ON THE ABSORPTION OF LINEARLY POLARIZED RADIATION IN A SEMICONDUCTOR-BASED NANOSTRUCTURE WITH HOLE CONDUCTION

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The absorption of linearly polarized radiation in a dimensionally quantized semiconducting well, which is connected with optical transitions both between the light and heavy hole branches and between dimensionally quantized subbands (DQSBs), has been considered. The principal features of light absorption in an indefinitely deep symmetric well have been established. This absorption is characterized by the intraband light absorption caused by the direct optical transitions of holes between the subbands which are formed due to dimensional quantization. The spectral dependence of the light absorption coefficient has been obtained.

Nowadays, the more and more intent attention is paid to the absorption of light in semiconductors with DQWs caused by interband optical transitions between DQSBs of the conduction and valence bands (see, e.g., works [1–8]). The intraband absorption of light in structures with quantum holes has been studied experimentally in works [4–7]; at the same time, works [1, 3] were devoted to the theoretical consideration of the light absorption in a quantum well of a p-semiconductor with a participation of a third body (phonons, impurities, and so on).

In work [9], the interband photoluminescence and light-induced light absorption by free current carriers, provided the optical interband excitation of electron-hole pairs in step-like InGaAs/AlGaAs quantum wells, have been investigated. In work [10], the spectrum of the equilibrium intersubband absorption caused by optical transitions between the dimensionally quantized states in the conduction band has been studied both experimentally and theoretically. The interband light absorption in the quasi-zero-dimensional structures representing spherical semiconducting microcrystals has been researched theoretically in work [11] in the framework of the dipole approximation.

In this work, we have considered the absorption of linearly polarized radiation in a semiconducting DQW of width a, connected with optical transitions between the branches of light and heavy holes, as well as between DQSBs. To simplify the problem, we considered the case of an indefinitely deep symmetric well. We took into account that there appears an additional mechanism of intraband light absorption in the quantum well, which is connected with the direct optical transitions of free carriers between subbands that are formed due to the dimensional quantization. When calculating the contribution of these transitions to the light absorption coefficient, the partial occupation of states in the formula for the absorption coefficient of bulk crystals was taken into account. We have also determined the symmetric and antisymmetric wave functions of holes, derived the expression for the energy spectrum of holes in a DQW of the type (111), and analyzed the range of low temperatures, when the current carriers can be considered accumulated at the center of the Brillouin zone.

At present, the energy spectrum of current carriers in dimensionally quantized semiconducting structures of the (001) type has been investigated in the main (see, e.g., works [12–14]). However, in our opinion, the issue concerning the band structure in a DQW, fabricated on the basis of multivalley semiconductors, or in indirect-band-gap p-semiconductors remains disputable, which is the topic of our consideration. In the Luttinger–Kohn approximation [15] for DQWs of the types (001) and (111), we have [1]

$$(E-F)\,(E-G)-|R|^2=0, \tag{1}$$

where the expressions for F, G, and R are presented in Table 1 of work [15]. From this equation, we get

$$E^2-2\gamma_1\left(k_\perp^2+q^2\right)\cdot E+\gamma_1^2\left(k_\perp^2+q^2\right)^2-$$

$$-\gamma_3^2\left(13k_\perp^4+20k_\perp^2q^2+4q^4\right)=0, \tag{2}$$

where q is the unknown z-component of the wave vector to be determined from relationship (2), $\vec{k}_\perp=\{k_x,\,k_y\}$ is the two-dimensional wave vector of holes on the plane of the DQW surface, and γ_n $(n=1,\,2,\,3)$ are the Luttinger parameters.

Following works [8, 14, 15], we adopt that, in a one-dimensional indefinitely deep DQW of a semiconductor with cubic symmetry, the wave functions of four Bloch states in the branches of light and heavy holes are described by the expression

$$\Psi_{\vec{k},q_\mu}=e^{i\vec{k}_\perp\vec{r}_\perp}e^{iq_\mu z}U_{\vec{k}q_\mu}(\vec{r}_\perp), \tag{3}$$

where $U_{\vec{k}_\perp q_\mu}$ is the Bloch amplitude, $\vec{r}_\perp=\{x,\,y\}$,

$$U_{\vec{k}q_\mu}(\vec{r}_\perp)=\sum_\mu U_\mu(\vec{r}_\perp)f_\mu(\vec{k}_\perp,q_\mu), \tag{4}$$

the symbol $\mu=\pm3/2$ and $\pm1/2$ numbers four states described by the wave functions U_μ (see Table 1), the coefficients $f_\mu(\vec{k}_\perp,q_\mu)$ form a unitary matrix $\hat{F}$,

$$\hat{F}'_{n\vec{k}_\perp}=\left[(E-F)\,(E_n-E_{\bar{n}})\right]^{-1/2}\hat{F}_{n\vec{k}_\perp}, \tag{5}$$

T a b l e 1. Wave functions of structures (001) and (111) in the Luttinger—Kohn basis [15]

	DQW of the (001) type	DQW of the (111) type
$U_{3/2}$	$-X_+\cdot S_+$	$X_+\cdot S_+$
$U_{-1/2}$	$3^{-1/2}(X_-\cdot S_++2^{1/2}ZS_-)$	$3^{-1/2}(X_-\cdot S_++2^{1/2}ZS_-)$
$U_{+1/2}$	$3^{-1/2}(-X_+\cdot S_-+\sqrt{2}ZS_+)$	$-i3^{-1/2}(-X_+\cdot S_-+2^{1/2}ZS_+)$
$U_{-3/2}$	$X_-\cdot S_-$	$i\cdot X_-\cdot S_-$

$n\neq\bar{n}=1,2$ in basis (3) in the representation $(3/2,-1/2,+1/2,-3/2)$,

$$F_{1\vec{k}_\perp,q}=\left\|\begin{matrix}H\\0\\E-F\\I*\end{matrix}\right\|,\quad F_{2\vec{k}_\perp,q}=\left\|\begin{matrix}-I\\F-E\\0\\H*\end{matrix}\right\| \tag{6}$$

and the parameters F, G, H, and I are quoted in Table 2.

