# EFFECT OF NEUTRON EXCESS ON ISOVECTOR DIPOLE RESPONSE OF HEAVY NUCLEI

**V.I. ABROSIMOV, O.I. DAVIDOVSKAYA**

**Institute for Nuclear Research, Nat. Acad. Sci. of Ukraine**
*(47, Nauky Prosp., Kyiv 03028; e-mail: oid@kinr.kiev.ua)*

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Isovector dipole excitations of spherical nuclei with neutron excess are studied within a semiclassical approach based on the Vlasov kinetic equation for finite systems with a moving surface. It is shown that the account of the surface degrees of freedom allows one to exclude spurious excitations related to the center-of-mass motion. It is found that the neutron excess leads to the appearance of a force in the energy range of a dipole pygmy-resonance.

## 1. Introduction

The recent experimental investigations of dipole excitations produced in the inelastic scattering of photons by nuclei with neutron excess [1—3] have revealed isovector dipole excitations in the region of the binding energy of nucleons (5—8.5 MeV for middle and heavy nuclei) that take 3—6 % of the energetically weighted sum rule. These excitations are called dipole pygmy-resonances. The dipole pygmy-resonance is very interesting for theoreticians (see, for example, [4—7]), because this is a new type of the collective motion related to the neutron-proton asymmetry of nuclei. The dipole pygmy-resonance represents the vibrations of the neutron excess with respect to the symmetric core (where the number of neutrons $N$ is equal to the number of protons $Z$), see, for example [3]. In the theoretical study of the dipole pygmy-resonance, the problem of separation of the center-of- mass motion from the internal excitations of the system arises, because, in the asymmetric system, there is a relation between isovector and isoscalar vibrations ([8], see also [9]). In the case of dipole vibrations, this relation can cause the appearance of spurious excitations (caused by the center-of-mass motion) in the region of a dipole pygmy-resonance.

In this work, the isovector dipole excitations in spherical nuclei with neutron excess are studied with the use of the semiclassical approach [9, 10, 11] which is based on the solution of the Vlasov kinetic equation for a finite Fermi-system with moving surface. Within this approach, due to the consistent account of the interrelation between the motion of nucleons and the surface vibrations, we can restore the translational invariance of the system upon dipole vibrations [12]. First, we consider the isovector dipole response function for the neutron-proton asymmetric system with a moving surface. In contrast to work [13], we obtain the analytical expression for the isovector dipole response function using the exact solution of the linearized Vlasov equation with the residual interaction in the separable approximation.

## 2. Response of the Asymmetric System with Movable Surface

The isovector dipole response function of the two-component system bounded by a moving surface is determined by the following expression [9]:

$$\tilde{R}(\omega) = \sum_{q=n,p} \tilde{R}_q(\omega) = \frac{1}{\beta} \sum_{q=n,p} \int d\vec{r} a_q r Y_{10}(\theta) \delta \bar{\varrho}_q(\vec{r}, \omega). \tag{1}$$

Here, $\delta \bar{\varrho}_q(\vec{r}, \omega)$ are the Fourier transforms, with respect to time, of the change in the neutron density at $q = n$ and the proton density at $q = p$ induced by the external field

$$V_q(\vec{r}, t) = \beta \delta(t) a_q r Y_{10}(\theta), \tag{2}$$

where $a_n = 2Z / A$ and $a_p = - 2N / A$. In the semiclassical (kinetic) approach, the particle density changes $\delta \bar{\varrho}_q(\vec{r}, \omega)$ of the spherical system with a moving surface have the form [10]

$$\delta \bar{\varrho}_q(\vec{r}, \omega) = \frac{2}{h^3} \int d\vec{p} \delta \tilde{n}_q(\vec{r}, \vec{p}, \omega) +$$

$$+ \delta(r - R) \rho_q \delta R_q(\omega) Y_{10}(\theta). \tag{3}$$

In this expression, $\rho_q = (A_q/A)\rho_0$ are the equilibrium densities of neutrons ($q = n$) and protons ($q = p$), where $\rho_0$ is the equilibrium density of the nuclear matter, and $\delta R_q(\omega)$ is the change of the equilibrium radius $R$ of the system induced by the external field (2). The variations of the particle distribution functions (neutrons and protons) in the phase space, $\delta\tilde{n}_q(\vec{r},\vec{p},\omega)$, obey the linearized Vlasov equation with the boundary conditions on the moving surface [13].

