DYNAMIC CHAOS IN THE MOTION AND SCATTERING OF FAST CHARGED PARTICLES IN CRYSTALS

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This paper is dedicated to Victor G. Bar'yakhtar on the occasion of his 75 birthday.

We consider a number of effects related to the dynamic chaos that appears in the motion of fast charged particles which move in the periodic field of an atomic string within a crystal. A possibility for particles to move either regularly or chaotically in such a field is demonstrated. Some consequences of such a dynamics for the processes of dechanneling and scattering of relativistic particles and radiation emission by them in the crystal have been discussed. A possibility of anomalous diffusion, when particles channel in the crystal, has been demonstrated.

1. Introduction

For a plenty of years, Victor Grygorovych Bar'yakhtar gave lectures on statistical physics at the Kharkiv State University. A number of important fundamental issues of statistical physics, such as the substantiation of the statistical physics method, the entropy, the Maxwell's demon, and so on, had often been considered at his lectures. Those problems, in this or that form, do arise in various branches of physics.

One of the most interesting and intriguing directions of statistical physics is the problem of dynamic chaos in the motion of particles in external fields. It is connected with the fact that if one talks about statistical physics, systems with a huge number of the degrees of freedom are usually implied. However, the following question arises: What is the minimal number of the degrees of freedom for the laws of statistical physics to be still valid? It has been demonstrated, in the last decades, that the minimal dimension of the systems, for which the laws of statistical physics are still applicable, is 2. The necessary condition in this case is that such a system should possess only a single integral of motion; it is possible if the system is nonlinear and is described by an asymmetric potential. The motion of a particle in such systems would be perceived as stochastic for prolonged time intervals. It allows one, in some cases, to take advantage of the methods of statistical physics in order to describe the physical processes which accompany the motion of particles.

A possibility for the dynamic chaos to emerge in a vast number of problems has been demonstrated in the last decades (see, e.g., works [1–4]). One of those problems is a transmission of high-energy particles through oriented crystals [3,4]. The dynamic chaos may occur in such a system owing to the following reasons. If a fast charged particle moves in a crystal along one of the crystal axes, there appear correlations between consecutive impacts of the particle with lattice atoms. Due to these correlations, the motion of the particle in the crystal is mainly governed by a continuous potential of those chains of crystal atoms which are oriented along that crystal axis (z-axis). Such a field preserves the component of the particle momentum parallel to the z-axis, with the motion in the transversal plane (xy-plane) obeying a two-dimensional equation of motion of a particle in the periodic potential field of crystal's atom chains.

The potential of every atomic chain is a substantially nonlinear function of coordinates x and y; therefore, nonlinear interaction effects are essential while describing the particle motion in the crystal. At first glance, any particle motion in such a periodic field of crystal's atom chains may appear to behave always regularly. Actually, it is not so, because a substantially irregular chaotic motion of particles in the crystal may take place alongside with a regular one.

Thus, the problem of the motion of particles in a crystal is a typical problem of the theory of nonlinear systems, where the phenomenon of dynamic chaos is allowable. The authors of works [5, 6] drew attention to this circumstance, while studying the motion of high-energy particles in oriented crystals. In this work, we report a number of results obtained in the course of researches of the motion and scattering of fast particles in crystals. In so doing, the main attention is paid to the manifestations of the dynamic chaos phenomenon in physical processes which accompany the transmission of high-energy particles through oriented crystals.

2. Regular and Chaotic Dynamics of Fast Charged Particles in Crystals

If a fast charged particle moving in a crystal is scattered at small angles, the effective constant of its interaction with lattice atoms is large [4]. In this case, the particle motion within the crystal can be considered in the framework of classical mechanics.

As is well known in classical electrodynamics, the trajectory of a relativistic particle in an external field is determined by the equation of motion [4]

$$\frac{d\mathbf{p}}{dt} = -\vec{\nabla}U(\mathbf{r}), \tag{1.1}$$

where $\mathbf{p}(t) = m\mathbf{v}(1 - \mathbf{v}^2/c^2)^{-1/2}$ is the particle momentum, m its mass, $\mathbf{v}(t)$ its velocity, and c the speed of light. If the particle moves in a crystal, this equation can be written down in the form

$$m\frac{d\mathbf{v}}{dt} = -(1 - \mathrm{v}^2/c^2)^{1/2}\left[\vec{\nabla}U_c(\mathbf{r}) - \frac{\mathbf{v}}{c^2}\left(\mathbf{v}\cdot\vec{\nabla}U_c(\mathbf{r})\right)\right], \tag{1.2}$$

where

$$U_c(\mathbf{r}) = \sum_n u(\mathbf{r} - \mathbf{r}_n) \tag{1.3}$$

is the potential energy of interaction between the particle and crystal's atoms, and $u(\mathbf{r} - \mathbf{r}_n)$ the potential energy of interaction between the particle and the lattice atom located at the point $\mathbf{r}_n$. The summation in Eq. (1.3) is carried on over all crystal atoms.

