POLARIZATION DEPENDENCES OF LIGHT EMITTED BY FREE ELECTRONS IN MULTIVALLEY SEMICONDUCTORS

V.M. BONDAR, A.E. LEVSHIN, P.M. TOMCHUK

Institute of Physics, Nat. Acad. Sci. of Ukraine
(46, Nauky Ave., Kyiv 03028, Ukraine)

UDC 535.341

The polarization dependences of light emitted by free electrons in n-Ge at liquid helium temperatures have been studied. The general expression for the intensity of spontaneously emitted light in multivalley semiconductors has been obtained, taking into account the mechanism of spontaneous emission of phonons by hot electrons. The results of numerical calculations of the polarization dependences of the light emission are in a qualitative agreement with the experiment.

1. Introduction

The phenomena of absorption and emission of light by free electrons have been studied for rather a long time (see, e.g., works [1, 2]). Nevertheless, these researches dealt mainly with the isotropic law of electron dispersion and single-valley semiconductors. On the other hand, multivalley semiconductors possess their own specificity. The features of the phenomena concerned in such semiconductors stem from the strong anisotropy of the electron dispersion law, from the fact that electrons populate several valleys, and from the anisotropy of scattering. It is known that, for the event of phonon absorption or emission by a free electron to occur, a "third body"is needed in addition to an electron and a photon, which ensures that the conservation laws for the energy and the momentum are satisfied during collisions. The role of a "third body" can be played by lattice vibrations (phonons), impurities, or interfaces. In this way, various mechanisms of scattering (and, therefore, scattering anisotropy) can affect the processes of absorption and emission of light.

In the state of thermodynamic equilibrium, free electrons absorb as many photons as they emit. The principle of detailed balance holds true. If the number of photons exceeds their equilibrium value, i.e. a semiconductor is irradiated by an external electromagnetic field, free current carriers absorb photons. If there is no external electromagnetic field and the electron gas is hot (e.g., is heated with the help of a dc electric field), free current carriers emit light.

A new regularity in the light emission by free electrons in multivalley semiconductors, in contrast with single-valley ones, is the dependence of the emitted light intensity on its polarization. The theory of this phenomenon has been developed in works [2, 3]. The scattering by impurities and uniformly distributed acoustic phonons has been considered. The phenomenon of the polarization dependence of the emitted light intensity was studied experimentally at temperatures of liquid helium in works [4,5]. At such temperatures, there appears a necessity to take into account the influence of the processes of spontaneous emission of acoustic phonons on the polarization dependence of the intensity of emitted light. Such a task, as well as a comparison of the theory with experiment, constitutes the purpose of this work.

2. Polarization Dependence of the Emitted Light Intensity upon Its Scattering by Spontaneously Emitted Acoustic Phonons

Consider the anisotropic scattering of electrons by acoustic vibrations of the lattice in multivalley semiconductors such as n-Ge. The collision integral of electrons with acoustic phonons, taking into account the influence of the high-frequency electromagnetic field of a wave on a collision event, looks like

$$If = \sum_s\sum_{\vec q}\sum_{\ell=-\infty}^{\infty} W^{(s)}(\vec q)\; I_\ell^2\left(\frac{e\gamma}{m_\perp \omega\, c}\right)\left\{\left[f(\vec p+\hbar\vec q)\times\right.\right.$$
$$\left.\times\left(N_{\vec q}^{(s)}+1\right)-f(\vec p)N_{\vec q}^{(s)}\right]\delta\left(\varepsilon_{\vec p+\hbar\vec q}-\varepsilon_{\vec p}-\right.$$
$$\left.-\hbar\omega_{\vec q}^{(s)}-\ell\hbar\omega\right)+\left[f(\vec p-\hbar\vec q)N_{\vec q}^{(s)}-f(\vec p)(N_{\vec q}^{(s)}+1)\right]\times$$
$$\left.\times\delta(\varepsilon_{\vec p-\hbar\vec q}-\varepsilon_{\vec p}+\hbar\omega_{\vec q}^{(s)}-\ell\hbar\omega)\right\}. \qquad (1)$$

