EFFECTS OF DYNAMIC MAGNETOELASTIC INTERACTION IN MAGNETIC CRYSTALS

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This paper is dedicated to Victor G. Bar'yakhtar on the occasion of his 75th birthday

The method of calculation of the magnetoelastic interaction energy for crystals of a certain symmetry has been developed in the framework of the dynamic theory of magnetoelasticity which is valid for large-scale strains.

The spontaneous magnetization of magnetically ordered crystals is one of a few bright manifestations of collective quantum-mechanical properties of solids at the macroscopic level. A magnetic ordering arises owing to purely quantum-mechanical effects of the spin exchange interaction between electrons which compose, as a rule, the internal unfilled shells of atoms. However, in some cases, the phenomena in magnets admit the macroscopic consideration in terms of the average magnetization in a crystal. For ferromagnets, it is a single vector $\vec{M}$; for more complicated magnetic structures, one has to introduce several vector quantities $\vec{M}_\alpha$ called the magnetizations of sublattices. The dynamic properties of such magnets are described by the functions $\vec{M}_\alpha(\vec{r},t)$, and it is supposed that $\vec{M}_\alpha$'s change little at interatomic distances.

The deviations of the magnetization $\vec{M}$ from its equilibrium value propagate in the form of waves which are referred to as spin waves in a magnetically-ordered crystal; they have been discovered by F. Bloch [1]. The macroscopic equations of motion for the magnetization have been formulated by L.D. Landau and E.M. Lifshits [2], and the fundamentals of the quantum-mechanical theory of spin waves have been grounded by the works of T. Holstein and H. Primakoff [3] and A.I. Akhieser [4].

Spin waves govern both the high-frequency properties of magnets and their thermodynamic and kinetic properties in the low-temperature range [5]. Spin waves can interact with sound and electromagnetic waves, with beams of charged particles, and also with neutral particles that possess a magnetic moment. They can be scattered by impurities, dislocations, and other crystal defects. Moreover, they can form specifically bound states if there exist non-homogeneities in the magnetic subsystem.

The resonant interaction of spin and elastic waves in ferromagnets has been considered for the first time by A.I. Akhiezer, V.G. Baryakhtar, and S.V. Peletminsky [6] and by Ch. Kittel [7]. The coupling between elastic and spin waves was shown to be governed by magnetostriction and ponderomotive forces: the coupling parameter is of the order of $M_0^2/(\rho c_s^2) \sim 10^{-5}$, where ρ is the density of the substance, and c_s the speed of sound. The magnetoelastic interaction manifests itself most effectively, provided the conditions for the magnetoacoustic resonance are fulfilled, i.e. when the frequency and the wave vector of a spin wave coincide with those of a sound wave, respectively. In this case, coupled magnetoelastic waves propagate in the bulk of the crystal. The magnetoelastic interaction eliminates the degeneration in the cross point of branches of the sound spin wave spectrum, so that the branches of magnetoelastic waves become separated by a gap proportional to the square root of the coupling parameter, and a characteristic "hybridization" of the coupled wave branches takes place [8]. The interaction between magnetic and elastic waves brings about the change of the speed of sound in the resonant frequency range, the additional absorption of sound, and the rotation of its polarization plane [9].

The magnetoacoustic resonance in antiferromagnets has been considered by S.V. Peletminsky [10]. In work [11], the variation of the speed of sound and the attenuation of magnetoelastic waves have been calculated for antiferromagnets in various ground states. The quantum-mechanical theory of the coupled magnetoelastic waves has been developed in work [12].

In the early works dealing with the theory of coupled magnetoelastic waves, the energy of the

