedge e^{a_{n-1}} \wedge \ldots \wedge e^{a_1} \Omega^{[n]}_{a_1 \ldots a_{n-1} a_n} \wedge * \Omega^{[n]} +$$

$$+ \frac{1}{n!} \frac{1}{(D-n-1)!} \Omega^{[n]} \wedge \delta e^{b_{D-n}} \times$$

$$\times \wedge e^{b_{D-n-1}} \wedge \ldots \wedge e^{b_1} \epsilon_{b_1 \ldots b_{D-n}}{}^{a_1 \ldots a_n} \Omega^{[n]}_{a_1 \ldots a_n}. \tag{58}$$

Following this pattern, we arrive at

$$\delta_* S_{\rm PST} = \int\limits_{\mathcal{M}^{11}} \frac{1}{2} \hat{v} \wedge \delta \hat{e}^{\hat{c}_9} (\frac{1}{8!} \hat{e}^{\hat{c}_8} \wedge \ldots \wedge \hat{e}^{\hat{c}_1} \hat{\mathcal{F}}^{\hat{a}[9]}_{\hat{c}_1 \ldots \hat{c}_8 \hat{c}_9}) \times$$

$$\times\eta_{\hat a\hat b}\wedge i_{\hat v}\hat{\mathcal F}^{\hat b[2]}+\int\limits_{\mathcal M^{11}}\frac{1}{2}\hat v\wedge\hat{\mathcal F}^{\hat a[9]}\eta_{\hat a\hat b}\wedge\delta\hat e^{\hat c}(i_{\hat v}\hat{\mathcal F}^{\hat b[2]})_{\hat c}. \quad (59)$$

Taking into account the results of (55), (57), and (59), one can derive the following sets of special symmetries

$$\delta a(\hat x)=0,\quad \delta\hat e^{\hat a}=da\wedge\hat\varphi^{\hat a[0]},$$

$$\delta\hat A^{\hat a[8]}=da\wedge\hat\varphi^{\hat a[7]}-d^{-1}\delta(\hat{*}\hat{\tilde G}^{\hat a[2]}), \quad (60)$$

$$\delta a(\hat x)=\Phi(\hat x),\qquad \delta\hat e^{\hat a}=-\frac{\Phi}{\sqrt{-(\partial a)^2}}i_{\hat v}\hat{\mathcal F}^{\hat a[2]},$$

$$\delta\hat A^{\hat a[8]}=-\frac{\Phi}{\sqrt{-(\partial a)^2}}i_{\hat v}\hat{\mathcal F}^{\hat a[9]}-d^{-1}[\delta(\hat{*}\hat{\tilde G}^{\hat a[2]})+$$

$$+\frac{1}{2!8!}\delta\hat e^{\hat c_9}\wedge\hat e^{\hat c_8}\wedge\ldots\wedge\hat e^{\hat c_1}\hat{\mathcal F}^{\hat a[9]}_{\hat c_1\ldots\hat c_8\hat c_9}-\frac{1}{2}\hat{*}(\delta\hat e^{\hat c}\hat{\mathcal F}^{\hat a[2]})_{\hat c}]. \quad (61)$$

Here, we have introduced the operator inverse to $d$, whose action on an arbitrary form is defined by the use of a "Green" function in the equation

$$d(x)h(x-y)=\delta^{11}(x-y), \quad (62)$$

where $\delta^{11}(x)$ is the Dirac delta-function, and therefore

$$d^{-1}(x)\omega^{[p]}(x)=(-)^p\int d^{11}y\ h(x-y)\wedge\omega^{[p]}(y). \quad (63)$$

To make a sense, the latter expression should only deal with the "Green" functions that act on the causality-related region of space-time.

