ENERGY DEPENDENCE OF THE NN-AMPLITUDE PARAMETERS AND THE CALCULATION OF THE EXCITATION FUNCTION OF THE ^{4}He(p,d)^{3}He REACTION IN THE MANY-CENTER EIKONAL APPROXIMATION

V.V. DAVYDOVSKYY, A.D. FOURSAT

Institute for Nuclear Research, Nat. Acad. Sci. of Ukraine
(47, Nauky Prosp., Kyiv 03028, Ukraine; e-mail: odavi@kinr.kiev.ua)

UDC 539.17

Using a non-local separable potential of the second rank with the Gaussian form factors, the energy dependence of the parameters of NN-amplitudes in the Glauber—Sitenko theory of multiple scattering is found. On this basis, the excitation functions for the ^{4}He(p,d)^{3}He reaction are calculated for two angles of the deuteron escape: $\theta_{\rm c.m} = 0$ and $34.5°$. It is shown that, even without the use of exotic mechanisms involving the pion and isobar productions, the multiple scattering theory agrees with experimental data in the energy interval from 100 to 1000 MeV.

1. Introduction

In the present time, the elastic and inelastic scatterings of nucleons by nuclei at average and high energies are described with the use of the diffraction (eikonal) approximation. Diffraction phenomena take place when the wavelength of the relative motion of the particles is small in comparison to the interaction region size. For nucleons, the diffractive nature of the nuclear interaction is revealed during the scattering by average and heavy nuclei at energies of several tens of MeV. At these energies, the free path inside the nucleus is small in comparison to its dimensions and therefore the nucleus can be considered as an absorbing body (macroscopic diffraction). At higher energies of the order of several hundreds of MeV, when the free path length in the nucleus exceeds the nucleus dimensions and the wavelength of the incident nucleon is less than the nucleon-nucleon interaction radius, the scattering by a nucleus can be considered as the multiple diffractive scattering by individual nucleons (microscopic diffraction). In this case, the Glauber—Sitenko (GS) diffraction theory of multiple scattering (DTMS) [1] allows us to obtain the interaction amplitude of a nucleon with a nucleus in terms of the amplitudes of scattering by the individual nucleons and the nuclear form factors. As a result of the interference of the amplitudes related to different multiplicities of scattering, the angular dependence of the cross-section has a characteristic structure containing a series of maxima and minima. Due to this interference, the interaction processes become sensitive to the spatial structure of nuclei, and this opens up a wide prospect to their investigation.

As the binding energy of nucleons in a nucleus is small in comparison to the energy of the incident particle, the effect of the binding of nucleons can be neglected. In this case, the amplitude of scattering by nucleons inside a nucleus can be replaced by that by free nucleons.

During the collisions of high-energy nucleons and complex particles with nuclei, in addition to the elastic and inelastic scatterings, one can observe other complicated processes that are accompanied by the redistribution of particles in the colliding systems. One of the examples is the reaction of nucleon transfer, when one or few nucleons are transferred from one system to another. The simplest example of the inelastic process with the redistribution of particles is the reactions of stripping (d,p) and capture (p,d). These reactions are used for a long time at low energies as a spectroscopic tool for investigations of nuclear states.

The capture reactions A(p,d)B at intermediate energies are characterized by a large transferred momentum. These reactions are similar to the reactions (π^+,p) and (γ,p), where also one nucleon is removed from the nucleus, and the nucleus obtains a large momentum. It it natural to expect that the (p,d) reactions at high energies may provide a new information about the wave functions of a nucleus, in particular, about their high-momentum components. Thus, these reactions may be a promising tool for the determination of details of the nucleus structure. Moreover, these reactions at high energies seem to be more sensitive to the D-wave of the deuteron wave function than to the S-wave as compared to the case of low energies. The dominance of the D-state indicates the necessity of using the exact deuteron wave function.

The dramatic situation occurred during the attempt to describe the angular distribution of deuterons in the reaction ^{4}He(p,d)^{3}He at 770 MeV [2]. As was shown in the detailed investigations [2,3], the standard DWBA method was not able to describe even qualitatively the experimental data. This indicates that the mechanism of the (p,d) reaction at energies ∼700—800 MeV is not well understood. In particular, the behavior of the reaction excitation function [4,5] suggests that the simple pole mechanism of the nucleon capture is not dominant in this energy range even for the lightest nuclei. Therefore, a number of other reaction mechanisms was proposed that include the participation of intermediate π-mesons [5, 6]. The introduction of intermediate pions makes it possible to distribute the large transferred momentum among many nucleons in the bound nucleus system. It should be noted that the same effect can be obtained with the use of the mechanism of multiple scattering, which will be shown later in this paper.

The mechanisms involving π-mesons and the mechanism based on the triangular pd-diagram [4] allow us to explain qualitatively the behavior of the reaction excitation function. However, for the quantitative calculation of the corresponding cross-sections, one needs to know the amplitudes and vertices of the processes with the π-meson participation which are not studied well enough. Obviously, it is more natural to use, at high energies, the approach based on GS DTMS. In work [16], the theory of reactions with one-nucleon capture at high energies was developed in the many-center eikonal approximation [7] which proved to be useful for the quantitative explanation of the angular distributions of deuterons created in the reaction ^{4}He(p,d)^{3}He at 770 MeV. Earlier, the same reaction was investigated in work [8] with the use of the method of eigenstates which is widely used for studying the diffractive dissociation of hadrons and the processes of meson production. It was revealed that these two different approaches give equally good descriptions of experimental data without any additional fitting parameters. The present work is the continuation of work [16]. The aim of this work is the calculation of the energy dependence of parameters of the elementary NN-amplitudes and the explanation of the excitation function of the reaction ^{4}He(p,d)^{3}He within the developed formalism.