magnetostriction interaction was constructed from the components of the magnetization vector $\vec{M}$ and the symmetric tensor of deformation $u_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i}\right)$. This kind of the interaction energy does not ensure the conservation of the total angular momentum of a magnet. In Vlasov's [13] and Tiersten's [14] works, it has been shown that the magnetostriction energy is determined by the distortion tensor $\frac{\partial u_i}{\partial x_k}$ rather than the deformation one u_{ik}, and the potential energy of a ferromagnet has to be constructed from certain combinations of dynamic variables, among which the magnetic moment per unit mass of the ferromagnet $\vec{\mu}$, the displacement vector $\vec{u}$, as well as their derivatives $\frac{\partial \mu_i}{\partial x_k}$ and $\frac{\partial u_i}{\partial x_k}$, were chosen. In order to ensure the fulfillment of the conservation law for the total angular momentum, these variables should enter into the expression for the energy density in the form of definite combinations, which are invariant with respect to any rotations of the ferromagnet as a whole, together with the magnetic moments and the field of elastic deformations. For a ferromagnet, there exist 18 such independent invariant combinations: $\mu_j \frac{\partial x_j}{\partial \xi_i}$, $\frac{\partial \mu_j}{\partial \xi_k}\frac{\partial x_j}{\partial \xi_i}$, and $\frac{\partial x_j}{\partial \xi_i}\frac{\partial x_j}{\partial \xi_k}$, where ξ_i are the Lagrange coordinates of a point in the crystal with the actual Euler coordinates x_i. The energy density of a ferromagnet F is the function of these invariant combinations:

$$F = F\left(\mu_j \frac{\partial x_j}{\partial \xi_i},\ \frac{\partial \mu_j}{\partial \xi_k}\frac{\partial x_j}{\partial \xi_i},\ \frac{\partial x_j}{\partial \xi_k}\frac{\partial x_j}{\partial \xi_i}\right). \tag{1}$$

In the framework of this approach, it is possible to ensure the fulfillment of the conservation law of the angular momentum of the system which is the sum of the orbital and spin moments.

In this connection, there appeared a necessity to revise the concept of magnetoelastic interaction in antiferromagnets as well, taking the rotation of bulk elements of the crystal into consideration. This matter was dealt in work [15], where a complete system of equations of the macroscopic theory of magnetoelastic processes in antiferromagnets has been formulated. The magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ of sublattices per unit mass of the antiferromagnet, the displacement vector $\vec{u}$, the vectors of electric and magnetic fields, and the entropy of a unit mass of the magnet were chosen as dynamic variables. On the basis of the conservation laws of energy, momentum, and angular momentum, the expressions for the apparent magnetic fields, strain tensor, density of the energy flux, and relaxation terms in the equations of motion were obtained. The strain tensor in an antiferromagnet, similarly to that in a ferromagnet, turns out non-symmetric; and, owing to the conservation law of the total angular momentum, there emerges a correlation between the anti-symmetric part of the strain tensor and the magnetic anisotropy energy. The regular account of the magnetoelastic interaction leads to a change of the dependence of the magnetoelastic coupling parameters on the direction of the magnetoelastic wave propagation [16]. There also emerges a correlation between the anti-symmetric part of the viscosity tensor and the relaxation members in the equations for the magnetic moments. The latter circumstance brings about the occurrence of the non-resonant additional absorption of sound in magnetically ordered crystals, which is not connected to the magnetostriction interaction. It is significant that these effects are purely dynamic, and the strain tensor takes a usual symmetric form in the equilibrium state.

The expression for the energy density of an antiferromagnet F as a function of $\vec{\mu}_1$, $\vec{\mu}_2$, $\frac{\partial \mu_{1i}}{\partial x_k}$, $\frac{\partial \mu_{2i}}{\partial x_k}$, and $\frac{\partial u_i}{\partial x_k}$ has to be constructed making use of their combinations that are invariant with respect to arbitrary rotations of the antiferromagnet as a whole, together with the magnetic moments and the field of elastic deformations. While searching for such combinations, it is convenient to pass from the actual (Euler) coordinates x_i to the coordinates of the same points of the magnet in the initial state $\vec{\xi} = \vec{x} - \vec{u}(\vec{x}, t)$ (Lagrange coordinates) and to include the derivatives with respect to ξ_i rather than the derivatives with respect to x_i. Upon a rotation, the quantities ξ_i remain constant, because they correspond to a certain initial position of the antiferromagnet, and the quantities $\vec{\mu}_1$, $\vec{\mu}_2$, $\frac{\partial \vec{\mu}_1}{\partial \xi_k}$, $\frac{\partial \vec{\mu}_2}{\partial \xi_k}$, and $\frac{\partial \vec{u}}{\partial x_k}$ are transformed by the vector law. Those eleven vectors and two tensors, δ_{ik} and e_{ikl}, can compose 231 invariants. However, only 30 of them are independent invariants. Those are the following:

$$A_i^{(1)} = \mu_{1j}\frac{\partial x_j}{\partial \xi}, \quad A_i^{(2)} = \mu_{2j}\frac{\partial x_j}{\partial \xi}, \quad B_{ik}^{(1)} = \frac{\partial \mu_{1j}}{\partial \xi_i}\frac{\partial x_j}{\partial \xi_k},$$

$$B_{ik}^{(2)} = \frac{\partial \mu_{2j}}{\partial \xi_i}\frac{\partial x_j}{\partial \xi_k}, \quad C_{ik} = \frac{\partial x_j}{\partial \xi_i}\frac{\partial x_j}{\partial \xi_k}. \tag{2}$$