Substituting expression (3) in Eq. (1), we obtain a response function in the form

$$\tilde{R}(\omega)=\sum_{q=n,p}\left(\tilde{R}_q^v(\omega)+\frac{1}{\beta}a_q\rho_q R^3\delta R_q(\omega)\right), \quad (4)$$

where

$$\tilde{R}_q^v(\omega)=\frac{1}{\beta}\frac{2}{h^3}a_q\int d\vec{r}\,d\vec{p}\,rY_{10}(\theta)\delta\tilde{n}_q(\vec{r},\vec{p},\omega). \quad (5)$$

In order to obtain the explicit expression for the response function (4), it is necessary to find the explicit expressions for functions (5) and collective variables (the vibration amplitudes of the neutron and proton surfaces with respect to the equilibrium radius) $\delta R_q(\omega)$ that are defined by the solutions $\delta\tilde{n}_q(\vec{r},\vec{p},\omega)$ of the linearized Vlasov equation. The explicit solution of the linearized Vlasov equation for a finite system with moving surface can be obtained under assumption that the interaction between nucleons has a separable multipole-multipole form [14]. For the dipole interaction, we have

$$u_{qq'}(\vec{r},\vec{r}\,')=\kappa_{qq'}\sum_M rr'Y_{1M}(\theta,\varphi)Y_{1M}^*(\theta',\varphi'), \quad (6)$$

where $\kappa_{qq'}$ are the interaction parameters. In work [15] with the use of a polarization-related sum rule, the authors obtained the expression for the isovector interaction parameter $\kappa_{q'q'}-\kappa_{qq'}$ in terms of the isovector Landau parameter $F_0'$ for the (neutron-proton) symmetric system. In this work, we study the asymmetric systems with a small asymmetry parameter $(I=(N-Z)/A)<1$). Up to the terms of the order $I^2$ in analogy with work [15], we can obtain the following expressions for the interaction parameters $\kappa_{qq'}$:

$$\kappa_{nn}=\kappa_{pp}=\frac{40\pi}{9A}\frac{\varepsilon_{\rm F}}{R^2}(F_0+F_0'),$$

$$\kappa_{np}=\kappa_{pn}=\frac{40\pi}{9A}\frac{\varepsilon_{\rm F}}{R^2}(F_0-F_0'). \quad (7)$$

Here, $F_0'$ and $F_0$ are the Landau parameters that describe the isovector and isoscalar interactions, respectively, in the symmetric nuclear matter, and $\varepsilon_F$ is the Fermi energy of the nuclear matter.

Using the separable interaction (6), the solutions of the Vlasov kinetic equation $\delta\tilde{n}_q(\vec{r},\vec{p},\omega)$ can be written in the form

$$\delta\tilde{n}_q(\vec{r},\vec{p},\omega)=\delta n_q^0(\vec{r},\vec{p},\omega)\times$$

$$\times\left(1+\sum_{q'=n,p}\kappa_{qq'}\frac{\tilde{R}_{q'}^v(\omega)}{a_qa_{q'}}\right)+\delta\tilde{n}_q^s(\vec{r},\vec{p},\omega). \quad (8)$$

Here, $\delta n_q^0(\vec{r},\vec{p},\omega)$ is the solution of the Vlasov equation for the system of noninteracting neutrons at $q=n$ or protons at $q=p$ bounded by a fixed surface ([10], see also [16]), and $\delta\tilde{n}_q^s(\vec{r},\vec{p},\omega)$ is the additional term of the solution of the Vlasov equation caused by the presence of the moving surface [11].