The proposed method of analysis of the excitation function of the nucleon capture reaction is, in essence, the DWBA method, in which the distorted waves are constructed in the eikonal approximation with regard for a multi-nucleon structure of nuclei. The distorted waves are expressed through the free amplitude of NN-scattering in the same manner as the many-nucleon T-operator of scattering is constructed in the GS theory. Within this approach, in contrast to [9], the expressions that relate the distorted waves to the free amplitude of NN-scattering, take into account more correctly the longitudinal component of the transferred momentum, which is important in the case of reactions.

2. Energy Dependence of Parameters of Nucleon-nucleon Scattering Amplitude

Within the diffractive approximation, the NN-scattering amplitude in the center-of-mass system of two nucleons is related to the profile function $\omega(\mathbf{s})$, in which a two-dimensional vector $\mathbf{s}$ has meaning of a target parameter according to the formula

$$f(\mathbf{q}) = \frac{ik}{2\pi} \int d^2s\ \exp(i\mathbf{q}\cdot\mathbf{s})\omega(\mathbf{s}). \quad (1)$$

Here, the integration is carried out in the plane perpendicular to the direction of the relative momentum $\mathbf{k}$. The transferred momentum is $\mathbf{q} = \mathbf{k} - \mathbf{k}'$. The function $\omega(\mathbf{s})$ contains the information on the spatial structure of the scattering center.

In this work, the NN-scattering amplitude is parametrized by the following T-invariant method:

$$f(\mathbf{q}) = \frac{k\sigma}{4\pi}(i+\rho)\exp(-aq_\perp^2 - bq_z^2), \quad q_\perp = k\sin\theta,$$

$$q_z = k(1-\cos\theta), \quad (2)$$

where $q_\perp$ and q_z are the transverse and longitudinal (with respect to $\mathbf{k}$) components of the transferred momentum. It should be underlined that the value q_z

satisfies exactly the momentum conservation in the NN-collision. In the case $b < a$, this amplitude can describe the rise of the cross section for backscattering angles. Indeed, the minimum of the absolute value of (2) is reached at the scattering angle θ^*:

$$\cos\theta^* = -\frac{\varepsilon}{1-\varepsilon}, \quad \varepsilon = \frac{b}{a}.$$

In the case $0 \le \varepsilon \le 1/2$, this angle is located in the physical regeon θ^*, which agrees qualitatively with the experiment. At $b = 0$, expression (2) corresponds to the standard parametrization of the amplitude in the diffractive approximation (1).

In order to obtain the parameters σ, ρ, and a of the amplitudes of pp- and pn-scattering, we used a model non-local separable potential of the second rank with Gaussian form factors [10] which is in a good agreement with the data on nucleon-nucleon scattering.

The NN-scattering amplitude, the differential cross-section, and other scattering characteristics were found with the use of the antisymmetric two-nucleon t-matrix:

$$\langle \mathbf{p}'|t(k)|\mathbf{p}\rangle = \sum_{\alpha l l' M} i^{l-l'}[1-(-1)^{l'+S+T}]\times$$

$$\times Y^{jM}_{l'S}(\hat{\mathbf{p}}')t^{\alpha}_{l'l}(p',p;k)Y^{jM^+}_{lS}(\hat{\mathbf{p}})P_T, \tag{3}$$

where $\alpha = \{jST\}$, P_T is the projection operator to the state with isospin T, and $Y^{jM}_{lS}(\hat{\mathbf{p}})$ is the spin-angle function. The partial t-matrix is derived from the solution of the Lippmann—Schwinger integral equation

$$t^{\alpha}_{l'l}(p',p;k) = V^{\alpha}_{l'l}(p',p)+$$

$$+\sum_{\lambda=j-1}^{j+1}\int_0^{\infty} q^2\,dq\,\frac{V^{\alpha}_{l'\lambda}(p',q)t^{\alpha}_{\lambda l}(q,p;k)}{\varepsilon(k)-\varepsilon(q)+i\eta}. \tag{4}$$

Here,

$$V^{\alpha}_{l'l}(p',p) = C^{\alpha}_{1l'l}g^{\alpha}_{1l'}(p')g^{\alpha}_{1l}(p) + C^{\alpha}_{2l'l}g^{\alpha}_{2l'}(p')g^{\alpha}_{2l}(p)\,;$$

$$g^{\alpha}_{il}(p) = \frac{p^l}{(2a^2_{il})^{l+3/2}}\exp\left(-\frac{p^2}{4a^2_{il}}\right), \quad i=1,2.$$

For the states that are not mixed by the NN-forces, the summation over λ is absent. In this case, Eq. (4) contains only diagonal elements. For the potential [10], the solution of Eq. (4) can be found in the analytical form. In the general case, the NN-scattering amplitude has a complicated spin structure, and the scalar (spin independent) part of the amplitude becomes dominant only at high energies ($\sim$ 1 GeV). Therefore, parametrization (2) should be considered effective in a sense that both Eqs. (2) and (3) lead to the same differential and total cross-sections.