It should be emphasized that we cannot present the explicit variation of the "graviton dual" field without introducing the non-local expressions, since we have no explicit local expression for $\hat{*}\hat{\tilde G}^{\hat a[2]}$. But nevertheless, these non-local expressions do not spoil the job of the PST symmetries (60), (61) to extract the duality relations and to establish the pure auxiliary nature of the PST scalar field.

To demonstrate that, it has to be noted that the general solution to the equation of motion of $\hat A^{\hat a[8]}$,

$$d(\hat v\wedge i_{\hat v}\hat{\mathcal F}^{\hat a[2]})=0, \quad (64)$$

has the form [27]

$$\hat v\wedge i_{\hat v}\hat{\mathcal F}^{\hat a[2]}=da\wedge d\hat\xi^{\hat a[0]}. \quad (65)$$

Under the action of (60), the l.h.s. of the latter expression is transformed as

$$\hat v\wedge i_{\hat v}\hat{\mathcal F}^{\hat a[2]}\longrightarrow\hat v\wedge i_{\hat v}\hat{\mathcal F}^{\hat a[2]}+da\wedge d\hat\varphi^{\hat a[0]}, \quad (66)$$

and therefore, setting $\hat\xi^{\hat a[0]}=\hat\varphi^{\hat a[0]}$, one can reduce (65) to

$$i_{\hat v}\hat{\mathcal F}^{\hat a[2]}=0\qquad\rightsquigarrow\qquad\hat{\mathcal F}^{\hat a[2]}=0. \quad (67)$$

Taking the latter into account and using the same arguments, one can reduce the vielbein equation of motion to

$$\hat{\mathcal F}^{\hat a[9]}=0. \quad (68)$$

It becomes clear that the equation of motion of the PST scalar $a(\hat x)$ is identically satisfied as a consequence of the equations of motion for the other fields. Therefore, the PST scalar equation of motion is the Noether identity that is a remnant of an additional symmetry which is nothing but the symmetry under transformations (61).

Let us now extend the results to the bosonic sector of $D=11$ supergravity. To do that, we note first that, as soon as the three-index photon field $\hat A^{[3]}$ is taken into account, Eq. (12) following from (10) is replaced with

$$\hat F^{[9]}_{\hat a}=d\hat A^{[8]}_{\hat a}+\hat{*}\hat{\tilde{\mathbb G}}^{[2]}_{\hat a}\equiv\hat{\mathbb F}^{[9]}_{\hat a}+\hat{*}\hat{\tilde{\mathbb G}}^{[2]}_{\hat a}, \quad (69)$$

where $\hat{\tilde{\mathbb G}}^{[2]}_{\hat a}$ is defined by $d\hat{*}\hat{\tilde{\mathbb G}}^{[2]}_{\hat a}=\hat{*}\hat{\tilde{\mathbb J}}^{[1]}_{\hat a}$ with $\hat{*}\hat{\tilde{\mathbb J}}^{[1]}_{\hat a}=\hat{*}\hat{\tilde J}^{[1]}_{\hat a}+\hat M^{[10]}_{\hat a}$. Here, as in Eq. (31), we have introduced the $\hat A^{[3]}$ energy-momentum form $\hat M^{[10]}_{\hat a}$. After such a modification, the reduction ansätze (36), (37) are modified as follows:

$$\hat{\mathbb F}^{a[9]}=e^{\frac{4}{3}\phi}*\mathbf g^{a[1]}+e^{-\frac{1}{12}\phi}(dA^{a[7]}+*\tilde{\mathfrak G}^{a[2]}+$$

$$+\frac{1}{12}*(d\phi\wedge e^a)+e^{-\frac{1}{12}\phi}*\mathbb G^{a[2]})\wedge(dz+A^{[1]}), \quad (70)$$

$$\hat{\mathbb F}^{z[9]}=-e^{\frac{2}{3}\phi}(dA^{[8]}-\frac{3}{4}F^{[8]}\wedge A^{[1]}+\frac{1}{2}B^{[2]}\wedge dB^{[6]}-$$