Let us substitute solution (8) in the equation for the function $\tilde{R}_q^v(\omega)$. With the use of the explicit expression for $\delta\tilde{n}_q^s(\vec{r},\vec{p},\omega)$, we rewrite Eq. (5) as

$$\tilde{R}_q^v(\omega)=R_q^0(\omega)\left(1+\sum_{q'=n,p}\kappa_{qq'}\frac{\tilde{R}_{q'}^v(\omega)}{a_qa_{q'}}\right)-$$

$$-\frac{1}{\beta}[\chi_q^0(\omega)-\chi_q^0(0)]R^2\delta R_q(\omega). \quad (9)$$

Here,

$$\chi_q^0(\omega)=-\frac{a_q}{m\omega^2}\chi_q(\omega), \quad (10)$$

$$\chi_q(\omega)=\frac{3A_q}{4\pi}\frac{m\omega^2}{R^2}-\frac{m^2\omega^4}{R^2}\frac{R_q^0(\omega)}{a_q^2}, \quad (11)$$

$$\chi_q^0(0)=-\frac{3A_q}{4\pi}\frac{a_q}{R^2}, \quad (12)$$

where $A_q=N$ at $q=n$ and $A_q=Z$ at $q=p$. The function $R_q^0(\omega)$ in (9) is the response function of the system of noninteracting neutrons $(q=n)$ or protons $(q=n)$ bounded by a fixed surface ([10], see also Eqs. (18), (20) in [16]). In view of Eq. (9), the function $\tilde{R}_q^v(\omega)$ can be represented in terms of the collective variables $\delta R_q(\omega)$:

$$\tilde{R}_q^v(\omega)=R_q(\omega)-$$

$$-\frac{1}{\beta}\frac{R^2}{1-\kappa_{nn}\left(\frac{R_n^0(\omega)}{a_n^2}+\frac{R_p^0(\omega)}{a_p^2}\right)+(\kappa_{nn}^2-\kappa_{np}^2)\frac{R_n^0(\omega)}{a_n^2}\frac{R_p^0(\omega)}{a_p^2}}\times$$

$$\times\left[\left(1-\kappa_{nn}\frac{R^0_{q'}(\omega)}{a^2_{q'}}\right)[\chi^0_q(\omega)-\chi^0_q(0)]\delta R_q(\omega)+\right.$$

$$\left.+\kappa_{np}\frac{R^0_q(\omega)}{a_n a_p}[\chi^0_{q'}(\omega)-\chi^0_{q'}(0)]\delta R_{q'}(\omega)\right]. \qquad (13)$$

Here, $R_q(\omega)$ is the collective response function of the system of interacting neutrons ($q=n$) or protons ($q=p$) bounded by a fixed surface and has the following form [13]:

$$R_q(\omega)=$$

$$=\frac{R^0_q(\omega)\left(1-\kappa_{nn}\frac{R^0_{q'}(\omega)}{a^2_{q'}}+\kappa_{np}\frac{R^0_{q'}(\omega)}{a_n a_p}\right)}{1-\kappa_{nn}\left(\frac{R^0_n(\omega)}{a^2_n}+\frac{R^0_p(\omega)}{a^2_p}\right)+(\kappa^2_{nn}-\kappa^2_{np})\frac{R^0_n(\omega)}{a^2_n}\frac{R^0_p(\omega)}{a^2_p}}. \qquad (14)$$

The equations for the collective variables $\delta R_q(\omega)$ are defined by the boundary conditions which demand that a change in pressure at the surface caused by the external field (2) be zero (see Eqs. (9) and (10) in [13]). After the substitution of solution (8), these equations acquire the form

$$\delta R_q(\omega)=-\beta\{[k_{iv}-2\chi_+(\omega)+2\tau_q\chi_-(\omega)]X_q(\omega)+$$

$$+\sum_{q'=n,p}(1-\delta_{qq'})k_{iv}X_{q'}(\omega)\}/\{\chi_+(\omega)\times$$

$$\times[k_{iv}-\chi_+(\omega)]+[\chi_-(\omega)]^2\}, \qquad (15)$$

where

$$X_q(\omega)=\chi^0_q(\omega)+[\chi^0_q(\omega)-\chi^0_q(0)]\sum_{q'=n,p}\kappa_{qq'}\frac{\tilde{R}^v_{q'}(\omega)}{a_q a_{q'}}, \qquad (16)$$

$$\chi_\pm(\omega)=\chi_n(\omega)\pm\chi_p(\omega), \qquad (17)$$

$k^{iv}=\frac{Q}{\pi r_0^4}$ is the isovector hardness parameter [9], $Q$ is the coefficient of elasticity of the neutron skin [17], $\tau_q=1$ at $q=n$, and $\tau_q=-1$ at $q=p$. Equations (13) and (15) constitute the system of algebraic equations for the functions $\tilde{R}^v_q(\omega)$ and the collective variables $\delta R_q(\omega)$. The solution of this system is given in Appendix 1.