At the energies lower than the threshold of the inelastic processes of meson production, the total cross-section can be found by integration of the differential cross-section of elastic scattering:

$$\sigma = 2\pi\zeta\int_0^{\pi}(d\sigma/d\Omega)_{\mathrm{c.m}}\sin\theta\,d\theta\,, \quad \zeta = \begin{cases} 1/2 & \text{for pp} \\ 1 & \text{for pn}\,. \end{cases} \tag{5}$$

The factor $1/2$ for the *pp*-scattering is due to the identity of the two protons. With the use of the optical theorem, we also obtained the following expressions for the total cross-sections of pp- and pn-scatterings:

$$\sigma_{pp} = -\frac{1}{4}(2\pi)^3\frac{M}{k}\mathrm{Im}\left(2t^{11}_{11}+t^{11}_{00}+t^{01}_{00}\right),$$

$$\sigma_{pn} = -\frac{1}{4}(2\pi)^3\frac{M}{k}\times$$

$$\times\mathrm{Im}\left[t^{11}_{11}+t^{10}_{11}+\frac{1}{2}\left(t^{11}_{00}+t^{10}_{00}\right)+\frac{1}{2}\left(t^{01}_{00}+t^{00}_{00}\right)\right]. \tag{6}$$

Here, M in the nucleon mass, and $t^{S\,T}_{M'_S M_S}$ is the matrix element of the t-operator in the state with spin S and isospin T calculated between states with the spin projections M'_S and M_S and for the scattering angle $\theta = 0$. As was shown by numerical calculations, the cross-sections obtained from formulas (5) and (6) are exactly the same for the whole energy range under consideration.

The parameter $\rho = \mathrm{Re}\,f(0)/\mathrm{Im}\,f(0)$ was found with the use of the NN-potential and from the value of the elastic cross-section at the angle $\theta = 0$ from the expression derived from (2):

$$\rho^2 = \left(\frac{4\pi}{k\sigma}\right)^2\sigma(0)-1, \quad \sigma(0) = |f(0)|^2. \tag{7}$$

It follows from the calculations based on the dispersion relation [11] that ρ can change the sign at high energies. Also, the form parameter a was found by the integration of the differential cross-section as

$$\sigma(\theta) = \left(\frac{k\sigma}{4\pi}\right)^2(1+\rho^2)\exp(-2aq^2_{\perp}-2bq^2_z). \tag{8}$$

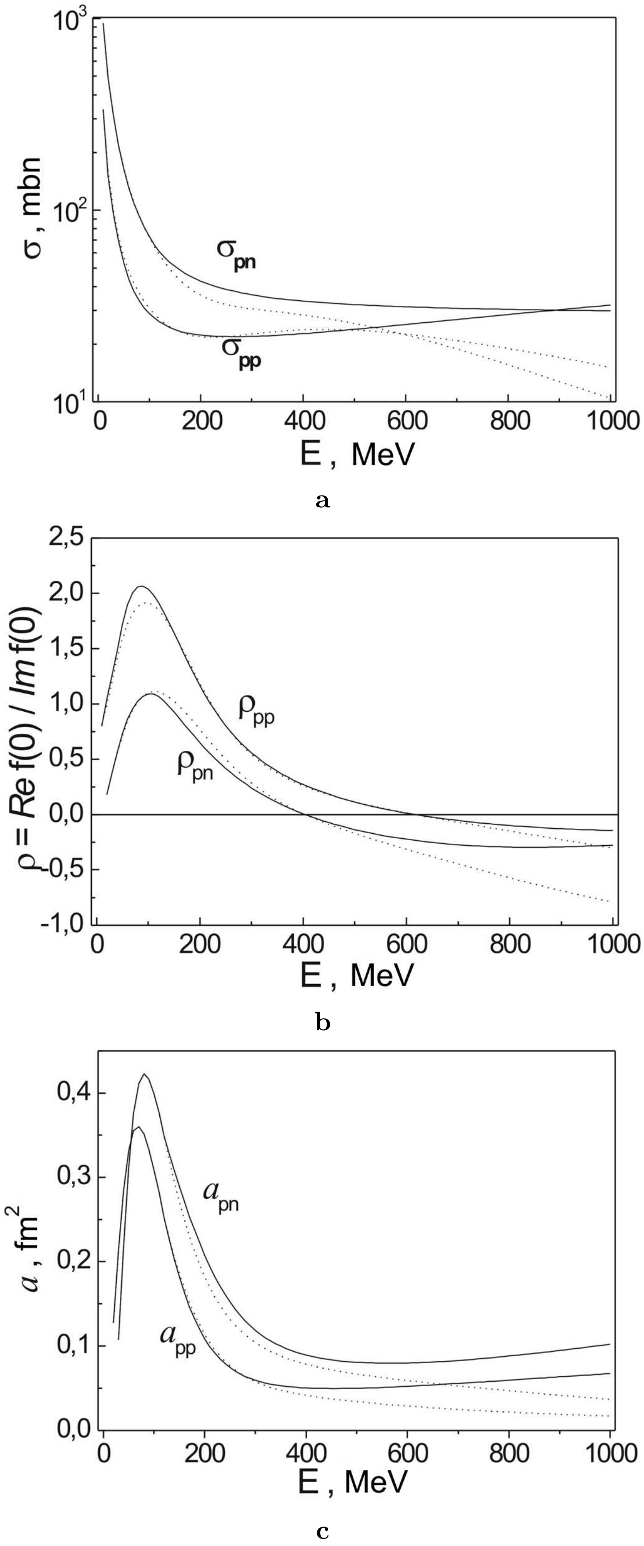


Fig. 1. Energy dependences of NN-amplitude parameters for pp- and pn-scatterings

As a result, at a given ratio $\varepsilon = b/a$, we obtain the following equation for a:

$$\frac{8\pi(1-\varepsilon)u}{k^2\sigma\,\zeta(1+\rho^2)} = D(u) - \exp[-4\varepsilon(1-\varepsilon)u^2]\,D[(2\varepsilon-1)u],$$

$$u = k\sqrt{\frac{2a}{1-\varepsilon}}, \qquad (9)$$

that was solved numerically. In this equation, $D(x)$ is the Dawson integral:

$$D(x) = \exp(-x^2)\int\limits_0^x \exp(t^2)\,dt.$$

Starting from the energy $E \sim 120$ MeV, we use the formula for a that holds in the diffraction limit:

$$a = \frac{\zeta}{16\pi}\sigma\,(1+\rho^2). \qquad (10)$$

It can be obtained by the integration of (8) over the two-dimensional vector $\mathbf{q}_\perp$ taking into account the expression $d\Omega = d^2q_\perp/k^2$ for the solid angle. The sewing of solutions (9) and (10) was smooth, without additional normalizing factors.

Figure 1 shows the parameters of amplitude (2) versus the incident proton energy in the laboratory coordinate system for the most typical case b =0. The solid lines represent the empirical total cross-sections σ [12]. The solid lines in the figures for ρ and a show the dependences, for which we used experimental values of the total cross-sections σ. The dashed lines in all the figures show the dependences that are derived with the use of the NN-potential. At high energies, the model potential systematically underestimates the total cross-sections in the case of open inelastic channels which are related to the pion production at energies higher than $\sim$ 600 MeV in the laboratory system.

The energy dependences of parameters of the NN-amplitudes obtained in this work are of great practical importance because the existing numerical calculations of the cross-sections of various nuclear processes initiated by protons, which are carried out within the DTMS and its generalization, require the knowledge of these parameters in the broad energy range. In addition, the available data on the parameters ρ and a are scarce and ambiguous.

3. Eikonal Model of the Capture Processes and the Reaction ^{4}He(p,d)^{3}He

The detailed theory of the nucleon capture at high energies is presented in work [16]. Here, we present only the main formulas and explain the notations.

Consider the process of capture of a neutron by a proton in the nuclear reaction A(p,d)B. Suppose that the incident proton has the coordinate $\mathbf{r}_p$ and captures the neutron with the coordinate $\mathbf{r}_n$ from the nucleus A. This neutron binds to the proton and creates a deuteron, while the remaining nucleus $B = A - 1$ can be in the ground or excited state. The transition matrix element is written in the form

$$\Im_{f\,i} = (\psi_f^{(-)}, V_{n\,p}\varphi_i^{(+)}), \tag{11}$$

where $V_{n\,p}$ is the operator of the proton-neutron interaction, and the distorted wave functions for the input and output channels have the following forms:

$$\varphi_i^{(+)} = \chi_i \prod_{j=1}^{B} \left[1 - \omega^{(+)}(\mathbf{r}_p - \mathbf{r}_j)\right] \quad ,$$

$$\psi_f^{(-)*} = \chi_f^* \prod_{j=1}^{B} \left[1 - \omega^{(-)}(\mathbf{r}_p - \mathbf{r}_j)\right] \left[1 - \omega^{(-)}(\mathbf{r}_n - \mathbf{r}_j)\right], \tag{12}$$

where the channel wave functions are

$$\chi_i = (2\pi)^{-3} \exp(i\mathbf{k}_p\mathbf{r}_p + i\mathbf{k}_A\mathbf{R}_A)\, \Phi_{J_AM_A}(\mathbf{r}_n, \xi)\chi_{\frac{1}{2}\mu_p},$$

$$\chi_f = (2\pi)^{-3} \exp(i\mathbf{k}_d\mathbf{R}_d + i\mathbf{k}_B\mathbf{R}_B)\times$$

$$\times\Phi_{J_dM_d}(\mathbf{r}_p, \mathbf{r}_n)\Phi_{J_BM_B}(\xi),$$

and $\omega^{(\pm)}$ are generalized nucleon-nucleon profile functions [7]. In the case of the parametrization of the elementary NN-scattering amplitude in the form (2), we obtain

$$\omega^{(\pm)}(\mathbf{r}) = \frac{1}{2}\omega(\mathbf{s})\left[1 \pm \mathrm{erf}\left(\frac{z}{2\sqrt{b}}\right)\right],$$

$$\omega(\mathbf{s}) = \frac{\sigma(1 - i\rho)}{8\pi a} \exp\left(-\frac{s^2}{4a}\right), \tag{13}$$

where $\omega(\mathbf{s})$ is the ordinary nucleon profile function in the GS theory related by the two-dimensional Fourier transformation (1) to the transverse part of the free NN-scattering amplitude. It should be underlined that the profile function $\omega(\mathbf{s})$ is due exclusively to the transversal momentum transfer. The appearance of the longitudinal (dependent on z) part in the profile $\omega^{(\pm)}$ is related to the explicit account of the longitudinal part of the transferred momentum in the NN-scattering amplitude. From the point of view of the eikonal approximation, this corresponds to consideration of the "off-shell" effects in the scattering amplitude.