One should bear in mind that the z-axis is selected to be normal to the surface of the DQW. For the fixed values of $\vec{k}_\perp$ and E, there are four independent solutions of the Schrödinger equation:

$$\hat{F}_n'^{(i)}(\vec{k}_\perp,q)e^{iq_iz}e^{i\vec{k}_\perp\vec{r}_\perp}, \tag{7}$$

where $n=1,2$ numbers two branches of the hole energy spectrum, q_i $(i=1,2)$ are two roots of Eq. (2) which correspond, in the spherical approximation, to heavy $(q_1=q_2=q_h)$ and light $(q_3=q_4=q_l)$ holes [12—14]. According to the boundary conditions $\Psi\,(\pm a/2)=0$, each row should become zero. Then, from the condition that the hole wave function becomes zero at both the DQW boundaries $z=\pm a/2$, we obtain the equation of the type

$$\sin(q_la)\cdot\sin(q_ha)\left(|R_2|^2+\beta^2\left|R_1^2\right|^2-2\beta|I|^2\right)=$$

$$=2\beta|H_1|\cdot|H_2|\left(1-\cos(q_ha)\cdot\cos(q_la)\right), \tag{8}$$

where $\beta=(E-F_1)^{-1}(E-F_2)$, $R_{1,2}=R(k_z=q_{h,l})$, and $H_{1,2}=H(k_z=q_{h,l})$. The analytical forms of the latter two quantities are quoted in Table 2. Note that in the spherical approximation, Eq. (8) changes to the equation obtained in work [14].

Following work [14], the symmetric wave function can be determined with the help of expression (4) as a

T a b l e 2. Expressions for the F, G, H, and I coefficients in terms of $\hbar^2/(2m_0)$

Coefficient	DQW (001)	DQW (111)$x\|[110]$, $y\|[112]$, $z\|[111]$)
F	$(\gamma_1+\gamma_2)\,k_\perp^2+(\gamma_1-2\gamma_2)\,k_z^2$	$(\gamma_1+\gamma_3)\,k_\perp^2+(\gamma_1-2\gamma_3)\,k_z^2$
G	$(\gamma_1-\gamma_2)\,k_\perp^2+(\gamma_1+2\gamma_2)\,k_z^2$	$(\gamma_1-\gamma_3)\,k_\perp^2+(\gamma_1-2\gamma_3)\,k_z^2$
H	$\sqrt{3}\gamma_3\,(k_x-ik_y)\,k_Z$	$2\sqrt{\frac{2}{3}}\;(\gamma_2-\gamma_3)\,k_+^2-(2\gamma_2+\gamma_3)\,k_zk_-$
I	$-\sqrt{3}\;\gamma_2\;\;k_x^2-k_y^2\;\;-2i\gamma_3k_xk_y$	$\frac{2}{\sqrt{3}}\;2\,(\gamma_2-\gamma_3)\,k_+k_z-(\gamma_2+2\gamma_3)\,k_-^2$

[1] the z-axis is supposed to be directed normally to the well surface

linear combination of the functions $\Psi_S^{(\mu)} = \left(\left(1+\hat{I}\right)/2\right)\Psi_{\vec{k},q_\mu}$, i.e. as

$$\Psi_S = \sum_\mu A_\mu \Psi_S^{(\mu)}, \tag{9}$$

while the antisymmetric one as

$$\Psi_a = \hat{J}\hat{K}\Psi_S, \tag{10}$$

where

$$\Psi_S^{(\mu)} = e^{i\vec{k}\vec{r}_\perp}\Big\{\Big[U_{3/2}\chi_{3/2}^{(\mu)} + U_{-1/2}^i\chi_{-1/2}^{(\mu)}\Big]\cos(q_\mu Z) +$$

$$+ i\sin(q_\mu Z)\Big[U_{-3/2}\chi_{-3/2}^{(\mu)} + U_{1/2}\chi_{1/2}^{(\mu)}\Big]\Big\}, \tag{11}$$

$\hat{I}$ is the operator of reflection with respect to the plane $Z = 0$ [14], $\hat{J}$ the operator of coordinate inversion, and $\hat{K}$ the operator of time inversion. It was also taken into account that the Bloch amplitudes $U_{3/2}$ and $U_{-1/2}$ are symmetric, while $U_{1/2}$ and $U_{-3/2}$ are antisymmetric with respect to the operation $\hat{I}$. Therefore, we obtain

$$\Psi_S^{(\mu)} = e^{i\vec{k}_\perp\vec{r}_\perp}\Big\{\Big[U_{3/2}\chi_{3/2}^{(\mu)} + U_{-1/2}\chi_{-1/2}^{(\mu)}\Big]\cos(q_\mu z) +$$

$$+ i\sin(q_\mu z)\Big[U_{-3/2}\chi_{-3/2}^{(\mu)} + U_{1/2}\chi_{1/2}^{(\mu)}\Big]\Big\}. \tag{12}$$

Below, the following notations are adopted: $C_{l,h} = \cos q_{l,h}$ and $S_{l,h} = \sin q_{l^*,h}$.[2] Then, for the DQW (111), using the boundary conditions $\Psi(\pm a/2) = 0$ and the explicit form of the matrix $\hat{F}_{1,2}$, we obtain

$$A_1/A_2 = v\xi^*, \quad A_3 = -\left(B_h S_h / B_l S_l\right) A_1,$$

$$A_4 = -A_2 B_h C_h / B_l S_l, \tag{13}$$

where $v = (B_l - B_h\alpha)/(B_l - \tau B_h)$, $\xi = I_h/H_h$, $\alpha = I_l/I_h$, $B_{l,h} = -E + F_{l,h}$, $\tau = (H_l\tan q_h)/(H_h\tan q_l)$, and $F_{l,h} = F(k_x, k_y, k_z = q_{l,h})$.