Using expression (13) for the function $\tilde{R}^v_q(\omega)$, we present the response function (4) as

$$\tilde{R}(\omega)=R(\omega)+\tilde{S}(\omega). \qquad (18)$$

Here, $R(\omega)=\sum_{q=n,p}R_q(\omega)$ is defined by formula (14), and the function $\tilde{S}(\omega)$ that describes the effects caused by the moving surface takes the form

$$\tilde{S}(\omega)=-\frac{1}{\beta}\times$$

$$\times\frac{R^2}{1-\kappa_{nn}\left(\frac{R^0_n(\omega)}{a^2_n}+\frac{R^0_p(\omega)}{a^2_p}\right)+(\kappa^2_{nn}-\kappa^2_{np})\frac{R^0_n(\omega)}{a^2_n}\frac{R^0_p(\omega)}{a^2_p}}\times$$

$$\times\sum_{\substack{q=n,p\\ q\neq q'}}\left[\chi^0_q(\omega)\left(1-\kappa_{nn}\frac{R^0_{q'}(\omega)}{a^2_{q'}}+\kappa_{np}\frac{R^0_{q'}(\omega)}{a_n a_p}\right)-\right.$$

$$-\chi^0_q(0)\left(\kappa_{nn}\frac{R^0_q(\omega)}{a^2_q}+\kappa_{np}\frac{R^0_{q'}(\omega)}{a_n a_p}-\right.$$

$$\left.\left.-(\kappa^2_{nn}-\kappa^2_{np})\frac{R^0_n(\omega)}{a^2_n}\frac{R^0_p(\omega)}{a^2_p}\right)\right]\delta R_q(\omega). \qquad (19)$$

The function $\tilde{S}(\omega)$ is proportional to the vibration amplitudes $\delta R_q(\omega)$ which have new poles in comparison to the response function $R(\omega)$. Moreover, the consistent interrelation between the vibration amplitudes $\delta R_q(\omega)$ and the variations of the nucleon density in the phase space which determines the response function $R(\omega)$ ensures the absence of spurious excitations.

## 3. Center-of-Mass Motion

It is known [8] that, in neutron-proton asymmetric nuclei, there is a relation between isovector and isoscalar excitations. Upon the study of the isovector dipole response of asymmetric nuclei to the external field (2), the account of this relation can lead to the spurious excitations caused by the center-of-mass motion.

In order to investigate this problem in our quasiclassical approach, we consider a displacement (response) of the center-of-mass induced by the external field (2) which is determined by the relation

$$\delta\tilde{R}_{\mathrm{c.m}}(\omega)=\int d\vec{r}\,rY_{10}(\theta,\varphi)\delta\bar{\varrho}(\vec{r},\omega), \qquad (20)$$

where $\delta\bar{\varrho}(\vec{r},\omega) = \sum_{q=n,p} \delta\bar{\varrho}_q(\vec{r},\omega)$, see (3). The displacement of the center-of-mass (20) can be represented in terms of the response function (1) as

$$\delta\tilde{R}_{\mathrm{c.m}}(\omega) = \beta \sum_{q=n,p} \frac{\tilde{R}_q(\omega)}{a_q}. \qquad (21)$$

With the use of (18), it can be written as

$$\delta\tilde{R}_{\mathrm{c.m}}(\omega) = \delta R_{\mathrm{c.m}}(\omega) + \delta S_{\mathrm{c.m}}(\omega), \qquad (22)$$

where the center-of-mass displacement for the system bounded by a fixed surface

$$\delta R_{\mathrm{c.m}}(\omega) = \beta \sum_{q=n,p} \frac{R_q(\omega)}{a_q}, \qquad (23)$$

and

$$\delta S_{\mathrm{c.m}}(\omega) = \beta \sum_{q=n,p} \frac{\tilde{S}_q(\omega)}{a_q}. \qquad (24)$$