The differential cross-section of the reaction in the center-of-mass system (the angular distribution of the created deuterons) is

$$\left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{c.m}} = \frac{k_d}{k_p}\frac{1}{2J_A + 1}\sum_{M_A}\frac{1}{2}\sum_{\mu_P}\sum_{M_dM_B}|F_{f\,i}|^2, \tag{14}$$

where the reaction amplitude is connected to the T-matrix element on the momentum surface by the relation

$$F_{f\,i} = -(2\pi)^2 (\mu_i\mu_f)^{1/2}\, T_{f\,i}. \tag{15}$$

Here, μ_i, μ_f are the relativistic reduced masses in the input and output channels, and the matrix element $T_{f\,i}$ is given below.

Let us apply the proposed formalism to the description of the reaction of capture of a neutron by a proton, ^{4}He(p,d)^{3}He. The internal nuclear wave functions are chosen antisymmetric with respect to the permutations of the proton (1,2) and neutron (3,4) coordinates:

$$\Phi_\alpha = \varphi_\alpha(\mathbf{r}_1...\mathbf{r}_4)\chi_{00}(1,2)\chi_{00}(3,4), \quad \Phi_p = \chi_{\frac{1}{2}\mu_P},$$

$$\Phi_\tau = \varphi_\tau(\mathbf{r}_1...\mathbf{r}_3)\chi_{00}(1,2)\chi_{\frac{1}{2}\mu_\tau}(3),$$

$$\Phi_d = \sum_{l=0,2}\varphi_l(r)\sum_{m,\mu_d} C_{lm\;1\mu_d}^{1M_d} Y_{lm}(\hat{\mathbf{r}})\chi_{1\mu_d}.$$

The spin overlap of these functions has the following form:

$$(\Phi_d\Phi_\tau, \Phi_\alpha\Phi_p) = \frac{1}{\sqrt{2}}(-1)^{1/2-\mu_\tau}\varphi_\tau^*\varphi_\alpha\times$$

$$\times\sum_{lm}\varphi_l^*(r)Y_{lm}^*(\hat{\mathbf{r}})\sum_{\mu_d} C_{lm\;1\mu_d}^{1M_d} C_{\frac{1}{2}\mu_p\;\frac{1}{2}\;-\mu_\tau}^{1\mu_d}.$$

With regard for the antisymmetrization factor $\sqrt{2}$ (because the ^{4}He nucleus contains two neutrons), the T-matrix element can be written in the form (we dropped the unessential phase factor) which contains the coefficients of the vector summation C:

$$T_{f\,i} = \sum_{lm} T_{lm} \sum_{\mu_d} C^{1M_d}_{lm\;1\mu_d} C^{1\mu_d}_{\frac{1}{2}\mu_p\;\frac{1}{2}\;-\mu_\tau}. \qquad (16)$$

Here, the partial matrix element that corresponds to the transition into the deuteron state with the quantum numbers l and m is determined by the expression

$$T_{lm} = \frac{1}{(2\pi)^3} \int d\mathbf{r}_p d\mathbf{r}_n \exp\left(i\mathbf{Q}_p\mathbf{r}_p + i\mathbf{Q}_n\mathbf{r}_n\right) D^*_{lm}(\mathbf{r}) \times$$

$$\times \int \prod_{j=1}^{B} d\mathbf{r}_j \delta(\mathbf{R}_\tau) \varphi^*_\tau \Omega \varphi_\alpha, \qquad (17)$$

$$D_{lm}(\mathbf{r}) = \left(-\varepsilon_d + \frac{\nabla^2}{M}\right) \varphi_l(r)\, Y_{lm}(\hat{\mathbf{r}}).$$

The transferred momenta are $\mathbf{Q}_p = \mathbf{k}_p - \frac{\mathbf{k}_d}{2}$, $\mathbf{Q}_n = -\frac{\mathbf{k}_p}{A} - \frac{\mathbf{k}_d}{2}$. The distortion operator Ω in (17) is related to the generalized profile functions of nucleons by the relation

$$\Omega(\mathbf{r}_p, \mathbf{r}_n; \xi) = \prod_{j=1}^{B} \left[1 - \omega^{(+)}(\mathbf{r}_p - \mathbf{r}_j)\right] \times$$

$$\times \left[1 - \omega^{(-)}(\mathbf{r}_p - \mathbf{r}_j)\right] \left[1 - \omega^{(-)}(\mathbf{r}_n - \mathbf{r}_j)\right], \qquad (18)$$

where $\xi = (\mathbf{r}_1 ... \mathbf{r}_B)$ is the totality of the coordinates of nucleons of the residual nucleus.