Here, we quote a useful relationship

$$v = \frac{B_l - B_h(I_l/I_h)}{B_l - B_h\tau} = \frac{B_l - B_h|H_l|^2/\left(\tau\cdot|H_h|^2\right)}{B_h\left(I_l^*/I_h^*\right) - B_l}. \tag{14}$$

In the spherical approximation for the DQW (111), $\gamma_2 = \gamma_3$, $H = -2\sqrt{6}\gamma_2 k_- q$, $I = -2\sqrt{3}\gamma_2 k_-^2$, $k_\pm = (k_x \pm ik_y)/\sqrt{2}$, so that $\alpha = 1$, $\tau = (q_l\tan q_h)/(q_h\tan q_l)$, $\xi = k_-/(\sqrt{2}q_h)$, and

$$v = \frac{b_l - b_h}{b_l - b_h\tau} = \frac{b_l - b_h q_l^2 q_h^{-2}/\tau}{b_l - b_h}|\xi|^{-2}, \tag{15}$$

where $b_i = (\gamma_1 + \gamma_2)k^2 + (\gamma_1 - 2\gamma_2)q_i^2 - \varepsilon$, $\varepsilon = 2m_0E/\hbar^2$, and m_0 is the free electron mass. This result coincides with that for the DQW (111) [14].

From Eq. (15), it is possible to obtain a quadratic equation for τ:

$$\tau^2 + 2\Phi\tau + \gamma^2 = 0. \tag{16}$$

Here, $\gamma = |H_l|/|H_h|$, and

$$-2\Phi = \frac{B_l}{B_h}\left[|\xi|^2\left(1 - \alpha\frac{B_h}{B_l}\right)\left(1 - \alpha\frac{B_h}{B_l}\right) + 1\right] + \frac{B_h}{B_l}\gamma^2. \tag{17}$$

Equations (16), (17), and (1), provided $\vec{k}$ is given, define the energy spectrum of light and heavy holes in the branches of the dimensionally quantized band. The equations for the hole spectrum in the DQW (001) case have been obtained earlier in work [12], as well as in works [13, 14]. We note that Eq. (16) has two solutions:

$$\tau_1 = -\Phi - \sqrt{\Phi^2 - \gamma^2},$$

$$\tau_2 = -\Phi + \sqrt{\Phi^2 - \gamma^2}, \tag{18}$$

whence, as well as in work [14], $\tau_1\tau_2 = \gamma^2$. The solution τ_1 corresponds to the light-hole branches with even numbers, and τ_2 to the heavy-hole branches and the light-hole branches with odd numbers. Surely, the equation for the hole energy spectrum in the cases of the (100)- and (111)-type structures is a complicated expression which is described by formulae (1), (16), and (17) and has several solutions. The same solution can describe the branches of both light and heavy holes which may differ from one another only by the value of n. This means that, at certain values of the two-dimensional wave vector of current carriers, the light and heavy holes are not distinguishable, i.e. light holes change into heavy ones and vice versa [16].

Using relationship (14), one can readily derive a relation for the DQW (111) which connects (at fixed $\vec{k}_\perp$ and q) the quantities v_1 and v_2:

$$v_1 v_2 = -|\xi|^{-2}\frac{B_l - B_h(I_l/I_h)}{B_l - B_h\left(I_l^*/I_h^*\right)}. \tag{19}$$

[2]Hereafter, similarly to work [14], the quantity $a/2$ is taken as a length unit.

In the spherical approximation, it reads

$$v_1 v_2 = -|\xi|^{-2}, \tag{20}$$

being also valid in the same approximation for the DQW (001).

Taking the above into account, the wave functions can be represented in the form

$$\Psi_S = N e^{i\vec{k}_\perp \vec{r}_\perp}\Big\{-\left(U_{3/2}+\sqrt{3}\tilde{W}_+ U_{-1/2}\right)C(z)+$$

$$+i\eta S(z)\left[U_{-3/2}+\sqrt{3}\tilde{W}_- U_{1/2}\right]\Big\}, \tag{21}$$

where $C(z) = \cos(q_h z) - (C_h/C_l)\cos(q_l z)$, $S(z) = \sin(q_h z) - (S_h/S_l)\sin(q_l z)$, $\tilde{W}_+ = \frac{B_h/\sqrt{3}}{I_h - v\xi H_h}$, $\tilde{W}_- = \frac{B_h/\sqrt{3}}{I_h^* + H_h^*(v\xi)^{-1}}$, and $\eta = \frac{v\xi I_h^* + H_h^*}{I_h - v\xi H_h} = \frac{1+v|\xi|^2}{1-v}\frac{I_h^* H_h^*}{|I_h|^2}$. If one supposes that $\tilde{W}_\pm = W_\pm e^{\pm 2i\varphi}$, $W_+ = \frac{B_h}{\sqrt{3}(1-v)|I_h|}$, and $W_- = \frac{B_h}{\sqrt{3}\;1+\left[v|\xi|^2\right]^{-1}\;|I_h|}$, then $|\eta|^2 = W_+\left(|I_h|W_+ - B_h\right)/\left(W_-\left(B_h - W_-|I_h|\right)\right)$. The constant N is determined from the condition that the wave functions have to be orthonormalized:

$$|N|^2 = \left[\left(1+3W_+^2\right)P_c + |\eta|^2\left(1+3W_-^2\right)P_s\right]^{-1}, \tag{22}$$

where, for the DQW (100),

$$P_c = \left(1+C_h^2 C_l^{-2}\right)+\frac{C_h}{C_l}\frac{\left[S_h C_l\left(q_l^2+3q_h^2\right)q_l-(l\leftrightarrow h)\right]}{q_l q_h\left(q_l^2-q_h^2\right)},$$

$$P_s = \left(1+S_h^2 S_l^{-2}\right)+\frac{S_h}{S_l}\frac{\left[S_h C_l\left(q_h^2+3q_l^2\right)q_h-(l\leftrightarrow h)\right]}{q_l q_h\left(q_l^2-q_h^2\right)}.$$

The latter two relations correspond to the results obtained in work [14].