Here, $f(\vec p)$ is the distribution function of electrons over the momentum $\vec p$; $N_{\vec q}^{(s)}$ the distribution function of phonons in the s-th branch of the spectrum; $\hbar\omega_{\vec{q}}^{(s)}$ the energy of phonons; $\hbar\omega$ the energy of a light quantum; $W^{(s)}(\vec{q})$ the probability of scattering; I_ℓ the Bessel function of the ℓ-th order; $m_\parallel$ and $m_\perp$ are the longitudinal and transverse electron masses, respectively; c the speed of light; and e the electron charge. Defining the vector potential of the electromagnetic wave in the form

$$\vec{A} = \vec{A}_0 \cos(\omega\, t - \vec{\chi}\, \vec{r}),$$

we obtain the following expression for the parameter γ in Eq. (1):

$$\gamma = \vec{A}_0\, \vec{q} + \left(\frac{m_\parallel}{m_\perp} - 1\right) (\vec{A}_0\, \vec{q})\, (\vec{q}\, \vec{i}_0), \tag{2}$$

where $\vec{i}_0$ is the unit vector directed along the rotation axis of the mass ellipsoid of the i-th valley. Provided that the temperature of the lattice is low or the electrons are strongly hot, the condition

$$\frac{\hbar\omega_q^{(s)}}{\theta_0} >> 1,$$

where θ_0 is the lattice temperature (in the energy units), is satisfied; whence,

$$N_{\vec{q}}^{(s)} + 1 \approx 1. \tag{3}$$

The estimations show [6] that, at the temperatures of liquid helium, there exists a broad interval of electron energies (0.0014 eV $< \varepsilon < \hbar\omega_0 =$ 0.037 eV), where the dominating mechanism of momentum and energy scattering of an electron is the spontaneous emission of acoustic phonons.

The total probability of electron scattering by the phonons of all the three acoustic branches is

$$W(\vec{q}) = \sum_{s=1}^{3} \frac{\pi\, q}{4\,\rho\, V} \left\{ \frac{1}{s_\parallel} \left[\Xi_d + \Xi_u \left(\frac{\vec{i}_0\, \vec{q}}{q}\right)^2 \right]^2 + \right.$$
$$\left. + \frac{\Xi_u^2}{s_\perp} \left[1 - \left(\frac{\vec{i}_0\, \vec{q}}{q}\right)^2 \right] \left(\frac{\vec{i}_0\, \vec{q}}{q}\right)^2 \right\}. \tag{4}$$

Here, $s_\parallel$ and $s_\perp$ are the longitudinal and transverse speeds of sound, respectively; V is the volume; ρ the density; and Ξ_d and Ξ_u are the constants of the deformation potential. According to Eq. (1), the energy that electrons transfer to the lattice per unit time in the presence of an electromagnetic wave is

$$P = \int \varepsilon\,(\vec{p})\; I f(\vec{p})\, d\,\vec{p} = \mathrm{P}_0 - \frac{V}{(2\,\pi\,\hbar)^3} \hbar\omega \times$$
$$\times \sum_{\ell=-\infty}^{\infty} \ell \int d\vec{p}\, f(\vec{p}) \int d\vec{p}'\, W(\vec{q})\, I_\ell^2 \left(\frac{\mathrm{e}\,\gamma}{\mathrm{m}_\perp\, \omega\, c}\right) \times$$
$$\times\, \delta\, \left(\varepsilon(\vec{p}\,') - \varepsilon(\vec{p}) - \ell\, \hbar\, \omega\right), \tag{5}$$

where $\vec{p}\,' = \vec{p} + \hbar\vec{q}$, and P_0 is the energy transferred in the absence of an electromagnetic wave. The estimations show that, at reasonable values of the frequency and the strength of the electromagnetic wave field, it is possible to confine the consideration in Eq. (5) to only the quantum-mechanical processes with $\ell = \pm 1$, where the value $\ell = -1$ describes the processes that are connected to the emission, while $\ell = -1$ does the processes that are connected to the absorption of a light quantum.