Expression (17) is the exact theoretical result. However, its direct calculation is quite complicated. After a number of simplifying approximations described in [16], the differential cross-section of the reaction can be presented in the form

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}} = \frac{3}{4} \mu_i \mu_f \frac{k_d}{k_p} |I(\mathbf{Q})|^2 \left(\varepsilon_d + \frac{Q_p^2}{M}\right)^2 \sum_{l=0,2} \varphi_l^2(Q_p). \qquad (19)$$

In this expression,

$$I(\mathbf{Q}) = \int d\mathbf{r} \exp(i\mathbf{Q}\cdot\mathbf{r})\, F(\mathbf{r})\, \Phi(\mathbf{r}),$$

$$F(\mathbf{r}) = \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \delta(\mathbf{R}_\tau) \left|\varphi_\tau(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3)\right|^2 \times$$

$$\times \prod_{j=1}^{3} \left[1 - \omega^{(+)}(\mathbf{r} - \mathbf{r}_j)\right] \left[1 - \omega^{(-)}(\mathbf{r} - \mathbf{r}_j)\right]^2,$$

$$\Phi(\mathbf{r}) = \int \prod_{j=1}^{4} d\mathbf{r}_j \delta(\mathbf{R}_\tau)\, \delta(\mathbf{r} - \mathbf{r}_4 + \mathbf{R}_\tau) \times$$

$$\times \varphi^*_\tau(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3)\, \varphi_\alpha(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3\mathbf{r}_4),$$

$$\varphi_l(Q_p) = \left(\frac{2}{\pi}\right)^{1/2} \int_0^\infty dr\, r^2 j_l(Q_p r)\, \varphi_l(r),$$

$\mathbf{Q} = \mathbf{Q}_p + \mathbf{Q}_n = \frac{A-1}{A}\mathbf{k}_p - \mathbf{k}_d$, $\varepsilon_d = 2.224$ MeV is the binding energy of a deuteron, and M is the nucleon mass.

In the calculations, we used the values of NN-amplitude parameters averaged over protons and neutrons: $A\sigma = Z\sigma_{pp} + N\sigma_{pn}$, $A\sigma\rho = Z\sigma_{pp}\rho_{pp} + N\sigma_{pn}\rho_{pn}$, and an analogous expression for the average value of the form parameter a. This approximation is especially good for the nuclei with $Z \sim N$. This averaging does not significantly influence the results but it essentially simplifies the computation.

4. Numerical Calculations of the Excitation Function of the Reaction ^{4}He(p,d)^{3}He

The calculations of the excitation function were carried out for energies in the interval from 100 to 1000 MeV for two angles of the deuteron escape $\theta_{\text{c.m}} = 0$ and 34.5°. The spatial parts of the internal wave functions of the ground states of nuclei ^{4}He and ^{3}He were chosen in the form of the superposition of Gaussians:

$$\varphi_\alpha = N_\alpha \left\{ \exp\left[-\frac{\alpha_1}{2} \sum_{j=1}^{4} (\mathbf{r}_j - \mathbf{R}_\alpha)^2\right] - \right.$$

$$\left. -D_\alpha \exp\left[-\frac{\alpha_2}{2} \sum_{j=1}^{4} (\mathbf{r}_j - \mathbf{R}_\alpha)^2\right] \right\},$$

$$\varphi_\tau = N_\tau \left\{ \exp\left[-\frac{\beta_1}{2} \sum_{j=1}^{3} (\mathbf{r}_j - \mathbf{R}_\tau)^2\right] - \right.$$

$$\left. -D_\tau \exp\left[-\frac{\beta_2}{2} \sum_{j=1}^{3} (\mathbf{r}_j - \mathbf{R}_\tau)^2\right] \right\}, \qquad (20)$$

where N_α and N_τ are the normalizing constants. The parameters of the phenomenological functions (20) were found in [13] from the condition that they describe the charge form factors of nuclei in a wide range of the transferred momenta:

$$\alpha_1 = 0.85 \quad \text{Fm}^{-2}, \quad \alpha_2 = 0.93 \quad \text{Fm}^{-2}, \quad D_\alpha = 1.1,$$

$$\beta_1 = 0.70 \quad \text{Fm}^{-2}, \quad \beta_2 = 2.24 \quad \text{Fm}^{-2}, \quad D_\tau = 1.9. \tag{21}$$

The same values of the parameters were used in work [8] for the interpretation of the reaction ^{4}He(p,d)^{3}He by the method of eigenstates.

For a deuteron, we took the wave functions related to the realistic Hamada—Johnston NN-potential [14] with a hard repulsive core. The approximations for S- and D-wave parts of these functions in the form of the superposition of exponentials are presented in [15].

From the form of the distorted function $F(\mathbf{r})$, it can be seen that the function accounts all possible virtual excitations of the ^{3}He nucleus during the reaction. Because the ^{3}He nucleus has no bound excited states which would noticeably contribute to the reaction cross-section, the contribution of these states can be neglected. Moreover, it is seen from the form of Ω (18) that if, for example, the neutron from the escaping deuteron excites the ^{3}He nucleus, the proton must de-excite it in order to obtain ^{3}He in the ground state. Obviously, such processes will promote the decay of the deuteron, and the probability of this decay will increase with the degree of the nucleus excitation caused by the large momentum transfer to various particles. Therefore, the main contribution to the elastic re-scattering of deuterons will be given by the ground state of ^{3}He. The above-said allows us to simplify the expression for the distortion function and to obtain it in the form

$$F(\mathbf{r}) = [1 - f(\mathbf{r})]^3,$$

$$f(\mathbf{r}) = \int d\mathbf{r}_1 \rho_\tau(\mathbf{r}_1) \left\{1 - \left[1 - \omega^{(+)}(\mathbf{r}-\mathbf{r}_1)\right] \times \right.$$

$$\left. \times \left[1 - \omega^{(-)}(\mathbf{r}-\mathbf{r}_1)\right]^2 \right\}, \tag{22}$$

where the one-particle mass density of the ^{3}He nucleus is

$$\rho_\tau(\mathbf{r}) = \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \delta(\mathbf{R}_\tau) \frac{1}{3}\sum_{j=1}^{3} \delta(\mathbf{r}-\mathbf{r}_j) |\varphi(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3)|^2. \tag{23}$$