It should be noted that the absorption of polarized radiation in structures with quantum wells differs strongly from that in bulk semiconductors both in the linear and nonlinear, with respect to the light intensity, cases. In the structures with quantum wells, light is absorbed in the space of the two-dimensional wave vector $\vec{k}_\perp$, which is analogous to the case of bulk semiconductors, and between the states of dimensional quantization. It is in the latter case where the selection rules for optical transitions vary, provided nonlinear light absorption. Therefore, there appears an additional mechanism of light absorption in a quantum well, which is connected with the direct optical transitions of free carriers between the states that are formed owing to the dimensional quantization. When calculating the contribution of these transitions to the light absorption coefficient, it is necessary to take into account the partial occupation of both two-dimensional subbands and dimensionally quantized states in the relevant formula. Then, the expression for the coefficient of one-photon light absorption in a DQW can be written down as follows:

$$K^{(1)} = \frac{2\pi\omega}{I}\sum_{\substack{\vec{k}_\perp;\,mm',\\ l,\,l',\,\nu\nu'}}|M^{(1)}_{l'm'\nu',lm\nu}|^2\times$$

$$\times\Delta_{l'l}\,\delta(E_{l'\nu'\vec{k}_\perp}-\hbar\omega-E_{l\nu\vec{k}_\perp}), \tag{23}$$

where

$$\Delta_{l'l} = \frac{f^{(0)}_{\ln\vec{k}_\perp}-f^{(0)}_{l'n'\vec{k}_\perp}}{[1+4\hbar^{-2}T_l T_{l'}|M^{(1)}_{l'n'm',lmn}|]^{1/2}},$$

$M^{(1)}_{l'n'm',lmn}$ is the matrix element of the one-photon optical transition $|l'n'm'\rangle \to |lnm\rangle$, T_l the time for the holes of the l-th branch to escape from the resonant region, $E_{ln\vec{k}_\perp}$ the energy spectrum, $f^{(0)}_{ln\vec{k}_\perp}$ the equilibrium hole distribution function (making allowance for the dimensional quantization), and δ-function describes the energy conservation law. Here, it was taken into account that the states of current carriers are dimensionally quantized along the z-axis, remaining Bloch ones along the other directions.[3]

If $k_\perp a \ll \sqrt{\pi n}$, the description in the framework of the effective mass approximation [11,12] is valid, so that the energy spectrum of holes can be presented in the form

$$E_{ln\vec{k}\perp} = \frac{\hbar^2 k_\perp^2}{2m_l^{(n)}a^2}+\varepsilon_l n^2, \tag{24}$$

where

$$\frac{1}{m_{lh}^{(n)}} = \frac{1}{m_l}\left[1+\frac{3\tilde{\beta}}{\pi n}\frac{\cos\left(\pi n\tilde{\beta}\right)-(-1)^n}{\sin\left(\pi n\tilde{\beta}\right)}\right] \tag{25}$$

is the reciprocal of the transverse effective mass of light holes ($l = lh$) in a quantum well; $\varepsilon_l = \frac{\hbar^2\pi^2}{2m_l a^2}$; $n = 1, 2, 3 \ldots$; $\tilde{\beta}^2 = m_{lh}/m_{hh}$; and m_{lh} and m_{hh} are the bulk effective masses of light and heavy holes, respectively (see Table 3). The effective mass of heavy holes ($l = hh$)

[3]Note that the summand proportional to the quantity T_l describes the contribution of the Rabi effect (see, e.g., works [17,18]) to the phenomenon under consideration. This contribution is not taken into account below. It is allowable in the range of low light intensities, where the perturbation theory is applicable. Taking such contributions into account requires a separate consideration.

can be calculated from relation (25), by replacing $\tilde{\beta}^{-1}$ for $\tilde{\beta}$ and m_{hh} for m_{lh}. From Table 3, one can see that the effective masses possess both negative and positive values in approximation (25). This means that, because of the complex functional dependence of the hole energy on its quasi-momentum, the curvature of the energy dispersion changes its geometrical shape, which depends on the model choice (e.g., the finiteness of the potential well depth) and the specimen properties (e.g., on the symmetry of the semiconductor, in which the quantum well is located).

We note that the Dirac δ-function, which describes the law of energy conservation for the optical transition considered, divides the region of the hole quasi-momentum values into allowed and forbidden ones. For the direct optical transitions between the subbands of heavy and light holes, provided that dimensional quantization is taken into account, the following condition is satisfied:

$$k_\perp = \frac{1}{\hbar}\left[2\mu_{ll'}^{n'n}\left(\hbar\omega - \varepsilon_{l'}n'^2 + \varepsilon_l n^2\right)\right]^{1/2}. \quad (26)$$

Here, $\left(\mu_{nn'}^{ll'}\right)^{-1} = (m_l^n)^{-1} - \left(m_{l'}^{n'}\right)^{-1}$ is the reciprocal of the reduced effective mass of holes. In relation (26), the energy difference $\varepsilon_{l'}n'^2 - \varepsilon_l n^2$ plays a role of the energy gap between the subbands of a complex band (or between the branches of a single band) in a semiconducting quantum well. It constitutes the selection rules for both the intrasubband (between the dimensionally quantized states with different numbers) and intersubband (between the dimensionally quantized states with different and identical numbers) optical transitions.