Making use of the electron distribution function in the form

$$f(\vec{p}) = \frac{n_i}{(2\,\pi\,\theta_i)^{\frac{3}{2}}\, m_\perp \sqrt{m_\parallel}}\; e^{-\varepsilon/\theta_i} \tag{6}$$

and a standard procedure (see, e.g., work [4]), we obtain the following expression for the radiation power emitted by electrons of the i-th valley:

$$\mathrm{P}_i^{(-)} = \frac{\Xi_d^2 \sqrt{m_\parallel\, m_\perp}\, \hbar\, \omega\, n_i}{\pi\, \rho\, \hbar^4\, s_\parallel} \left(\frac{\mathrm{e}}{c}\right)^2 \times$$
$$\times \left[1 + 6\, \frac{\theta_i}{\hbar\,\omega} + 12 \left(\frac{\theta_i}{\hbar\,\omega}\right)^2 \right] \times$$
$$\times \left[A_\perp^2\, M_\perp + A_\parallel^2\, \frac{m_\perp}{m_\parallel}\, M_\parallel \right] \mathrm{e}^{-\frac{\hbar\,\omega}{\theta_\mathrm{i}}}, \tag{7}$$

where

$$A_\perp^2 = A_0^2 - A_0^2 \left(\vec{i}_0 \vec{j}_0\right)^2, \quad A_\parallel^2 = A_0^2 \left(\vec{i}_0 \vec{j}_0\right)^2, \tag{8}$$

and $\vec{j}_0$ is the unit vector which characterizes the polarization of the wave.

The radiation power of spontaneous emission can be obtained from Eq. (7) by normalizing the vector potential of the wave $\vec{A}$ in such a way that the volume V should contain N_{ph} photons, i.e. using the condition

$$\frac{1}{V}\, N_{\mathrm{ph}}\, \hbar\, \omega = \frac{\mathrm{E}^2}{4\,\pi} = \frac{1}{8} \left(\frac{\omega}{c}\right)^2 A^2,$$

whence, provided $N_{\mathrm{ph}} = 1$,

$$A = 2c \left(\frac{2\,\pi\,\hbar}{V\,\omega}\right)^{\frac{1}{2}}. \tag{9}$$

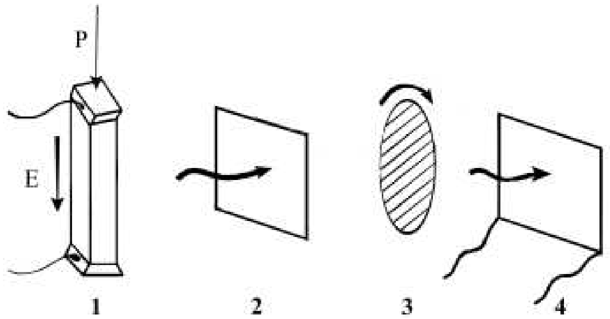

Fig. 1. Experimental setup for studying the emission polarization: (*1*) germanium specimen, (*2*) filter made up of black polyethylene for cutting off the short-wave range of radiation with $\lambda < 50$ μm, (*3*) polarizer, and (*4*) radiation detector

Substituting expression (9) into Eq. (7), taking into account Eqs. (8), and multiplying the obtained result by the density of final field states in a solid angle $d\Omega$, $d\,\rho(\omega) = \frac{V}{(2\,\pi\,c)^3}\,d\Omega$, we obtain

$$W_a^{c(-)} = \sum_{i=1}^{4} \frac{\Xi_d^2 \mathrm{e}^2 \sqrt{m_\| m_\perp}\,(\hbar\omega)^2\; n_i}{\pi^3\,\rho\hbar^4 s_\| c^3} \times$$

$$\times \left[1 + 6\frac{\theta_i}{\hbar\omega} + 12\left(\frac{\theta_i}{\hbar\omega}\right)^2\right] \times$$

$$\times \left[M_\perp + \left(M_\| \frac{m_\perp}{m_\|} - M_\perp\right)\left(\vec{i}_0\vec{j}_0\right)^2\right] \mathrm{e}^{-\frac{\hbar\omega}{\theta_i}}\,d\Omega, \qquad (10)$$

where $M_\|$ and $M_\perp$ are the coefficients, which are connected with the mass anisotropy and equal to

$$M_\| = \frac{1}{8\alpha\,b}\big\{4b^4 - 2\alpha^2 b^2 - \alpha^4 b\,b_1 + 2b_2 b^3 \times$$