Substituting the expression for the response function $R_q(\omega)$ [see (14)] in (23), we obtain

$$\delta R_{\mathrm{c.m}}(\omega) = = \beta \frac{\sum\limits_{\substack{q=n,p \\ q\neq q'}} \frac{R_q^0(\omega)}{a_q}\left(1 - \kappa_{nn}\frac{R_{q'}^0(\omega)}{a_{q'}^2} + \kappa_{np}\frac{R_{q'}^0(\omega)}{a_n a_p}\right)}{1 - \kappa_{nn}\left(\frac{R_n^0(\omega)}{a_n^2} + \frac{R_p^0(\omega)}{a_p^2}\right) + (\kappa_{nn}^2 - \kappa_{np}^2)\frac{R_n^0(\omega)}{a_n^2}\frac{R_p^0(\omega)}{a_p^2}}. \qquad (25)$$

It is seen from (25) that, in the symmetric ($N = Z$) system bounded by a fixed surface, the external field (2) does not cause the center-of-mass displacement [$\delta R_{\mathrm{c.m}}(\omega) = 0$]. However, in the case of the asymmetric system, we have $\delta R_{\mathrm{c.m}}(\omega) \neq 0$, and the isovector dipole response of this system exhibits spurious excitations.

Substituting (19) in (24), using the explicit form for the vibration amplitudes $\delta R_q(\omega)$ (see Appendix 1), and performing some transformations, we obtain (see Appendix 2)

$$\delta S_{\mathrm{c.m}}(\omega) = -\delta R_{\mathrm{c.m}}(\omega). \qquad (26)$$

Therefore, we obtain [see (22)

$$\delta\tilde{R}_{\mathrm{c.m}}(\omega) = 0. \qquad (27)$$

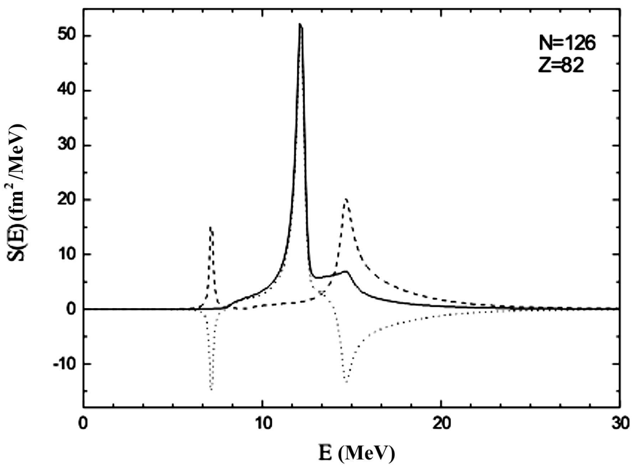


Fig. 1. Force function (28) for the isovector dipole excitations of the asymmetric system with $N$=126 neutrons and $Z$=82 protons: the solid line corresponds to the system with moving surface (18), and the dashed line to the system with fixed surface (14). The dotted line shows the imaginary part of the function $\tilde{S}(\omega)$ (19)

Thus, in the asymmetric system bounded by a moving surface, the external field (2) causes no center-of-mass displacement, and, therefore, the isovector dipole excitations of this system do not contain spurious excitations related to the center-of-mass motion.

The quantitative evaluation of the spurious phenomena is presented in Fig. 1. There, we show the results of numerical calculations of the force function that is determined by the imaginary part of the response function in the form

$$S(E) = -\frac{1}{\pi}\mathrm{Im}\tilde{R}(E), \qquad (28)$$

where $E = \hbar\omega$. The dashed line in Fig. 1 shows the force function of the system bounded by a fixed surface [see (14) and (23)]. It can be seen that the force distribution has, apart from the giant resonance with a maximum at 14.7 MeV, a resonance structure in the low-energy region which is caused by the center-of-mass motion. The position of the maximum of this force is determined by the isoscalar parameter $F_0$. The solid line in Fig. 1 displays the force function of the system bounded by a moving surface [see (18) and (22)]. In order to represent the effects related to the dynamical surface, the dotted line in Fig. 1 shows the imaginary part (28) of the function $\tilde{S}(\omega)$ [see (19) and (24)]. It is seen that the surface effects compensate the force related to the spurious excitations.