It is known that the direct optical transitions between the subbands of heavy and light holes are allowed in a bulk semiconductor with the degenerate

T a b l e 3. Effective masses of light and heavy holes in a semiconducting quantum well for two pairs of the bulk effective masses

$m_{lh} = 0.082m_0$, $m_{hh} = 0.51m_0$			$m_{lh} = 0.083m_0$, $m_{hh} = 0.605m_0$		
N	$m_{lh}^{(n)}/m_0$	$m_{hh}^{(n)}/m_0$	N	$m_{lh}^{(n)}/m_0$	$m_{hh}^{(n)}/m_0$
1	0.059	0.119	1	0.070	0.109
2	0.019	–0.186	2	0.067	–0.371
3	0.096	0.693	3	0.047	0.641
4	0.083	0.356	4	0.077	0.303
5	0.078	0.008	5	0.085	–0.261

valence band Γ_8 (or Γ_8^+) for the arbitrary polarization of light. In the approximation of the effective Luttinger–Kohn Hamiltonian [15], the operator of velocity is defined by the relation

$$\vec{e}\,\vec{\mathrm{v}} = \frac{1}{\hbar}\vec{e}\,\vec{\nabla}_{\vec{k}}\mathrm{H} = Q_{\alpha\beta}e_\alpha k_\beta, \quad (27)$$

where $Q_{\alpha\beta} = \frac{1}{\hbar m_0}\left\{\left[\left(\gamma_1 + \frac{5}{2}\gamma_2\right) - 2\gamma J_\alpha^2\right]\delta_{\alpha\beta} - (1-\delta_{\alpha\beta})2\gamma[J_\alpha J_\beta]\right\}$, and $J_\alpha\,(\alpha = x, y, z)$ are the 4×4-matrices corresponding to a moment of 3/2. Then, in the limit of indefinitely high barriers, the matrix element of the velocity operator for the intersubband optical transitions $(l,\,n,\,m) \to (l',\,n',\,m')$ in a structure with $z \parallel [001]$ and a quantum well $\vec{k}_\perp = 0$ looks like [4]

$$\vec{e}\vec{V}^{(0)}_{l'n'm',\ln m} = \frac{2}{\hbar}k_z^{(nn')}\{(A\pm B)e_z\delta_{mm'} + +B[J_z,\vec{J}_\perp\vec{e}_\perp]_{m'm}\} \quad (28)$$

or

$$\left|\vec{e}\vec{V}_{l'n',\,ln}\right|^2 = \frac{1}{\hbar^2}\times$$

$$\times\begin{cases} 4\,(A\pm B)^2\left|k_z^{(n',\,n)}\right|^2|e_z'|^2\delta_{l'l} & \text{for}\quad \vec{e} = (0,\,0,\,e_z)\;, \\ D^2\left|k_z^{(n'n)}\right|^2\left|e_+'\right|^2(1-\delta_{l'l}) & \text{for}\quad \vec{e} = (e_x,\,e_y,\,0)\;. \end{cases}$$

Here, $\hat{k}_z = -i\frac{\partial}{\partial z}$; $\vec{e}$ is the vector of light polarization; $e'_\pm = e_{x'} \pm ie_{y'}$; $e_{x'}$, $e_{y'}$, and $e_{z'}$ are the projections of the light polarization vector $\vec{e}$ on the axes x', y', and z', respectively, which are connected to the direction of the two-dimensional wave vector $\vec{k}_\perp$; ω is the frequency; I the intensity of exciting light; $k_z^{(nn')} = 2i\left[1-(-1)^{n+n'}\right]\frac{nn'}{n^2-n'^2}$; A and B are the semiconductor band parameters determined by the Luttinger ones [14]; $m, m' = \pm 3/2$ (at $l, l' = hh$) and $\pm 1/2$ (at $l, l' = lh$). So, one can see that allowed are the transitions $(hh,\,n) \to (hh,\,n')$ and $(lh,\,n) \to (lh,\,n')$, provided the polarization $\vec{e} \parallel z$, and the transitions $(hh,\,n) \to (lh,\,n')$, provided the polarization $\vec{e} \perp z$, where n and n' have different parities.

[4]Note that the matrices $[J_z J_\alpha]$ $(\alpha = x, y)$ in the effective Hamiltonian are preceded by the multipliers $k_\alpha \hat{k}_z$. Therefore, for the matrices $[J_z J_\alpha]$, different from zero are the matrix elements with $(l = hh,\, m_l = +3/2;\; l' = lh,\, m' = +1/2)$ and $(l = hh,\, m = -3/2;\; l' = lh,\, m' = -1/2)$. Then, for the s-polarization of light $(\vec{e} \perp z)$, we have $\left[\vec{e}\,\vec{v}_\perp(\vec{k}_\perp = 0)\right]_{hh,\pm 3/2,\nu_1;lh,\pm 1/2,\nu} = \frac{\sqrt{3}B}{\hbar}e_z k_z^{(\nu\nu_1)}$, where $e_\pm = e_x + ie_y$, $|B| = \frac{\hbar^2}{4\mu_-}$, $\mu_- = \frac{m_{hh}m_{lh}}{m_{hh}-m_l}$, $m_{hh} = \frac{m_0}{\gamma_1+2\gamma}$, $m_{lh} = \frac{m_0}{\gamma_1-2\gamma}$, and $\gamma = \frac{1}{5}(2\gamma_2 + 3\gamma_3)$.