$$\times[4b - 6b\alpha^2 + 3b_1\alpha^4] + 4b_3 b_2^2 \alpha^2 b^2\left[2b^2 + 4\alpha^2 -\right.$$

$$\left.-3b\alpha^2 b_1\right] + b_2^2[b^4 - b_3\alpha^2 b^2][4 - 10\alpha^2 - 30\alpha^4 + 15\alpha^4 b_1]\big\},$$

$$M_\perp = \frac{1}{4\alpha b}\big\{2b^2 + bb_1\alpha^2 + 2b_2 b^3 \times$$

$$\times[2b - b_1\alpha^2] + 4\,b_3 b_2^2 b^2 \alpha^2[bb_1 - 1] +$$

$$+ b_2^2 b^2[b^2 - b_3\alpha^2][2b^2 + 4\alpha^2 - 3bb_1\alpha^2]\big\} - \frac{M_\|}{2}.$$

Here, $b = \sqrt{\alpha^2 + 1}$, $b_1 = \ln\left[\frac{b+1}{b-1}\right]$, $b_2 = \frac{\Xi_u}{\Xi_d}$, $b_3 = \frac{s_\|}{s_\perp}$, $\alpha^2 = \frac{m_\perp}{m_\| - m_\perp}$.

For $\theta_i \gg \hbar\omega$, i.e. in the case of classical emission, Eq. (10) reads

$$W_a^{c(-)} = \sum_{i=1}^{4} 12\frac{\Xi_d^2 \mathrm{e}^2 \sqrt{m_\| m_\perp}\,(\theta_i)^2\, n_i}{\pi^3 \rho\hbar^4 s_\| c^3} \times$$

$$\times\left[M_\perp + \left(M_\| \frac{m_\perp}{m_\|} - M_\perp\right)\left(\vec{i}_0\,\vec{j}\right)^2\right] d\Omega. \qquad (11)$$

Whereas, for $\theta_i \ll \hbar\omega$, i.e. in the case of quantum emission, we get

$$W_a^{c(-)} = \sum_{i=1}^{4} \frac{\Xi_d^2 \mathrm{e}^2 \sqrt{m_\| m_\perp}\,(\hbar\omega)^2\, n_i}{\pi^3 \rho\hbar^4 s_\| c^3} \times$$

$$\times\left[M_\perp + \left(M_\| \frac{m_\perp}{m_\|} - M_\perp\right)\left(\vec{i}_0\vec{j}\right)^2\right] \mathrm{e}^{-\frac{\hbar\omega}{\theta_i}}\,d\Omega. \qquad (12)$$

3. Experimental Part and the Results of Numerical Calculations

As was mentioned above, the experimental researches of the polarization dependence of the light emission intensity were fulfilled using n-Ge specimens [4, 5]. The experimental setup borrowed from work [4]is shown in Fig. 1. Specimens $1 \times 1 \times 6$ mm^3 in size were cut off in such a manner that the longest dimension was oriented along the $\langle 111\rangle$ crystallographic direction. The dc electric field and pressure were applied along this direction as well. The researches were carried on at a temperature of 5 K. The emitted wavelength was $\lambda = 100$ μm. In a 0°-(reference) position of the polarizer, its marks were aligned along the $\langle 111\rangle$ axis of the specimen. The intensity of light radiation, which had passed through the polarizer, was measured provided a dc electric field of 20—150 V/cm, a uniaxial deformation pressure of 0—4 kbar, and an electron concentration of 10^{15} cm^{-3}.