Thus, the invariant expression for the energy density of an antiferromagnet looks as

$$F = F\left(A_i^{(1)},\ A_i^{(2)}, B_{ij}^{(1)},\ B_{ij}^{(2)}, C_{ij}\right). \tag{3}$$

In this form, the function F defines the energy density not only for antiferromagnets but also for ferrites with two magnetic sublattices. The explicit expression of

the function F depends on the specific symmetry of the crystal.

In the case of small deformations, one can easily obtain the expression for the magnetoelastic interaction energy by expanding Eq. (3) in a power series of $\frac{\partial u_i}{\partial x_k}$:

$$F = a + \lambda_{ik} u_{ik} + \left(\frac{\partial a}{\partial \mu_{1i}} \mu_{1k} + \frac{\partial a}{\partial \mu_{2i}} \mu_{2k} \right) \varepsilon_{ik} + \dots , \quad (4)$$

where $a = F\left(\mu_{1i}, \mu_{2i}, 0, 0, \delta_{ij}\right)$ and $\varepsilon_{ik} = \frac{1}{2}\left(\frac{\partial u_k}{\partial u_i} - \frac{\partial u_i}{\partial u_k}\right)$. The first term in Eq. (4) describes the energy of magnetic anisotropy, the second the energy of magnetostriction connected with the tensor of deformation, and the third the magnetoelastic energy related to a rotation of elements in the bulk. The third term is expressed through the known energy of magnetic anisotropy.

However, the procedure of constructing the energy of an antiferromagnet can also be given in a more general form suitable for the description of large deformations. The methods of constructing the energies of the magnetic and elastic subsystems separately are known well in the physics of crystals; known are also the methods for constructing the magnetoelastic energy related to the symmetric tensor of deformation for crystals with a specific symmetry [17, 18]. Therefore, it is of practical interest to formulate a procedure, which would allow us to construct an invariant expression for the energy density suitable for the dynamic theory of magnetoelasticity, by using the explicit expressions for the energies of the magnetic and elastic subsystems, as well as the energy of static magnetoelastic interaction in a crystal with a specific symmetry.

With this purpose, let us analyze how the operations of crystal symmetry transform quantities (2). The Lagrange coordinates ξ_i remain constant when the body moves, because they correspond to the equilibrium state of the antiferromagnet. Under the transformations of the crystal symmetry, the coordinates ξ_i are transformed according to the vector law, and the dynamic invariants (2) according to some representations of the symmetry group of the antiferromagnet. The quantities $A_i^{(1)}$, $A_i^{(2)}$, $B_{ij}^{(1)}$, $B_{ij}^{(2)}$, and C_{ij} compose three groups which are transformed independently. The operation of time inversion $\hat{R}$ changes the signs of $A_i^{(1)}$, $A_i^{(2)}$, $B_{ij}^{(1)}$, and $B_{ij}^{(2)}$. The quantities C_{ij} are transformed according to a tensor representation. The transformation character of $A_i^{(1)}$, $A_i^{(2)}$, $B_{ij}^{(1)}$, and $B_{ij}^{(2)}$ is more complicated. If the elements of a crystallographic group of symmetry include such an element which results in a permutation of atoms of the magnetic sublattices (an odd element), the quantities $A_i^{(1)}$ and $A_i^{(2)}$ (as well as $B_{ij}^{(1)}$ and $B_{ij}^{(2)}$) are transformed through each others. If such elements are absent from the group (all elements are even), the quantities $A_i^{(1)}$, $A_i^{(2)}$, $B_{ij}^{(1)}$, and $B_{ij}^{(2)}$ are transformed independently: $A_i^{(1)}$ and $A_i^{(2)}$ according to a vector representation, and $B_{ij}^{(1)}$ and $B_{ij}^{(2)}$ according to a tensor one.