The quantity $I(\mathbf{Q})$ is calculated numerically in the cylindrical coordinates as

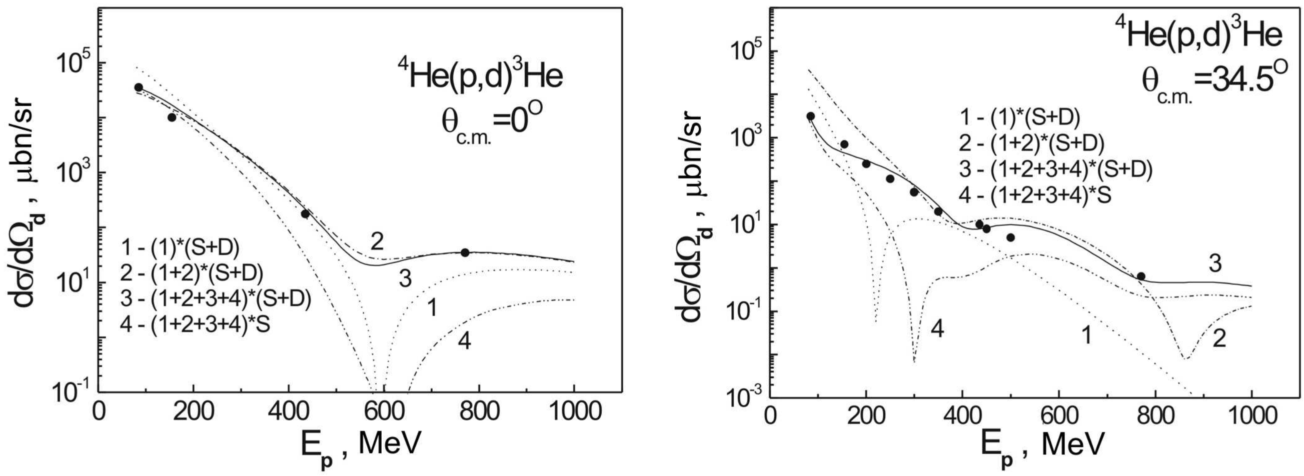
$$I(\mathbf{Q}) = 2\pi \int_0^\infty db\, b J_0(Q_\perp b) \times$$

$$\times \int_{-\infty}^{\infty} \exp(iQ_z \cdot z) F(b,z)\, \Phi(b,z) \equiv \sum_{\lambda=1}^{4} I_\lambda(\mathbf{Q}), \tag{24}$$

where the four terms in the last equality correspond to four terms in the expansion of (22), $F = 1-3f+3f^2-f^3$, in a series in the increasing multiplicity of scattering. The term $\lambda = 1$ in the cross section (19) corresponds to the Born plane wave approximation. Within the Feynman diagram formalism, this term is represented by a pole diagram. In this approximation, the cross section is proportional to the square of the Fourier transform of the neutron wave function $\Phi(Q)$:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m}}^{\text{PWBA}} = 3\pi\, \mu_i \mu_f \frac{k_d}{k_p} \left[D_0^2(Q_p) + D_2^2(Q_p)\right] \Phi^2(Q). \tag{25}$$

Figure 2 shows the results of calculations of the excitation function for the reaction ^{4}He(p,d)^{3}He at $\theta_{\text{c.m}} = 0$ and 34.5° along with the experimental data [4,5]. Various curves show the contributions to the cross-section corresponding to the different terms of expansion (24) in the increasing multiplicity of the re-scattering of the incident proton and the nucleons of the escaping deuteron by the residual nucleus. Curve *1* corresponds to the plane-wave Born approximation (25) ($\lambda = 1$ in (24)). It is seen that the one-pole mechanism of the reaction cannot explain the excitation function in the whole energy range and leads to the underestimated cross-section. Curve *2* takes into account the single scattering ($\lambda = 1$ and 2). Solid curve *3* takes into account all three multiplicities of the re-scattering ($\lambda = 1$, 2, 3, and 4). Curve *4* shows the contribution from the S-state of the deuteron only, which is not enough for the quantitative description of the experiment. Only taking into account all the distorting re-scattering events and the exact wave function of the deuteron, we can describe quantitatively the experiment in the whole range of the energies under study. The interference of the terms corresponding to different scattering multiplicities leads to a noticeable diffraction structure in the excitation function which is related to the energy change (and, hence, to the change in transferred momenta) and to the energy dependence of the parameters σ, ρ, and a of nucleon-nucleon scattering amplitudes. It should be



a **b**

Fig. 2. Excitation functions of the reaction ^{4}He(p,d)^{3}He for two angles of the deuteron escape: $\theta_{\rm c.m} = 0$ and 34.5°

noted that the minima of the excitation function at $\theta_{\rm c.m.} = 0$ obtained for only the S-state of a deuteron and in the pole approximation are of different nature. Their filling is related to the D-wave of the deuteron or to the effects of the multiple re-scattering, respectively. The calculation of the excitation function with the use of the pd-triangular diagram [4] also leads to a slower decay of this function with increase in the energy in comparison with the purely pole mechanism, which proves the importance of re-scattering effects. In the case of the excitation function at $\theta_{\rm c.m} = 34.5°$, the minima shift to smaller energies, because the energy decrease in this case is compensated by the increase of the angle at the constant transferred momenta.