While calculating the polarization dependence of radiation, the influence of the acoustic spontaneous emission and impurity-induced scattering was taken into account. The expressions for the energy power, which is emitted into a solid angle $d\Omega$ at the impurity-induced scattering of energy by the conduction electrons, were obtained earlier [3] and look like

$$W_{\mathrm{p}}^{(-)} = \frac{e^6\, n_a \sqrt{m_\|}}{(2\pi)^{\frac{3}{2}}\, \varepsilon_0^2 c^3 \left(m_\| - m_\perp\right)^2} \times$$

$$\times \sum_i \frac{n_i}{\sqrt{\theta_i}} \Psi_i \ln\left(C_1\, x_{\min}\right)^{-1} d\Omega \qquad (13)$$

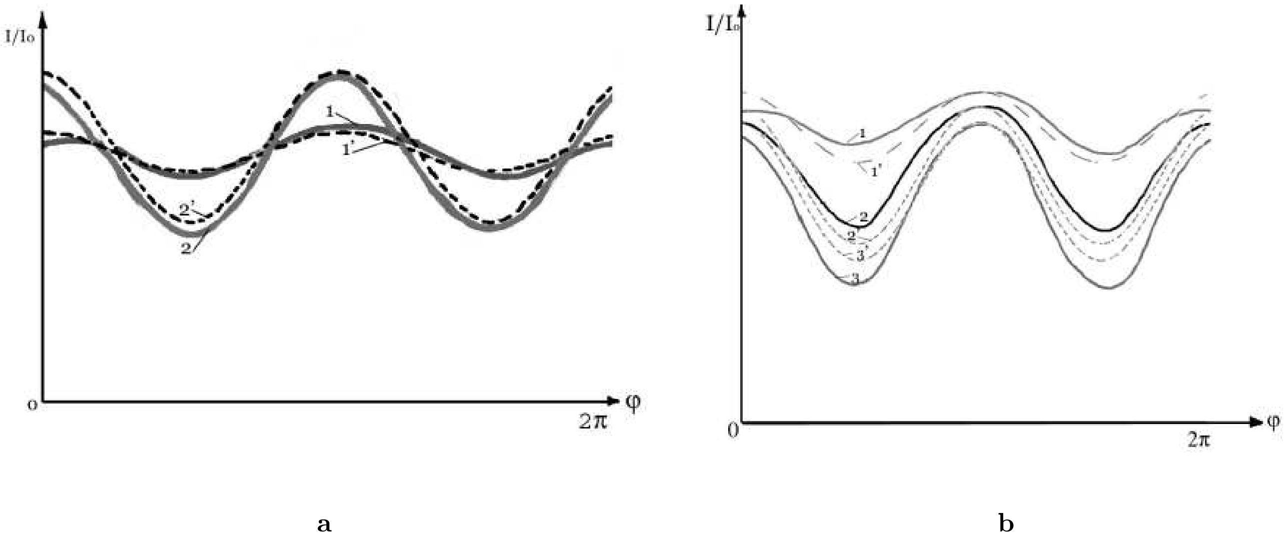

Fig. 2. Experimental (solid curves) and theoretical (dashed curves) dependences of the emission intensity of n-Ge specimens ($n = 10^{15}$ cm^{-3}) on the rotation angle of the polarizer at various amplitudes of the electrical field E and the uniaxial pressure p applied to the specimen: (a) $E = 15$ V/cm; $p = 0$ (curves *1* and *1'*) and 4 kbar (curves *2* and *2'*); (b) $E = 20$ V/cm; $p = 0$ (curves *1* and *1'*), 2 (curves *2* and *2'*), and 4 kbar (curves *3* and *3'*)

for the classical range of frequencies and

$$W_{\mathrm{p}}^{(-)} = \frac{e^6 n_a \sqrt{m_\parallel}}{\sqrt{2\,\hbar\omega}\pi\,\varepsilon_0^2 c^3 \left(m_\parallel - m_\perp\right)^2}\times$$

$$\times\sum_i n_i\,\Psi_i\,\mathrm{e}^{-\frac{\hbar\omega}{\theta_i}}\,d\Omega \qquad (14)$$

for the quantum-mechanical one, where

$$\Psi_i = \frac{1}{\alpha^3}\left[\alpha + \left(1-\alpha^2\right)\mathrm{arctg}\,\frac{1}{\alpha}\right] +$$

$$+\left[2\frac{m_\perp}{m_\parallel}\left(-\frac{1}{1+\alpha^2} + \frac{1}{\alpha}\,\mathrm{arctg}\,\frac{1}{\alpha}\right) -\right.$$

$$\left.-\frac{1}{\alpha^3}\left(\alpha + \left(1-\alpha^2\right)\mathrm{arctg}\,\frac{1}{\alpha}\right)\right]\left(\vec{i}_0\vec{j}_0\right)^2$$

$$x_{\min} = \frac{\hbar^2}{8\,m_\perp\theta_i\,r_{\mathrm{D}}^2},$$

r_{D} is the Debye radius, n_a the impurity concentration, ε_0 the dielectric permittivity of the substance, and ln $C_1 = 0.577\ldots$ the Euler constant.