If a fast charged particle moves in the vicinity of one of the crystal axes (e.g., the z-axis), the variation of the impact parameter between consecutive collisions of the particle with lattice atoms is small. In this case, the trajectory of the particle in the crystal varies smoothly in time. It allows one to take advantage of the approximation of a continuous potential of atomic chains in order to describe the particle motion in the crystal [7]. In this approximation, the particle motion in the crystal is governed mainly by a continuous potential of atomic chains which is the potential of the lattice averaged along the z-axis:

$$U(x,y) = \frac{1}{L}\int\limits_{-\infty}^{\infty} dz\, U_c(x,y,z), \tag{1.4}$$

where L is the crystal thickness.

In this field, the component of the particle momentum parallel to the z-axis remains constant (see Eq. (1.1)). The motion of the particle in the transversal plane is determined by the two-dimensional equation

$$\ddot{\rho} = -\frac{c^2}{\varepsilon_\parallel}\frac{\partial}{\partial\boldsymbol{\rho}}U(\boldsymbol{\rho}), \tag{1.5}$$

where $\boldsymbol{\rho} = (x,y)$ and $\varepsilon_\parallel = c\sqrt{p_z^2 + m^2c^2}$. If the angle of incidence ψ of the particle onto the crystal reckoned from its axis is small, one may assume, to the accuracy of ψ^2, that the quantity $\varepsilon_\parallel$ is the energy of the incident particle ε.

Taking the periodicity of the atomic arrangement in the lattice into account, the quantity $U(x,y)$ can be represented as a sum of the continuous potentials of individual atomic chains of the crystal, which are aligned in parallel to the z-axis:

$$U(x,y) = \sum_n U_R(\boldsymbol{\rho} - \boldsymbol{\rho}_n), \tag{1.6}$$

where $\boldsymbol{\rho}_n = (x_n, y_n)$ is the location of the n-th chain axis in the transversal plane,

$$U_R(\boldsymbol{\rho}) = \frac{1}{L}\int\limits_{-\infty}^{\infty} dz \sum_n u(\boldsymbol{\rho}, z - na), \tag{1.7}$$

and a the interatomic distance along the z-axis. Thus, we come to the problem of the two-dimensional motion of a particle in the periodic potential field of crystal's atom chains (1.5).

Fig. 1 exhibits a typical example of the continuous potential of the lattice (1.6) in the case where a positively charged particle moves in a silicon crystal along the $\langle 100 \rangle$ axis. The calculations were carried out in the framework of Molière's model for the potential of an individual lattice atom, taking into account thermal fluctuations of atoms at $T = 293$ K. The locations of the atomic chain axes (at room temperature, $U(x, y) = 92.2$ eV at those points) are marked by crosses in this figure.

The equipotential curves displayed in Fig. 1 illustrate the fact that the potential $U(x, y)$ possesses significant maxima at the points that correspond to the positions of the axes of every atomic chain. Moreover, this potential is a symmetric function of coordinates in the vicinity of every atomic chain. In the region between the atomic chains, the potential $U(x, y)$ has shallow two-dimensional potential wells. The particle motion in such a field can be finite (trajectories b and d) or infinite (trajectories a, c, e, and f) with respect to crystal's atom chains. The type of a trajectory is determined by the first integral of the equation of motion (1.5) which is the traversal energy of the particle:

$$\varepsilon_{\perp} = \frac{1}{2}\varepsilon\dot{\boldsymbol{\rho}}^2 + U(x, y). \tag{1.8}$$

It is clear that the finite motion would take place, if $\varepsilon_{\perp} < V_c$, where V_c is the potential energy at the saddle point [the integration constant of Eq. (1.5) was chosen so that the quantity $\varepsilon_{\perp}$ should be equal to zero at the center of the region between crystal's atom chains]. Such a motion of the particle in the crystal is called hyperchanneling [8].

If $\varepsilon_{\perp} > V_c$, the particles would move infinitely (over the barrier) with respect to crystal's atom chains. Any particle would interact, in this case, in series with various chains of atoms.

Concerning the second integral of motion of Eq. (1.5), its existence is by no means obligatory owing to the essential nonlinearity of the problem under consideration [5]. Nevertheless, if the second integral of motion of Eq. (1.5) does exist, two variables in this equation are separable, and the particle will move quasi-periodically in the transversal plane. Otherwise, the particle will move chaotically in this plane. The latter case is coined a phenomenon of dynamic chaos in the particle motion in the potential field $U(x, y)$. It is essential that the phenomenon of dynamic chaos is realized provided either the finite or infinite particle motion in the field $U(x, y)$ [5,6].