Thus, the proposed DTMS-based approach to the theory of transfer reactions provides a simple explanation of the behavior of the excitation function which plays an important role in elucidating the reaction mechanism. In particular, the characteristic sharp break in its behavior is naturally explained by the multiple scattering, and it is quite probable that this break is not related to intermediate pions and isobars.

It is known from the analysis of the data on high-energy elastic scattering that the amplitudes corresponding to different multiplicities of scattering have different decay rates versus the transferred momentum. At small q, the most important is the single scattering with the quickest decay. With increase in q, the double scattering becomes important, which has a more smooth dependence on q, etc. As it is obvious that the energy increase of the incident proton at a fixed angle of the deuteron escape is equivalent to the increase in transferred momenta, the above-given picture of the elastic scattering will be qualitatively observed in the reaction as well. This explains the characteristic break in the excitation function that appears if one passes from middle energies to high ones.

Thus, we can conclude that the mechanism of multiple scattering is important in the region of large transferred momenta. Moreover, the knowledge of the proper content of high-momentum components in the wave functions is necessary for the quantitative description of both the angular distribution of created deuterons and the reaction excitation function.

The developed formalism and the given analysis show that the investigation of the reactions with nucleon capture in the region of intermediate energies may be a very sensitive tool for the determination of details of the structure of nuclei, the wave functions of nuclei in the region of zeros of the form factors, the momentum distribution of nucleons in nuclei, and other characteristics of nuclei which cannot be determined at low energies.

1. *Sitenko A.G.* // Ukr. Fiz. Zh. — 1959. — **4**. — P.152; *Glauber R.J.* Lectures in Theoretical Physics. — New York: Interscience, 1959.— Vol. 1.— P.315.

2. *Bauer T., Boudard A., Catz H. et al.* // Phys. Lett. B. — 1977. — **67**. — P.265.

3. *Rost E., Shepard J.R., Sparrow D.A.* // Phys. Rev. C. — 1978. — **17**. — P.1513.

4. *Kallne J., Hutcheon D.A., McDonald W.J. et al.* // Phys. Rev. Lett. — 1978. — **41**. — P.1638.

5. *Kallne J., Gugelot P.C.* // Phys. Rev. C. — 1979. — **20**. — P.1085.
6. *Wilkin C.* // J. Phys. G: Nucl. Phys. — 1980. — **6**. — P.69.
7. *Levshin E.B., Foursat A.D.* // Yad. Fiz. — 1983. — **38**. — P.1572.
8. *Levshin E.B., Sailer K.G., Foursat A.D.* // Ibid. — 1982. — **36**. — P.1150.
9. *Tekou A.* // Nuovo cim. A. — 1979. — **54**. — P.25.
10. *Levshin E.B., Foursat A.D.* // Yad. Fiz. — 1987. — **46**. — P. 1614.
11. *Barashenkov V.S., Toneev V.D.* Interaction of High-Energy Particles and Atomic Nuclei with Nuclei. — Moscow: Atomizdat, 1972 (in Russian).
12. *Bildsten L., Wasserman I., Salpeter E.E.* // Nucl. Phys. A. — 1990. — **516**. — P.77.
13. *Levshin E. B., Foursat A. D.* // Izv. AN USSR. Ser. Phys. — 1981. — **45**. — P.48.
14. *Hamada T., Johnston I.D.* // Nucl. Phys. — 1962. — **34**. — P.382.
15. *Humberston J.W., Wallace J.B.G.* // Nucl Phys. A. — 1970. — **141**. — P.362.
16. *Davydovskyy V.V., Foursat A.D.* // Ukr. Fiz. Zh. — 2005. — **50**, №3. — P.219.

Received 23.03.05.
Translated from Ukrainian by M. Shendeleva

ЕНЕРГЕТИЧНА ЗАЛЕЖНІСТЬ ПАРАМЕТРІВ NN-АМПЛІТУД І РОЗРАХУНОК ФУНКЦІЇ ЗБУДЖЕННЯ РЕАКЦІЇ ^{4}He(p,d)^{3}He В БАГАТОЦЕНТРОВОМУ ЕЙКОНАЛЬНОМУ НАБЛИЖЕННІ

В.В. Давидовський, А.Д. Фурса

Р е з ю м е

За допомогою нелокального сепарабельного потенціалу другого рангу з гауссівськими форм-факторами знайдено енергетичну залежність параметрів NN-амплітуд теорії багаторазового розсіяння Глаубера—Сітенка і на цій основі розраховано функції збудження реакції ^{4}He(p,d)^{3}He для двох кутів вильоту дейтронів $\theta_{\rm c.m} = 0$ і 34,5° . Показано, що без залучення екзотичних механізмів з народженням піонів та ізобар теорія багаторазового розсіяння добре описує експериментальні дані в інтервалі енергій від 100 до 1000 МеВ.