In order to find the total intensity of emission, it is necessary to sum up the expressions for the intensities of spontaneous acoustic and impurity-induced emissions over all the valleys.

In the course of numerical calculations, the temperature and the concentration of electrons in the valleys were determined from the equations of energy balance for each of the valleys. The scattering of the momentum both by impurities and owing to the spontaneous emission of phonons, as well as the scattering of the energy upon the spontaneous emission of acoustic phonons and upon the collisions with optical ones, was taken into account. The results of numerical calculations showed that the main contribution to the emission is given by electrons which move along the light-mass axis in the valley. Warming the carriers with a dc electric field, as well as applying a uniaxial pressure to the specimen, results, in our case, in the repopulation of electrons into the $\langle 111\rangle$ valley. Being warmed, but without applying the pressure, the electrons become redistributed among valleys; but the differences between the carrier temperatures in the valleys are small, so that the degree of polarization of the light emission is low. If the pressure is applied along the $\langle 111\rangle$ axis, electrons pass to the $\langle 111\rangle$ valley, and the degree of polarization grows. The results of numerical calculations are presented in Fig. 2.

4. Conclusions

The results of numerical calculations of the polarization dependence of the light emission in n-Ge at low temperatures, which were obtained with regard for the spontaneous acoustic and impurity-induced scattering, are in a qualitative agreement with the experimental data measured at low values of the dc electric field $F < 70$ V/cm and the concentration of electrons $n = 10^{15}$ cm^{-3}. At greater amplitudes of the dc electric field, the calculated values of the polarization degree exceed the experimental ones. In our opinion, it may be related to the enhancement of the influence of the asymmetric part of the electron distribution function at low temperatures, which can become substantial and comparable with the contribution of the symmetric part even at low fields $F > 70$ V/cm. It may results in the appearance of a competing mechanism of light polarization with the orientation along the dc field. The influence of this phenomenon on the polarization dependence of the emitted light will be considered elsewhere.

The authors express their gratitude to O.G. Sarbei for the numerous and fruitful discussions of the materials of this work.

1. *Vas'ko F.T.* // Fiz. Tekhn. Polupr. — 1990. — **24**, N 4. — P. 682—686.
2. *Bondar V.M., Sarbei O.G., Tomchuk P.M.* // Fiz. Tverd. Tela. — 2002. — **44**, N 9. — P. 1540—1546.
3. *Tomchuk P.M.* // Ukr. Fiz. Zh. — 2004. — **49**, N 7. — P. 681—690.
4. *Bondar V.M., Chernomorets N.F.* // Ibid. — 2003. — **48**, N 1. — P. 51—55.
5. *Bondar V., Tomchuk P., Tulupenko V.* // Fiz. Nizk. Temp. — 2001. — **27**, N 7. — P. 767—768.
6. *Denis V., Pozhela Yu.* Hot Electrons. — Vilnus: Mintis, 1971 (in Russian).

Received 14.03.05.
Translated from Ukrainian by O.I. Voitenko

ПОЛЯРИЗАЦІЙНІ ЗАЛЕЖНОСТІ ВИПРОМІНЮВАННЯ СВІТЛА ВІЛЬНИМИ ЕЛЕКТРОНАМИ В БАГАТОДОЛИННИХ НАПІВПРОВІДНИКАХ

В.М.Бондар, О.Є.Левшин, П.М.Томчук

Р е з ю м е

Досліджено поляризаційні залежності випромінювання світла вільними електронами в n-Ge при температурах рідкого гелію. Отримано загальний вираз для інтенсивності спонтанного випромінювання світла в багатодолинних напівпровідниках із врахуванням механізму спонтанного випромінювання фононів гарячими електронами. Отримано якісне узгодження чисельних розрахунків поляризаційної залежності випромінювання з експериментом.