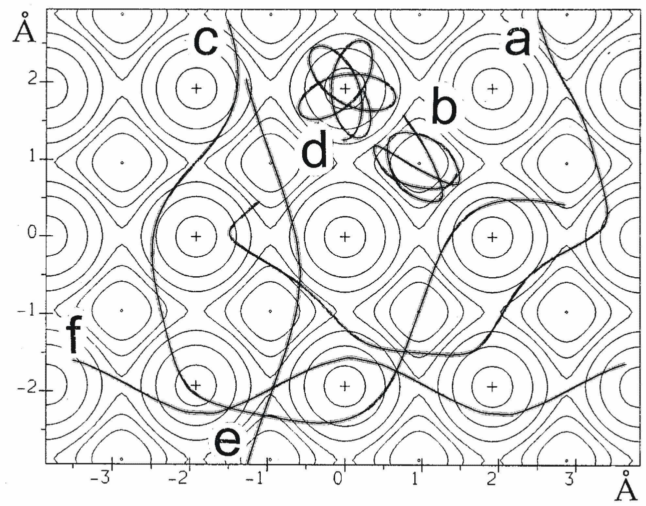

Fig. 1. Equipotential surfaces of the continuous potential energy of the interaction between a positively charged particle (positron) and a silicon crystal, if the particle moves at a small angle ψ with respect to the $\langle 100 \rangle$ axis; and typical trajectories of the positively (a, b, and e) and negatively (c, d, and f) charged particles in the field $U(x, y)$. Points correspond to the values $U(x, y) = 0$; the equipotential curves are drawn for the values $U(x, y) = 1$, 2, 3, 5, 10, and 30 eV

Thus, the character of the motion of particles in a crystal is rather complicated. Therefore, keeping finding the particle trajectory in view, the method of the numerical solution of Eq. (1.5) takes on a special significance. Fig. 1 exposes the typical trajectories of particles in a crystal which possess either a regular or chaotic character. The calculations were carried out, provided various initial conditions for Eq. (1.5).

The regular and chaotic trajectories in a periodic field of crystal's atom chains are possible for both positively and negatively charged particles. For electrons, the minus sign should be assigned to the equipotential surfaces as compared to those exhibited in Fig. 1. In this case, the deep potential wells, where the finite motion of an electron is possible, are located near all the axes of the atomic chains. The typical trajectories of negatively charged particles in the potential field $U(x, y)$, which correspond to the regular and chaotic characters of the particle motion, are designated by letters c, d and f in Fig. 1.

The study of the character of motion is rather important while analyzing the stability of various trajectories in the crystal. For example, when analyzing the process of dechanneling of relativistic electrons from

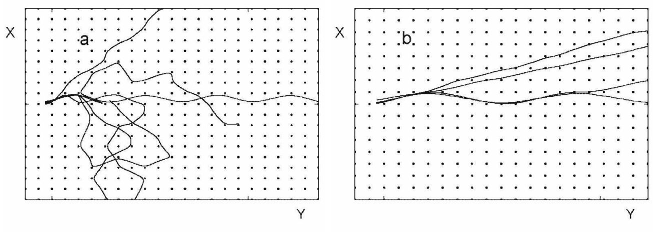

Fig. 2. Typical trajectories of positively charged particles that move in the overbarrier mode in a silicon crystal along the $\langle 100 \rangle$ axis at $\psi = \psi_c$ (a) and $2\psi_c$ (b)

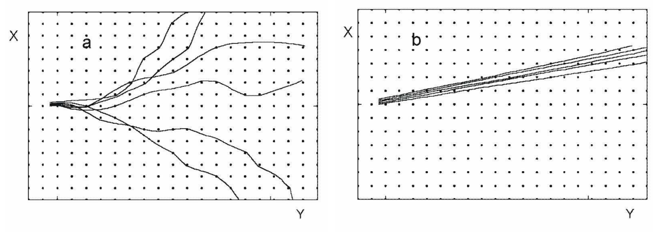

Fig. 3. The same as in Fig. 2, but for negatively charged particles at $\psi = \psi_c$ (a) and $5\psi_c$ (b)

axial channels, the model, where the potential of each axial channel is supposed axisymmetric, is frequently used (see, e.g., work [9]). The process of dechanneling, i.e. the escape of a particle from the channel, occurs mainly owing to the multiple scattering of an electron by thermal fluctuations of atoms' positions in the crystal. Thereto, the electron must find itself in this region. If the electron channeling occurs in an axially symmetric field, there exist two integrals of motion: the transversal energy of the particle and its angular momentum. In this case, the particle motion is always regular and quasiperiodic. Moreover, there are the trajectories of channeled electrons, which do not pass through the region of thermal fluctuations of lattice atoms, so that the process of dechanneling is suppressed for such electrons. The account of the asymmetry of the axial channel brings about the opportunity for the dynamic chaos to arise in the motion of a channeled electron in the channel, owing to which the electron passes through the region of thermal fluctuations of atoms and quickly dechannels. Such a situation takes place, e.g., if an electron channels in a silicon crystal along the $\langle 110 \rangle$ axis. In this case, the channel is formed by the potential of coupled atomic chains and is rather asymmetric (see Fig. 8 in work [3]). The lengths of the particle dechanneling in the models of the symmetric and real potentials of a + channel can differ from one another by several orders of magnitude. This circumstance appeared extremely important while studying the characteristics and mechanisms of radiation by relativistic electrons in thick crystals (see works [10, 11]). We note that the account of the multiple scattering of an electron by the electron subsystem of the lattice also results in the violation of its regular motion in the channel. Owing to that, the angular momentum of the particle changes, which is accompanied by small variations of its transversal energy; this allows the particle to pass through the region of thermal fluctuations of lattice atoms and, hence, to dechannel quickly.