At $\vec{k}_\perp \neq 0$, the hole wave functions comprise the mixture of all the four states with the momenta $m = \pm 3/2$ and $\pm 1/2$, so that the indicated simple rules of selection become violated. At a low temperature, the distribution function has a step-like shape: $\Theta(E - E_\mathrm{F})$, where E_F is the chemical potential of holes to be determined from the equation

$$e^{\frac{E_\mathrm{F}}{k_\mathrm{B}T}} = \sum_{l=lh,\ hh;\, n=1,2,3,} \frac{\hbar^2 p}{k_\mathrm{B} T m_l^{(n)}} \exp\left[-\frac{\varepsilon_l}{k_\mathrm{B}T} n^2\right], \quad (29)$$

and p is the two-dimensional concentration of holes. In this case, the spectrum of intersubband absorption represents a collection of relatively narrow peaks corresponding to the transitions $hl \to hn, ln$ (Fig. 1). Each of the peaks is confined to the energy interval $\hbar\omega$ between the $E_h^{(n)} - E_h^{(1)}$ (or $E_l^{(n)} - E_l^{(1)}$) and $E_h^{(n)}(k_\mathrm{F}) - E_\mathrm{F}$ (or $E_l^{(n)}(k_\mathrm{F}) - E_\mathrm{F}$) values, where k_F is the Fermi quasi-momentum.

The calculations, carried out in the framework of the Boltzmann statistics, show that, in the range of low temperatures and for a quantum hole with $a = 200$ Å, if the index of light refraction $n_\omega = 4$ at the frequency ω, $m_{lh} = 0.082 m_0$, and $m_{hh} = 0.51 m_0$ $\left(\tilde{\beta} = 0.16078\right)$, the coefficient of single-photon absorption of the linearly polarized light with $\vec{e} = (e_x, e_y, 0)$ and the energy $\hbar\omega = \varepsilon_{lh}(n' = 2) - \varepsilon_{hh}(n = 1) = 44.41$ meV caused by the optical transition $(hh,\ n' = 1) \to (lh,\ n = 2)$ at the extremum point $\left(\vec{k}_\perp = 0\right)$ is equal to 32 cm^{-1}, and by the optical transition $(hh,\ n' = 2) \to (lh,\ n = 1)$ to 682 cm^{-1}.

It is appropriate to mention here that the contribution of light holes to the total coefficient of one-photon absorption of the light propagating along the well plane is three times as much as the contribution of heavy holes.

The values of the absorption coefficient for the light propagating across the axis of quantization are presented in Table 4 for the parameter indicated above and for various types of optical transitions.

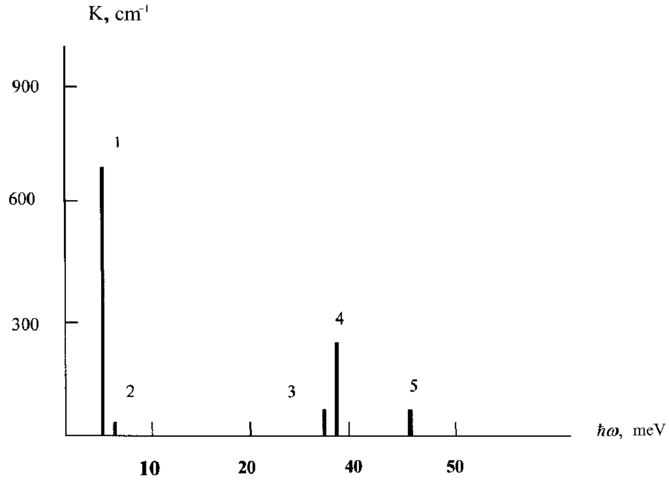

Fig. 1. Spectral dependences of the coefficient of intersubband absorption of light in the p-GaAs/AlGaAs structure with a quantum well possessing indefinitely high barriers at $p = 2 \times 10^{11}$ cm^{-2}, $T = 78$ K, and $a = 200$ Å: (*1*) $\vec{e} \perp z$, $|h2\rangle \to |l1\rangle$; (*2*) $\vec{e} \parallel z$, $|h1\rangle \to |h2\rangle$; (*3*) $\vec{e} \parallel z$, $|h1\rangle \to |h4\rangle$; (*4*) $\vec{e} \parallel z$, $|l1\rangle \to |l2\rangle$; and (*5*) $\vec{e} \perp z$, $|h1\rangle \to |l2\rangle$

In the general case, the coefficient of light absorption in p-GaAs/AlGaAs structures with a quantum well possessing indefinitely high barriers is determined with the help of the wave functions [19]

$$\Psi_S = e^{i\vec{k}_\perp \vec{r}_\perp} \hat{R}_S,$$

$$\Psi_a = e^{i\vec{k}_\perp \vec{r}_\perp} \hat{R}_a, \quad (30)$$

where

$$\hat{R}_S = \left\| \begin{array}{c} -C(z) \\ i\sqrt{3}\tilde{W}_- \eta S(z) \\ -\sqrt{3}\tilde{W}_+ C(z) \\ i\eta S(z) \end{array} \right\|, \quad \hat{R}_a = \left\| \begin{array}{c} i\eta^* S(z) \\ -\sqrt{3}\tilde{W}_+^* C(z) \\ i\sqrt{3}\tilde{W}_-^* \eta^* S(z) \\ -C(z) \end{array} \right\|. \quad (31)$$