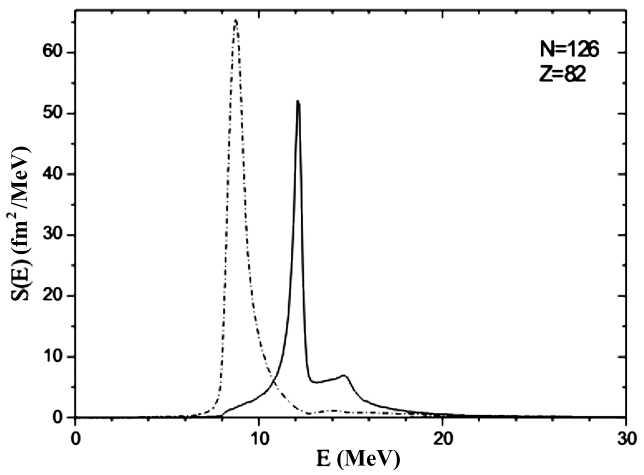


Fig. 2. Force function (28) for the isovector dipole excitations of the asymmetric system with a moving surface in two approximations: the dash-dotted line corresponds to the system with the residual interaction neglected [see (28) at $\kappa_{qq'} = 0$, and the solid line corresponds to the system with the residual interaction

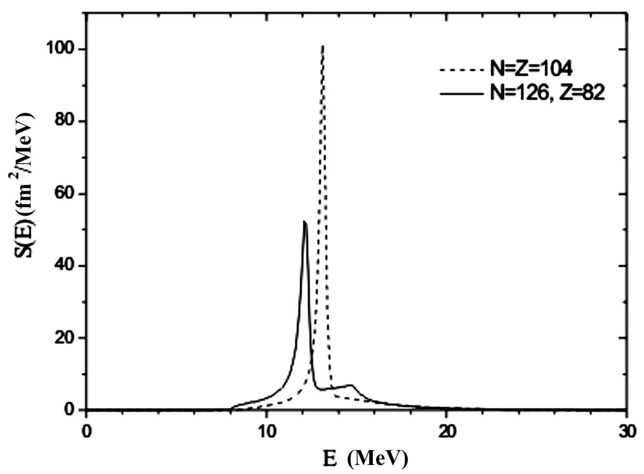


Fig. 3. Force function (28) for the isovector dipole excitations of the system with a moving surface for the symmetric ($N = Z = 104$) system (dashed line) and asymmetric ($N = 126$, $Z = 82$) one (solid line)

The numerical calculations of the force function were carried out for the asymmetric system with the number of neutrons $N$=126 and that of protons $Z$=82. In these calculations, we used the parameters of the nucleus $r_0$ =1.12 fm, $\varepsilon_{\rm F}$ = 40 MeV$\times(10^{-22}$ s$)^2$/fm$^2$, and $m$ =1.04 MeV and the parameters $Q$ = 75 MeV, $F_0'$ =1.25, and $F_0 = -0.42$ [18].

## 4. Neutron Excess Effects in the Force Distribution

Let us apply the response function (18) to the investigation of isovector dipole excitations in a spherical neutron-proton asymmetric system. For this purpose, we use the force function (28).

Fig. 2 shows the force function for the isovector dipole vibrations of the asymmetric system that corresponds to a Pb$^{208}$ nucleus in two approximations. The dash-dotted line shows the force function for the system bounded by a moving surface without taking into account the residual interaction [see (18) at $\kappa_{qq'} = 0$]. In this approximation, we observe the giant resonance with a maximum at an energy of 9 MeV. The account of the residual interaction shifts the giant resonance to higher energies with a maximum at 12.2 MeV (the solid line in Fig. 2). This resonance reproduces the properties of the giant dipole resonance for a Pb$^{208}$ nucleus.