$$-\frac{1}{4}F^{[6]}\wedge A^{[3]})-e^{\frac{2}{3}\phi}(1+\frac{2}{3})*d\phi+e^{\frac{4}{3}\phi}*\mathbf g^{z[1]}+$$

$$+e^{\frac{2}{3}\phi}(dA^{[7]}+F^{[6]}\wedge B^{[2]}+B^{[2]}\wedge B^{[2]}\wedge dA^{[3]}+$$

$$+e^{-\frac{5}{6}\phi}*\mathbb G^{z[2]})\wedge(dz+A^{[1]}). \quad (71)$$

To present (70) and (71), we have used the reduction rule $\hat{\tilde{\mathbb G}}^{[2]}_{\hat a}=\mathbb G^{[2]}_{\hat a}+\mathbf g^{[1]}_{\hat a}\wedge(dz+A^{[1]})$, $*\tilde{\mathfrak G}^{a[2]}$ is the analog of the form defined previously in (30) and extended with the appropriate contribution from the energy-momentum tensor of RR 3-form $A^{[3]}$ and of NS 2-form $B^{[2]}$. The latter forms come from the reduction of $\hat A^{[3]}$. Finally, $F^{[6]}$ and $F^{[8]}$ are the field strengths dual to the $F^{[4]}=dA^{[3]}-dB^{[2]}\wedge A^{[1]}$ and $F^{[2]}=dA^{[1]}$ (see [26] for their explicit expressions).

Let us end up with the action for the complete duality-symmetric bosonic sector of $D = 11$ supergravity (cf. [23, 26]). The action reads as

$$S = \int\limits_{\mathcal{M}^{11}} \Big[\hat{R}^{\hat{a}\hat{b}} \wedge \hat{\Sigma}_{\hat{a}\hat{b}} + \frac{1}{2}\hat{F}^{[4]} \wedge \hat{*}\hat{F}^{[4]} - -\frac{1}{3}\hat{A}^{[3]} \wedge \hat{F}^{[4]} \wedge \hat{F}^{[4]}\Big] + + \int\limits_{\mathcal{M}^{11}} \frac{1}{2}\left[\hat{v} \wedge \hat{\mathcal{F}}^{\hat{a}[9]} \wedge i_{\hat{v}}\hat{\mathcal{F}}^{\hat{b}[2]}\eta_{\hat{a}\hat{b}} - \hat{v} \wedge \hat{\mathcal{F}}^{[7]} \wedge i_{\hat{v}}\hat{\mathcal{F}}^{[4]}\right] \quad (72)$$

with

$$\hat{\mathcal{F}}^{\hat{a}[2]} = d\hat{e}^{\hat{a}} - \hat{*}(d\hat{A}^{\hat{a}[8]} + \hat{*}\hat{\hat{\mathbb{G}}}^{\hat{a}[2]}), \quad (73)$$

$$\hat{\mathcal{F}}^{\hat{a}[9]} = -\hat{*}\hat{\mathcal{F}}^{\hat{a}[2]}, \quad (74)$$

$$\hat{F}^{[4]} = d\hat{A}^{[3]}, \qquad \hat{F}^{[7]} = d\hat{A}^{[6]} + \hat{A}^{[3]} \wedge \hat{F}^{[4]}, \quad (75)$$

$$\hat{\mathcal{F}}^{[4]} = \hat{F}^{[4]} - \hat{*}\hat{F}^{[7]}, \qquad \hat{\mathcal{F}}^{[7]} = -\hat{*}\hat{\mathcal{F}}^{[4]}. \quad (76)$$