Therefore, the analysis of the stability of the motion of particles in a crystal is extremely important for the consideration of the physical processes which accompany the transmission of particles through the crystal.

3. Motion and Scattering of Particles in a Crystal in the Overbarrier Mode

In the mode of overbarrier motion, the particle consistently collides with various crystal's atom chains. Such a motion of a particle is characteristic of all particles in the incident beam, provided $\psi \geq \psi_c$, where ψ_c is the critical angle for the axial channeling [3, 7]. In the periodic field of atomic chains, the particle can move either regularly or chaotically with respect to those chains [6].

In the regular mode of the particle motion, there emerge correlations between consecutive impacts of the particle with atomic chains. Such a motion takes place, e.g., if particles participate in the plane channeling along the open channels formed by a regular arrangement of atomic chains in the crystal. If the motion is chaotic, collisions between the particle and various atomic chains can be regarded as stochastic. In this case, the methods of statistical physics can be applied to describe a number of interaction processes between the particle and the atomic chains.

The features of the motion of particles that were marked above are illustrated in Figs. 2 and 3, where the results of numerical calculations of the trajectories of positively and negatively charged particles that move in the overbarrier mode in the transversal plane are depicted. The calculations consisted in solving Eq. (1.5) numerically. Various trajectories in the figures correspond to various initial conditions and to various values of the angle of incidence ψ of the particles onto the silicon crystal with respect to the $\langle 100 \rangle$ crystal axis.

The results of calculations testify to that the motion of positively or negatively charged particles can be

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regular or chaotic in the transversal plane, which depends on initial conditions. The positively charged particles moves regularly between crystal's atom planes made up of periodically arranged atomic chains. Such a motion is rather stable, because positively charged particles cannot approach closely, provided the plane channeling, to the axes of atomic chains, where the potential is highly non-uniform, even, may be, at $\psi \leq \psi_c$. For negatively charged particles, the mode of plane channeling is less stable than that for positively charged ones. This is related to the fact that electrons, provided their plane channeling, always pass through the region, where the axes of atomic chains are located. The increase of the angle ψ results in enhancing the stability of the plane channeling modes of particles.

The determination of the motion modes of particles in the crystal is extremely important while analyzing the angular distributions of particles scattered by the crystal. In particular, provided that a beam of particles strikes the crystal at a small, with respect to one of the crystal axes, angle ψ, the angular distributions of scattered particles are determined by the features of their scattering by a separate atomic chain and the features of their multiple scattering by various atomic chains [12].

The scattering of a particle in the field of the continuous potential of a separate atomic chain is possible only along a certain azimuthal angle φ in the transversal plane. The multiple scattering by various atomic chains results in the redistribution of particles over the angle φ. Nevertheless, in the case of the plane channeling of particles, the scattering angles cannot grow as the target thickens. In this case, all scattering angles of the particles must be confined within the range $\theta \leq \theta_p$, where θ_p is the critical angle of the plane channeling [4,7]. At the same time, the scattering angles for particles, which move chaotically in the field of atomic chains, should grow with the enlargement of the target thickness.

Fig. 4 displays the results of our calculations of the angular distributions of 1-GeV relativistic electrons and positrons scattered by the continuous potential field of crystal's atom chains. The points correspond to the results of calculations of individual trajectories, when particles strike the silicon crystal at the angles $\psi = 0.5\psi_c$, ψ_c, and $2\psi_c$ with respect to the $\langle 100\rangle$ crystal axis. The results of calculations illustrate the features inherent to the scattering of fast charged particles by atomic chains, which were indicated above. They testify to that, in the continuous potential field of atomic chains, particles are scattered along the azimuthal angle