Therefore, the construction of the energy density of an antiferromagnet $F\left(A_i^{(1)}, A_i^{(2)}, B_{ij}^{(1)}, B_{ij}^{(2)}, C_{ij}\right)$ consists in searching for all the combinations of the dynamic invariants (2) which also remain invariant under the action of all the elements of the symmetry group of the crystal (taking into account their parity with respect to the permutation of sublattice atoms) and under the action of the time inversion operator.

It is easy to see that the quantities $A_i^{(1)}$, $A_i^{(2)}$, $B_{ij}^{(1)}$, $B_{ij}^{(2)}$, and C_{ij} are transformed in the same manner as M_{1i}, M_{2i}, $\frac{\partial M_{1i}}{\partial x_j}$, $\frac{\partial M_{2i}}{\partial x_j}$, and u_{ik}, respectively, upon all the indicated transformations. Hence, the construction of the antiferromagnet's energy comprises the known procedure of finding the combinations of M_{1i}, M_{2i}, $\frac{\partial M_{1i}}{\partial x_j}$, $\frac{\partial M_{2i}}{\partial x_j}$, and u_{ik}, which are invariant with respect to the transformations of the crystal symmetry, and the subsequent replacement of those quantities by $A_i^{(1)}$, $A_i^{(2)}$, $B_{ij}^{(1)}$, $B_{ij}^{(2)}$, and C_{ij}, respectively. The energy F is an arbitrary function of the invariant combinations obtained.

In order to illustrate the aforesaid, we construct, as an example, the part of the magnetoelastic energy, which is connected to a rotation of the bulk elements, for a uniaxial antiferromagnet of the $MnCO_3$ type. Its magnetic structure has the symmetry axis of the third order C_3^+ (an even element) and a slip plane parallel to it σ_y^- (an odd element) [17], while the density of the magnetic anisotropy energy is defined by the expression

$$F_{ma} = \delta \vec{M}_1 \vec{M}_2 - \tfrac{1}{2}\beta(M_{1z}^2 + M_{2z}^2) -$$

$$-\beta' M_{1z} M_{2z} - d(M_{1x} M_{2y} - M_{1y} M_{2x}), \quad (5)$$

where δ is the exchange constant, β and β' are the constants of anisotropy, z is the axis of anisotropy, and d is the Dzyaloshinskii constant.

Let us substitute $A_i^{(1)}$ and $A_i^{(2)}$ for M_{1i} and M_{2i}, respectively, in Eq. (5). The expressions for $A_i^{(1)}$ and $A_i^{(2)}$ are taken from Eq. (2). The result obtained contains the following term proportional to ε_{ik}:

$$F_{me}^{(a)} = \left[\beta\left(M_{1i} M_{2z} + M_{1i} M_{2z}\right) +\right.$$

$$+\beta'(M_{1i}M_{2z}+M_{1i}M_{2z})+d\,e_{ijk}M_{1i}M_{2z}\big]\,\varepsilon_{iz}. \qquad (6)$$

Expression (6) for the antisymmetric part of the magnetoelastic energy in antiferromagnets of the $MnCO_3$ type was obtained for the first time in work [19].

The sequential account of the magnetoelastic interaction brings about the loss of symmetry of the tensor of elastic constants and to the emergence of terms depending on the rotation tensor ε_{ik} in the expression for the antiferromagnet's energy. Quantitatively, the corresponding additives to the elastic constants are proportional to the small parameter $\delta M_0^2/(\rho c_s^2)$. However, they can become substantial while considering such phenomena as the rotation of the sound polarization plane.

The contributions to the energy of magnetoelastic interaction in magnets, which are caused by a rotation of bulk elements, are proportional to the ratio between the constant of anisotropy and the constant of magnetostriction. This ratio can vary within rather wide limits, depending on a specific material: from 10^{-3} for garnets up to 0.1 for $MnCO_3$.

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ДИНАМІЧНІ ЕФЕКТИ МАГНІТОПРУЖНОЇ ВЗАЄМОДІЇ У МАГНІТОВПОРЯДКОВАНИХ КРИСТАЛАХ

В.В. Ганн

Р е з ю м е

У рамках динамічної теорії магнітопружності розвинуто метод побудови енергії магнітопружної взаємодії для кристалів з визначеною симетрією. Цей метод придатний для опису великих деформацій.