In order to present the influence of the neutron excess on the dipole response of spherical nuclei, we show the force functions for the symmetric system ($N = Z = 104$) and the asymmetric one that corresponds to a Pb$^{208}$ nucleus ($N = 126$ and $Z = 82$) in Fig. 3. It is seen that the neutron excess slightly shifts the position of the giant resonance, but also creates the dipole excitations in the region of a dipole pygmy-resonance (8—10 MeV). In addition, there is a resonance structure in the energy region above the giant resonance. However, the force distribution in the region of the dipole pygmy-resonance does not show the resonance behavior. This may be related to the fact that, in the considered approximation, we do not take into account the effects of the neutron skin in the ground state (the equilibrium radii of the neutron and proton surfaces are assumed to be the same). Comparing the force functions of the asymmetric system (the solid lines in Figs. 1—3), symmetric system (the dashed line in Fig. 3), and the asymmetric system with a fixed surface (the dashed line in Fig. 1), it can be seen that the resonance structure in the energy region above the giant resonance is due to both the neutron excess and the dynamic surface effects. It is known that a similar resonance structure was obtained in the quantum calculations (see, for example, [19]) and in the semiclassical approach based on the Landau—Vlasov equation with hydrodynamic boundary conditions [20].

## 5. Conclusions

Within the semiclassical approach based on the kinetic Vlasov equation, the analytical expression for the isovector dipole response function is obtained for a neutron-proton asymmetric system. Taking into account the dynamic surface effects, we can exclude the spurious excitations related to the center-of-mass motion. The neutron excess leads to the dipole excitations in the region of the binding energy of nucleons in heavy nuclei, but we cannot obtain, within the considered approximation, the resonant structure corresponding to the dipole pygmy-resonance. It can be expected that the account of properties of the neutron skin of a nucleus in the ground state with neutron excess will reproduce the dipole pygmy-resonance within our semiclassical approach.

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### APPENDIX 1

Solving the system of algebraic equations (13) and (15), we find the expression for the collective variables $\delta R_q(\omega)$ in the form

$$\delta R_q(\omega) = \frac{d_{q'} c_q - b_q c_{q'}}{d_n d_p - b_n b_p}, \tag{A1.1}$$

where $q \neq q'$,

$$d_q = \quad -\chi_+(k_{iv} - \chi_+) - \chi_-^2 \quad + \frac{\alpha_q(\omega) R^2}{a_q z} \times$$

$$\times \left\{ \; k_{iv} - 4\chi_{q'} \;\; \frac{\alpha_q(\omega)}{a_q} \quad \kappa_{nn} - \frac{R^0_{q'}(\omega)}{a^2_{q'}} (\kappa^2_{nn} - \kappa^2_{np}) \right) +$$

$$+ k_{iv} \frac{\alpha_{q'}(\omega)}{a_{q'}} \kappa_{np} \Bigg\},$$

$$b_q = \frac{\alpha_{q'}(\omega) R^2}{a_{q'} z} \Bigg\{ \; k_{iv} - 4\chi_{q'} \;\; \frac{\alpha_q(\omega)}{a_q} \kappa_{np} + k_{iv} \frac{\alpha_{q'}(\omega)}{a_{q'}} \times$$

$$\times \quad \kappa_{nn} - \frac{R^0_q(\omega)}{a^2_q} (\kappa^2_{nn} - \kappa^2_{np}) \Bigg) \Bigg\},$$

$$c_q = \;\; k_{iv} - 4\chi_{q'} \;\; f_q(\omega) + k_{iv} f_{q'}(\omega),$$

$$f_q(\omega) = \left\{ \chi^0_q(\omega) + \;\; \chi^0_q(\omega) - \chi^0_q(0) \quad\; \kappa_{nn} \frac{R_q(\omega)}{a^2_q} + \kappa_{np} \frac{R_{q'}(\omega)}{a_q a_{q'}} \right) \Bigg\},$$

$$\alpha_q(\omega) = \chi^0_q(\omega) - \chi^0_q(0),$$

$$z = 1 - \kappa_{nn} \quad \frac{R^0_n(\omega)}{a^2_n} + \frac{R^0_p(\omega)}{a^2_p} \Bigg) + (\kappa^2_{nn} - \kappa^2_{np}) \frac{R^0_n(\omega)}{a^2_n} \frac{R^0_p(\omega)}{a^2_p}.$$

Substituting the expression for $\delta R_q(\omega)$ [see (A1.1)] to Eq. (13), we obtain the explicit expression for the functions $\tilde{R}^v_q(\omega)$.