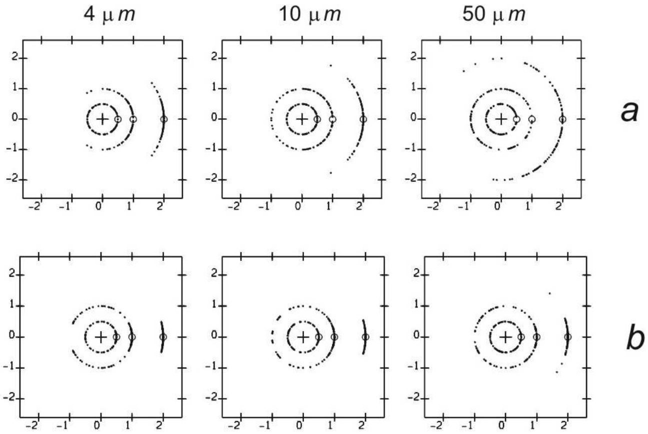
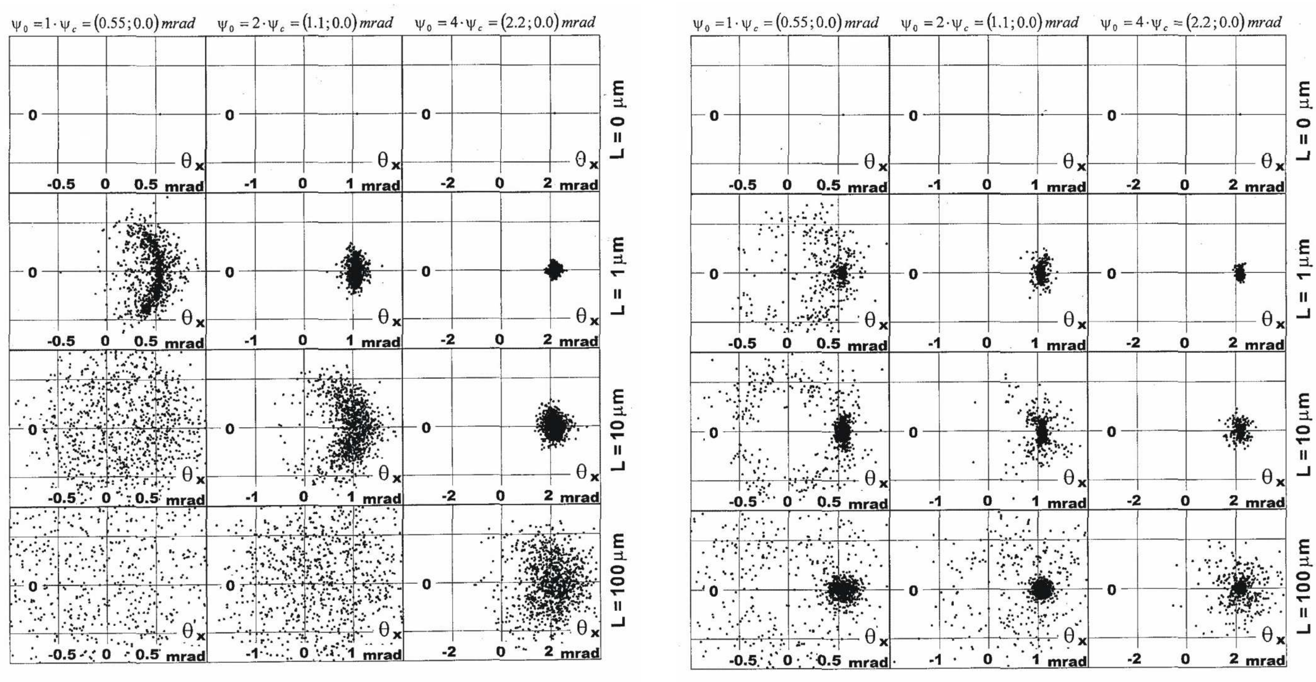


Fig. 4. Multiple azimuthal scattering of negatively (a) and positively (b) charged particles in the continuous potential field of atomic chains of a silicon crystal in cases where the incidence angle of a particle with respect to the $\langle 100\rangle$ axis is equal to $\psi = 0.5\psi_c$, ψ_c, and $2\psi_c$. Crosses denote the direction of the $\langle 100\rangle$ axis, and hollow circles denote the direction of the incident beam; points are the results of the modeling; figures indicate the values of the input thickness parameter of the crystal (in μm) for calculations; the axes are graded in the degrees of the angles of scattering θ_x and θ_y referred to the angle ψ_c

φ in the transversal plane, and the angular distributions for electrons and positrons differ at $\psi \geq \psi_c$ rather considerably.

Taking the non-coherent scattering effects into account results in the dechanneling of particles and their rechanneling into plane channels, the additional scattering of particles along the azimuthal angle φ in the transversal plane, and the opportunity of their scattering along a certain polar angle, i.e. it results in the variation of the angle ψ amplitude. The motion of particles in the crystal, making allowance for this effect, can be described on the basis of the numerical simulation of the transmission of particles through the crystal. Such a method has been developed in work [13]. In this method, the whole path of a particle is divided into a great number of small intervals. Within every interval, the particle's path is determined by solving Eq. (1.5). At the end of the interval, the additional increments to scattering angles, which are caused by the non-coherent scattering of the particle in this interval, are selected randomly.

Fig. 5 exhibits the results of the modeling of the angular distributions of electrons (a) and positrons (b), calculated taking into account the non-coherent scattering effects. These calculations were carried out



a b

Fig. 5. Angular distributions of scattered electrons (a) and positrons (b) with the energy $\varepsilon = 500\,\text{MeV}$ calculated taking into account the non-coherent scattering effects

for particles with the energy $\varepsilon = 500\,\text{MeV}$ that strike silicon crystals of various thicknesses at the angles $\psi = \psi_c$, $2\psi_c$, and $4\psi_c$ with respect to the $\langle 100\rangle$ axis. The results obtained testify to that thin crystals reveal a significant asymmetry of the angular distribution of particles, which is caused by the features of their scattering in the continuous potential field of atomic chains. As the energy of particles grows, the contribution of the non-coherent scattering process to the formation of the angular distributions of scattered particles decreases. In particular, for particles with energies of about 100 GeV, the significant asymmetry in scattering is preserved even in crystals, the thickness of which is about several centimeters [13].