$C(z) = \cos(q_h z) - \frac{c_h}{c_l}\cos(q_l z)$, $c_{h,\,l} = \cos\ q_{h,\,l}$, $S(z) = \sin(q_h z) - \frac{s_h}{s_l}\sin(q_l z)$, $s_{h,\,l} = \sin\ q_{h,\,l}$, $\eta = \sqrt{\frac{W_+(W_+ - 1)}{W_-(1 - W_-)}}$, $N = \left[\left(1 + 3W_+^2\right)\Re_{cc} + \left(1 + 3W_-^2\right)\Re_{ss}\right]^{-1/2}$ is the normalizing factor, $\Re_{cc} = \frac{1}{a}\int_{-a/2}^{+a/2} C^2(z)\, dz$, and $\Re_{ss} =$

T a b l e 4. Coefficients of light absorption in a quantum hole of the (001) type for various types of optical transitions

Optical transition	$(hh,\ n' = 1) \to (hh,\ n = 2)$	$(hh,\ n' = 1) \to (hh,\ n = 4)$	$(lh,\ n' = 1) \to (lh,\ n' = 2)$
Absorption coefficient, cm^{-1}	0.501	1.225	7.57

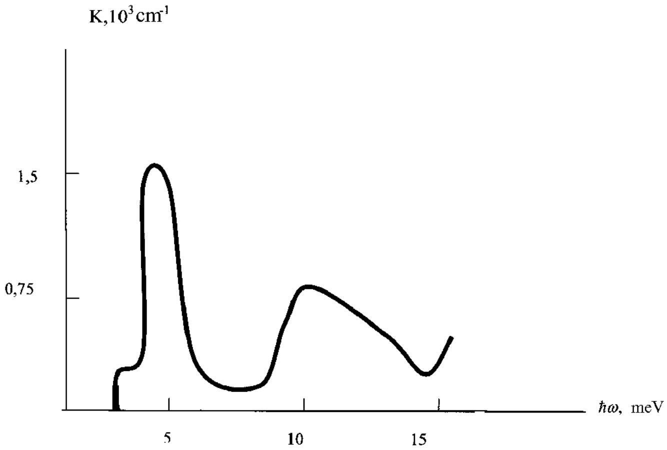

Fig. 2. Spectral dependence of the absorption coefficient of light with the polarization $\vec{e} \perp z$ in the p-GaAs/AlGaAs structure with a quantum well possessing indefinitely high barriers at $p = 2 \times 10^{11}$ cm^{-2}, $T = 78$ K, and $a = 200$ Å

$= \frac{1}{a}\int_{-a/2}^{+a/2} S^2(z)\ dz$. Then, at $\vec{e} \perp z$, the absorption coefficient of linearly polarized radiation is determined by the expression

$$K = \sum_{l=lh,\ hh;\, n,n'=1,\,2,\,3,} \frac{8\pi^2 e^2}{c n_\omega a S_0} \Big[|[\vec{e}_\perp \mathrm{v}_\perp]_{l,n;\,l',n'}|^2 \times$$

$$\times (f_{l,n} - f_{l'n'}) \delta \Big[E_{l'n'}(\vec{k}_\perp) - E_{ln}(\vec{k}_\perp) - \hbar\omega \Big] \Big], \qquad (32)$$

$$\left| [\vec{e}_\perp \mathrm{v}_\perp]_{l,\nu;\,l',\nu'} \right| = \frac{2B}{\hbar} k \Big\{ e_+ o_- \Big[\Re_{cc}^{\nu'\nu} \left(1 - 3W_+^{(\nu')}\right) +$$

$$+ \eta^{(\nu)} \eta^{(\nu')} \Re_{ss}^{\nu'\nu} \left(1 - 3W_-^{(\nu)}\right) + 3\eta^{(\nu)} \Re_{sc}^{\nu''\nu} \times$$

$$\times \frac{1}{k} \left(W_+^{(\nu')} - W_-^{(\nu)} \right) \Big] + e_- o_+ \Big[\Re_{cc}^{\nu'\nu} \left(1 - 3W_+^{(\nu)}\right) +$$

$$+ \eta^{(\nu)} \eta^{(\nu')} \Re_{ss}^{\nu'\nu} \left(1 - 3W_-^{(\nu')}\right) - 3\eta^{(\nu')} \Re_{sc}^{\nu''\nu} \times$$

$$\times \frac{1}{k} \left(W_+^{(\nu')} - W_-^{(\nu)} \right) \Big] \Big\} N^{(\nu)} N^{(\nu')},$$

where $o_\pm = \frac{k_x \pm i k_y}{k_\perp \sqrt{2}}$, S_0 is the area of the well plane, $\Re_{LQ}^{\nu'\nu} = \frac{1}{a}\int_{-a/2}^{+a/2} Q^{(\nu)}(z)\, L^{(\nu')}(z)\ dz$, $L^{(\nu)}(z)$ and $Q^{(\nu)}(z)$ are one of the expressions $C^{(\nu)}(z) \equiv [C(z)]_{q_{h,\,l}=q_{h,\,l}^{(\nu)}}$ or $S^{(\nu)}(z) \equiv [S(z)]_{q_{h,\,l}=q_{h,\,l}^{(\nu)}}$, $q_{h,l}^{(\nu)} = q_{h,l}(E \to E^{(\nu)})$ (see Appendix). In Fig. 2, we present the frequency dependence of the absorption coefficient caused by the transitions between the subbands with the dimensional quantization of holes in p-GaAs and calculated by formula (31) taking into account Eq. (32) at $T = 78$ K and $p = 2\times10^{11}$ cm^{-2}. From this figure, one can see that, as the temperature rises up to $T = 200$ K, the heights of the peaks become several times lower, depending on the light frequency, and the coefficient of light absorption becomes different from zero in the intervals between the peaks. In contrast to the second and subsequent peaks, which broaden strongly, both the long- and short-wave edges of the first peak ($h1 \to h2$) remain abrupt. At the concentration $p = 2 \times 10^{11}$ cm^{-2}, the hole masses $m_{hh} = 0.51 m_0$ and $m_{lh} = 0.082 m_0$ in the bulk semiconductor, and the well width $a = 200$ Å, the center of the first absorption peak of light with the $\vec{e} \perp z$ polarization corresponds to the energy $\hbar\omega = 5$ meV, with the peak width being approximately 1 meV.