### APPENDIX 2

To simplify the calculations, we consider the center-of-mass displacement for a Fermi-system bounded by the moving surface without regard for interactions within the system volume, i.e. by setting $\kappa_{qq'} = 0$. Then the expression for the center-of-mass displacement of the system bounded by a fixed surface [see (25)] takes the form

$$\delta R_{\rm c.m}(\omega) = \beta \sum_{q=n,p} \frac{R^0_q(\omega)}{a_q}. \tag{A2.1}$$

Substituting (19) in (24) and putting $\kappa_{qq'} = 0$, we obtain

$$\delta S_{\rm c.m}(\omega) = -R^2 \sum_{q=n,p} \frac{\chi^0_q(\omega)}{a_q} \delta R_q(\omega). \tag{A2.2}$$

Substituting the explicit expression for $\delta R_q(\omega)$ [see (15) at $\kappa_{qq'} = 0$] in (A2.2), we get

$$\delta S_{\rm c.m}(\omega) = \beta \frac{R^2}{\chi_+(\omega)[k_{iv} - \chi_+(\omega)] + [\chi_-(\omega)]^2} \times$$

$$\times \sum_{q=n,p} \frac{\chi^0_q(\omega)}{a_q} \Bigg\{ [k_{iv} - 2\chi_+(\omega) + 2\tau_q \chi_-(\omega)] \chi^0_q(\omega) +$$

$$+ \sum_{q'=n,p} (1 - \delta_{qq'}) k_{iv} \chi^0_{q'}(\omega) \Bigg\}, \tag{A2.3}$$

Taking into account (17) and (10), we obtain, after some transformations,

$$\delta S_{\rm c.m}(\omega) = -\beta \frac{R^2}{m\omega^2} \sum_{q=n,p} \chi^0_q(\omega). \tag{A2.4}$$

Using (10) and (11), we obtain

$$\delta S_{\rm c.m}(\omega) = \beta \sum_{q=n,p} \quad \frac{3A_q a_q}{4\pi m\omega^2} - \frac{R^0_q(\omega)}{a_q} \Bigg). \tag{A2.5}$$

Since $A_n = N$, $A_p = Z$, $a_n = 2Z/A$, and $a_p = -2N/A$, we have $\sum\limits_{q=n,p} \frac{3A_q a_q}{4\pi m\omega^2} = \frac{3NZ}{2A\pi m\omega^2} - \frac{3ZN}{2A\pi m\omega^2} = 0$. Thus, we obtain

$$\delta S_{\rm c.m}(\omega) = -\beta \sum_{q=n,p} \frac{R^0_q(\omega)}{a_q}. \tag{A2.6}$$

Comparing the expressions (A2.1) and (A2.6), we obtain Eq. (26).

In the same manner, we obtain Eq. (26) with regard for interactions in the system volume.

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## ВПЛИВ НАДЛИШКУ НЕЙТРОНІВ НА ІЗОВЕКТОРНИЙ ДИПОЛЬНИЙ ВІДГУК ВАЖКИХ ЯДЕР

*В.І. Абросімов, О.І. Давидовська*

Р е з ю м е

Ізовекторні дипольні збудження сферичних ядер із надлишком нейтронів досліджено в рамках напівкласичного підходу, що спирається на кінетичне рівняння Власова для скінченних систем з рухливою поверхнею. Показано, що врахування поверхневих ступенів вільності дозволяє виключити духові збудження, що пов'язані з рухом центра мас. Знайдено, що врахування надлишку нейтронів приводить до появи сили в області енергій дипольного пігмі-резонансу.