The results presented testify to that, in wide ranges of the particle energy and the crystal thickness, the scattering is governed mainly by the continuous potential of atomic chains and by the circumstance that the average values of the particle scattering angles in such a field are much greater than the average scattering angles in an amorphous target of the same thickness. A description of the scattering process in such a field is rather complicated in the general case. Significant simplifications arise in the case where the phenomenon of dynamic chaos takes place. In this case, the collisions of particles with various atomic chains can be considered as stochastic, which allows one to take advantage of the methods of statistical physics in order to find the distribution function of particles over the scattering angle. In doing so, the scattering of a particle by an atomic chain as a single object, rather than its scattering in the field of an individual atom of the lattice should be regarded as an elementary act of the interaction between the particle and the crystal. The kinetic equation for the particle distribution function $f(\varphi, z)$ over the azimuthal angle of scattering φ at the depth z has the following form [3, 14, 15]:

$$\frac{df}{dz} = nd\psi \int_{-\infty}^{\infty} db[f(\varphi + \varphi(b), z) - f(\varphi, z)], \tag{2.1}$$

where n is the atomic density of the crystal; d the interatomic distance along the crystal axis z, along which the particle moves; and $\varphi(b)$ the function that describes a deviation of the particle in the continuous potential field of an atomic chain, being dependent on the impact parameter of the chain b (see work [15, formula (4.2)]). The solutions of this equation were analyzed in works [3, 15]. Among all the results obtained there, we will discuss only the dependence of the mean-square angle of the multiple scattering of particles in the crystal, which is determined by Eq. (2.1), on the crystal thickness and the angle ψ. This dependence is determined by the relation (see work [3, formula (5.4)])

$$\overline{\vartheta^2} = 2\psi^2\left\{1 - \exp\left[-2nd\psi z\int\limits_{-\infty}^{\infty} db\left(\sin\frac{\varphi(b)}{2}\right)^2\right]\right\}. \tag{2.2}$$

Let us compare the results of calculations of the quantity $\overline{\vartheta^2}$ by this formula with those obtained while modeling the motion of particles in the framework of the model of stochastic and periodic arrangements of atomic chain axes of the crystal in the transversal plane. The relevant relationships between the quantities $\overline{\vartheta^2}$ and ψ are presented in Fig. 6 for electrons and positrons moving with the energy $\varepsilon = 30$ GeV and striking a silicon crystal with the thickness $L = 50$ μm at small angles ψ's with respect to the $\langle 100\rangle$ crystal axis. The ordinates in the plots correspond to the quantity $f = \left(\overline{\vartheta^2}/\overline{\vartheta^2_{\rm am}}\right)^{1/2}$, where $\overline{\vartheta^2_{\rm am}} = \varepsilon_s^2 L/\varepsilon^2 L_{\rm rad}$ is the mean-square angle of the multiple scattering of particles in an amorphous environment, $\varepsilon_s^2 = 4\pi m^2/e^2$, and $L_{\rm rad}$ the radiation length [4]. The solid curves correspond to the results of calculations of $\overline{\vartheta^2}$ by formula (2.2). The solid squares are the simulation results in the model of atomic chains located randomly. The hollow circles and triangles are the results of the simulation of the scattering of particles in the field of crystal's atom chains, the axes of which are arranged periodically in the transversal plane.

The corresponding analysis demonstrates that the results of calculations in the model of stochastically arranged atomic chains and those of analytical calculations according to formula (2.2) are in a good agreement. At the same time, the results of numerical modeling of the motion of particles in the periodic field of atomic chains coincide with the corresponding results of calculations by the same formula only in the angle range $\psi \le \psi_c$, with the coincidence for electrons holding true in a wider interval of the angle ψ than that for positrons. It is related to a higher stochastization in the motion of negatively charged particles in the periodic field of atomic chains as compared to that for positively charged ones (see Fig. 4).