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APPENDIX

To simplify the problem, consider the oblique incidence of light with s-polarization and $\vec{e} = (e_x, 0, 0)$. Then, the operator of velocity is written down as

$$\vec{e}\vec{v} = \frac{1}{\hbar}\Big[2 \quad A + \frac{5}{4}B \quad k_x -$$

$$-2B \quad \hat{J}_x^2 k_x + [J_x J_y]\, k_y \quad \Big] e_x - \frac{2}{\hbar} B e_x \left[\hat{J}_x \hat{J}_z \right] \hat{k}_z. \qquad \text{(A1)}$$

We need to calculate the following matrix elements:

$$\langle S|A|S\rangle, \langle a|\hat{A}|a\rangle, \langle S|\hat{A}|a\rangle, \langle a|\hat{A}|S\rangle, \qquad \text{(A2)}$$

where $|S\rangle = \hat{R}_S$, $|a\rangle = \hat{R}_a$, and $\hat{A}$ is either a unit matrix or one of matrices $\hat{J}_x^2$, $\left[\hat{J}_x \hat{J}_y\right]$, or $\left[\hat{J}_x \hat{J}_z\right]$. Then, it is not difficult to obtain the following relations useful for further calculations:

$$\int_{-L/2}^{+L/2} \langle S|\hat{1}|S\rangle dz = \int_{-L/2}^{+L/2} \langle a|\hat{1}|a\rangle dz =$$

$$= \overline{C^2(z)} \quad 1 + 3\,\tilde{W}_+ \quad ^2 + |\eta|^2 \overline{S^2(z)} \quad 1 + 3\,\tilde{W}_- \quad ^2 \quad N^2, \qquad \text{(A3)}$$

$$\int_{-L/2}^{+L/2} \langle S|\hat{J}_x^2|S\rangle dz = \int_{-L/2}^{+L/2} \langle a|\hat{J}_x^2|a\rangle dz =$$

$$= \frac{3}{4}N^2\Big\{\overline{C^2(z)}\ \ 1+7W_+^2-2W_+2\cos 2\varphi_{\vec{k}}\ \ -$$

$$+|\eta|^2\overline{S^2(z)}\ \ 1+7W_-^2-2W_-2\cos(2\varphi_{\vec{k}})\ \ \Big\}, \tag{A4}$$

$$\int\limits_{-L/2}^{+L/2}\langle S|\hat{J}_x\hat{J}_y|S\rangle dz = \int\limits_{-L/2}^{+L/2}\langle a|\hat{J}_x\hat{J}_y|a\rangle dz =$$

$$= -3N^2\left\{\overline{C^2(z)}W_+\sin(2\varphi_{\vec{k}}) + |\eta|^2 W_-\overline{S^2(z)}\sin(2\varphi_{\vec{k}})\right\}, \tag{A5}$$

$$\int\limits_{-L/2}^{+L/2}\langle S|\hat{J}_y\hat{J}_z|S\rangle dz = -\frac{3}{2}N^2|\eta|\,(W_+-W_-)\ \ \overline{CS'}-\overline{S'C}\ \ , \tag{A6}$$

$$\int\limits_{-L/2}^{+L/2}\langle S|\hat{J}_y\hat{J}_z|S\rangle dz = \int\limits_{-L/2}^{+L/2}\langle a|\hat{J}_y\hat{J}_z|a\rangle dz =$$

$$= -\frac{3}{2}N^2|\eta|\,(W_+-W_-)\ \ \overline{CS'}-\overline{S'C}\ \ . \tag{A7}$$

Here,

$$\int\limits_{-L/2}^{+L/2} C_m(z)\,C_n(z)\,\frac{dz}{L} = \overline{C^2(z)},$$

$$\int\limits_{-L/2}^{+L/2} S_m(z)\,S_n(z)\,\frac{dz}{L} = \overline{S^2(z)}, \tag{A8}$$

$$-i\int\limits_{-L/2}^{+L/2} C_m(z)\,\frac{\partial}{\partial z}S_n(z)\,\frac{dz}{L} = \overline{CS'},$$

$$-i\int\limits_{-L/2}^{+L/2} S_m(z)\,\frac{\partial}{\partial z}C_n(z)\,\frac{dz}{L} = \overline{SC'}, \tag{A9}$$

and the relation $\eta = |\eta|e^{2i\varphi_{\vec{k}}}$ was used.

Thus, we have established that, at illumination of the DQW (001) with the s-polarized light, the optical transitions between the states with identical parities are allowed in a quadratic in $\vec{k}$ approximation for the effective Hamiltonian.

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ПРО ПОГЛИНАННЯ ЛІНІЙНО ПОЛЯРИЗОВАНОГО ВИПРОМІНЮВАННЯ У НАНОСТРУКТУРІ НАПІВПРОВІДНИКА З ДІРКОВОЮ ПРОВІДНІСТЮ

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Р е з ю м е

Розглянуто поглинання лінійно поляризованого випромінювання у напівпровідниковій розмірно-квантованій ямі, пов'язане з оптичними переходами як між вітками легких і важких дірок, так і між розмірно-квантованими підзонами. Встановлено основні риси поглинання світла у нескінченно глибокій симетричній ямі, яке характеризується внутрішньозонним поглинанням світла, пов'язаним з прямими оптичними переходами дірок між підзонами, що формуються за рахунок розмірного квантування. Отримано спектральну залежність коефіцієнта поглинання світла.