One can verify that, after the dimensional reduction with taking into account the ansätze (70) and (71), action (72) reproduces the following action of the completely duality-symmetric type IIA supergravity [cf. Eq. (69) in [26]]

$$S = -\int\limits_{\mathcal{M}^{10}} \left(R^{ab} \wedge \Sigma_{ab} + \frac{1}{2}v \wedge \mathcal{F}^{a[8]} \wedge i_v\mathcal{F}^{b[2]}\eta_{ab}\right) + +\frac{1}{2}\int\limits_{\mathcal{M}^{10}} \sum_{n=1}^{4}\Big[\frac{1}{3^{[\frac{n+1}{4}]}}F^{[10-n]} \wedge F^{[n]} + i_v\mathcal{F}^{[10-n]} \times \times \wedge v \wedge F^{[n]} + v \wedge F^{[10-n]} \wedge i_v\mathcal{F}^{[n]}\Big]. \quad (77)$$

## 4. Summary

To summarize, we have exploited a similarity between the gravity equation of motion and that of the Maxwell electrodynamics with an electric-type source. The use of this fact allowed us to write down the second-order equation of motion for the vielbein as the first-order Bianchi identity for its dual partner. Knowing the structure of the duality-symmetric action that follows from the reduction of gravity action from $D$ to $D-1$ space-time dimensions, we have deduced the reduction ansatz for the field strength of the "graviton dual" field that reproduces the correct $D-1$-dimensional duality relations between the dilaton and the Kaluza—Klein vector field and their duals. After establishing the ansatz, we have proposed the additional PST-like term entering the action of the duality-symmetric gravity and encoding the duality relation between the graviton and its dual partner, and have verified the relevance of this term by the dimensional reduction from eleven to ten comparing the results of the reduction with those previously obtained in studying the duality-symmetric formulation of type IIA supergravity. Finally, we have extended the ansatz to the bosonic sector of $D = 11$ supergravity and have established, after the reduction, the correct structure of the duality-symmetric type IIA supergravity action. Therefore, we have non-trivially tested the approach by Pasti, Sorokin, and Tonin and have confirmed the relevance of the approach to the construction of the duality-symmetric supergravity actions in diverse dimensions.

In the course of our studying, it has been also shown that the duality relations for the graviton and its dual partner come as equations of motion from the proposed action of the duality-symmetric (super)gravity and that the PST scalar field entering the action is an auxiliary field and does not spoil the field content of the original theory. To demonstrate that we have established the PST symmetries which are characteristic of the PST approach; one of these symmetries has been used to eliminate the auxiliary PST scalar field from a theory. A feature of these transformations consists in dealing with non-local terms. The same concerns the action we proposed. In general, there is no exact local expression for the Landau—Lifshits "pre-current" entering the action, so the action is also constructed out of the non-local terms. Having such terms is a feature of the self-interacting theory, since the double field approach to the Yang—Mills theory also reveals non-locality [36].

Since we have not explicitly studied the fermionic sector here, an extension of the model to involve fermions remains conjectural at this stage, but the experience in the immersion of the PST formalism into a supersymmetric theory lends credence to the conjecture.

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## ДУАЛЬНО-СИМЕТРИЧНА ГРАВІТАЦІЯ І СУПЕРГРАВІТАЦІЯ: ТЕСТ ДЛЯ PST-ПІДХОДУ

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Р е з ю м е

Використовуючи аналогію між динамічними рівняннями гравітаційного поля та рівняннями електродинаміки Максвелла в присутності джерела електричного типу, окреслено шлях для виникнення поля, дуального до поля гравітона. Запропоновано анзац для розмірної редукції напруженості дуального до гравітона поля, що приводить до правильних співвідношень дуальності для решти полів, що виникають за розмірної редукції $D$-вимірної гравітаційної дії до $(D-1)$-вимірної. За допомогою модифікації PST-підходу запропоновано новий доданок, що входить в дію $D = 11$ дуально-симетричної гравітації, і, використовуючи запропонований анзац, продемонстровано актуальність врахування цього доданка для відтворення правильних співвідношень дуальності в зредукованій теорії. Одержані результати узагальнено на бозонний підсектор $D = 11$ супергравітації.