A significant discrepancy between the results of the numerical modeling (hollow circles and triangles) and analytical calculations by formula (2.2) in the range

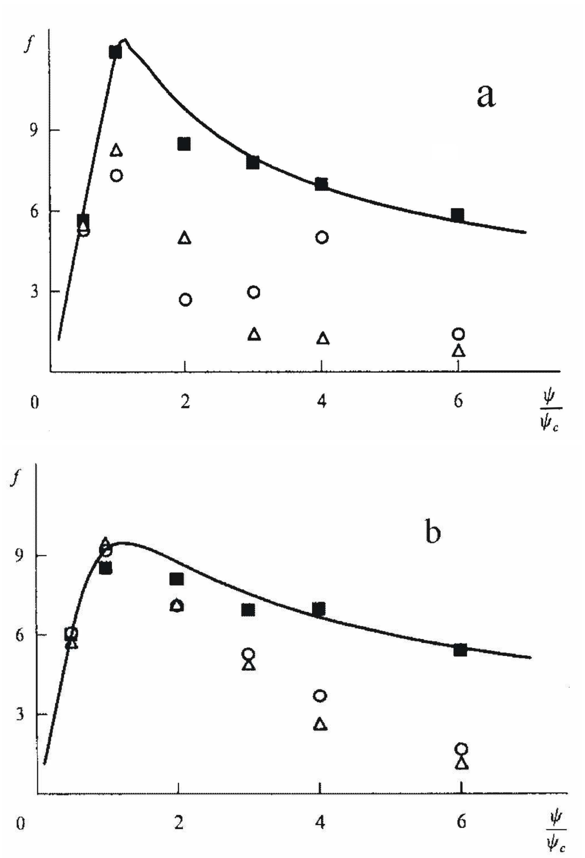


Fig. 6. Orientational dependences of the mean-square angle of the multiple scattering of positively (a) and negatively (b) charged particles by crystal's atom chains

$\psi \gg \psi_c$ is induced by the influence exerted by the regular modes of the particle motion in the periodic field of atomic chains on the scattering.

The features of the particle scattering in the crystal, which were discussed above, are extremely important while analyzing the spectral-angular distributions of the coherent radiation of relativistic electrons and positrons in oriented crystals [4]. Revealing the basic mechanisms of the radiation of particles in crystals requires studying not only the characteristics of the radiation but also the angular distributions of particles scattered by the crystal under the same conditions.

4. Phenomenon of Anomalous Diffusion upon Channeling

Alongside with the regular and chaotic motions of particles in the periodic field of atomic chains, the mode, at which regular intervals of the particle trajectory

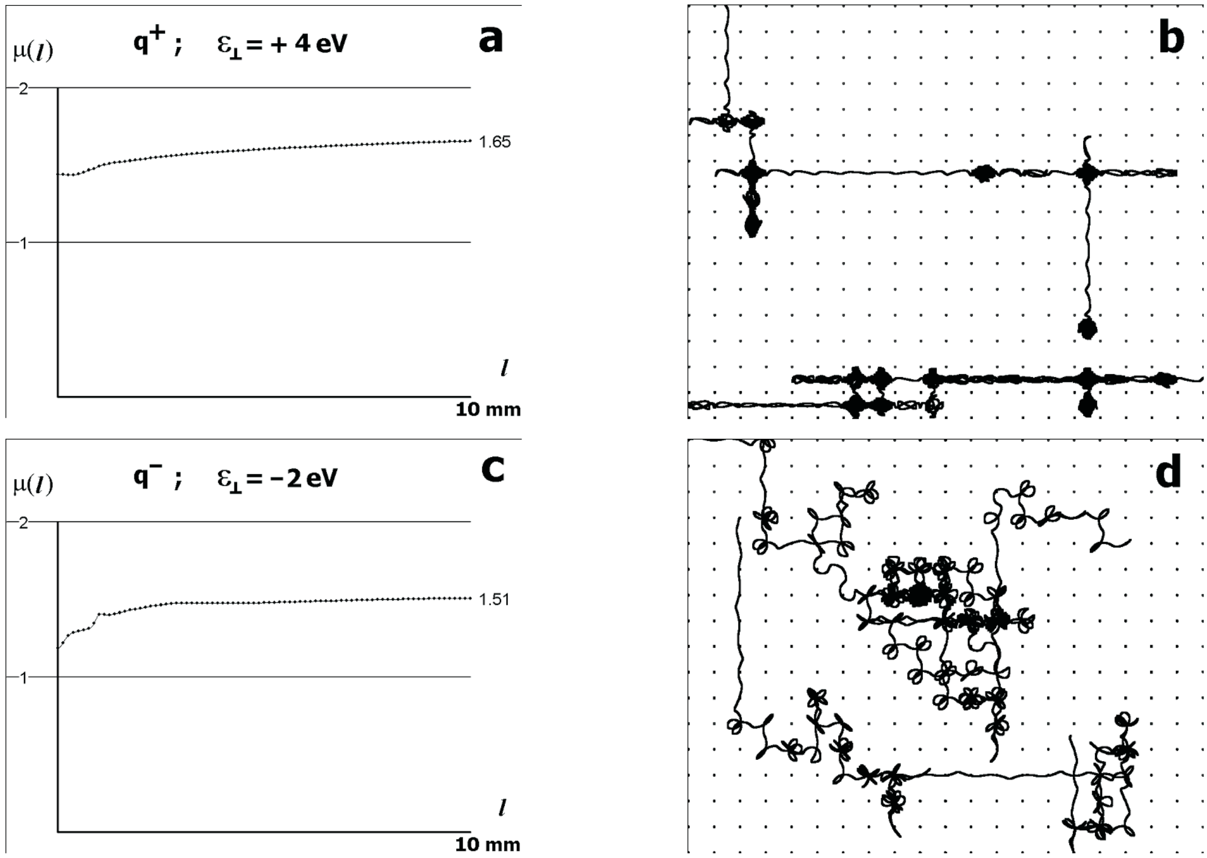


Fig. 7. Diffusion coefficients μ and typical trajectories of positively (a and b) and negatively (c and d) charged particles in the transversal plane of periodically located atomic chains of a $\langle 100\rangle$ silicon crystal. The points mark the positions of atomic chain axes in the transversal plane

alternate with chaotic ones, is possible. Such a situation takes place, if the particle motion in the plane channel is weakly unstable. In this case, the particle, after having passed some interval in the plane channel, owing to the instability of the particle motion which is related to the non-uniformity of the potential of the atomic chain system, escapes from this channel and begins to move chaotically in the periodic field of atomic chains. Then, having found another channel, the particle can be captured into it for some time, and so on. The capture of the particle by the channel is caused by the same non-uniformity of the continuous potential of atomic chains.

The motion of a particle through the crystal reminds, in this case, the so-called "Lévy flights" in statistical physics (see, e.g., works [16, 17]), i.e. such a motion, when the regular intervals of a path alternate with the chaotic ones. Such a mode of the motion is of a special interest, because it brings about the phenomenon of anomalous diffusion of particles. Ib this case, the super- or subdiffusion of particles, i.e. an enhancement or a reduction, respectively, of the diffusion process, is possible. In particular, it results in the replacement of the linear dependence of the mean-square fluctuation of any quantity on time by a more complex power dependence with a fractional power exponent. For example, concerning the time-dependence of the squared displacement of a particle, this new dependence is determined by the relation

$$\left\langle (\Delta\rho)^2 \right\rangle_t \sim t^{\mu}. \tag{3.1}$$

The case $\mu = 1$ corresponds to the normal diffusion and the usual Brown's motion of a particle. Provided $\mu > 1$, there occurs the superdiffusion, when an enhancement of the diffusion takes place in comparison with Brown's case. If $\mu < 1$, the diffusion becomes weaker, which is the case of the subdiffusion of particles.

It is this situation that arises while studying the motion of particles in the periodic field of atomic chains. This circumstance was emphasized in work [18]. To illustrate this, consider the dependence of the squared displacement of a particle in the transversal plane on time. Generally, this dependence is determined by relation (3.1). The considered effect manifests itself most brightly, if the transversal energy (1.8) of the particle is close to the potential energy at the saddle point of the continuous potential of atomic chains ($V_c \approx 3$ eV for the potential displayed in Fig. 1).

Fig. 7 exhibits the results of calculations of the typical trajectories of positrons (b) and electrons (d) with the energy $\varepsilon = 10$ GeV and the transversal energies $\varepsilon_\perp = 4$ or 2 eV in the periodic field of the continuous potential of atomic chains, provided the particles move near the $\langle 100 \rangle$ axis of a 1-cm silicon crystal. The dependence $\mu(t)$ in these cases is

$$\mu(t) = \frac{\ln\left(\left\langle (\Delta\rho)^2 \right\rangle_t / \left\langle (\Delta\rho)^2 \right\rangle_{t_0}\right)}{\ln\left(t/t_0\right)}, \tag{3.2}$$

where t_0 and $\left\langle (\Delta\rho)^2 \right\rangle_{t_0}$ are the normalizing values of the quantities t and $\left\langle (\Delta\rho)^2 \right\rangle_t$, respectively [18]. The calculations were carried out for 10^4 trajectories of particles, corresponding to various initial conditions and a fixed value of $\varepsilon_\perp$.

The results obtained testify to that, in the case concerned, the superdiffusion of particles caused by the “Lévy flights” upon their channeling takes place. The modeling brings about the value $\mu = 1$, provided that the axes of atomic chains are located chaotically in the transversal plane [18], which corresponds to Brown’s mode of the motion of particles.

Thus, the problem of the channeling of particles in the crystal near one of the crystal axes is a typical one in the theory of nonlinear systems, in which the phenomenon of anomalous diffusion is possible. It opens new opportunities for studying the transmission of particles through crystals.

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ЯВИЩЕ ДИНАМІЧНОГО ХАОСУ
В РУСІ ТА РОЗСІЮВАННІ ШВИДКИХ
ЗАРЯДЖЕНИХ ЧАСТИНОК У КРИСТАЛАХ

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Р е з ю м е

Обговорено ряд ефектів, зв’язаних з явищем динамічного хаосу в русі швидких заряджених частинок у періодичному полі атомних ланцюжків кристала. Показано можливість регулярного і хаотичного руху частинок у полі ланцюжків атомів. Розглянуто деякі наслідки такої динаміки для процесів деканалювання, розсіяння та випромінювання релятивістських частинок у кристалі. Показана можливість явища аномальної дифузії при каналюванні частинок у кристалі.