
PHENOMENOLOGICAL THEORY OF AN EXCITON CONDENSATION IN CRYSTALS OF DIFFERENT DIMENSIONS

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UDC 535-37
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The density distribution function and the structure factor for the exciton condensed phase in 1D, 2D, and 3D systems are founded as functions of the exciton pumping, exciton lifetime, exciton-exciton annihilation rate, fluctuations of the exciton density, and other system-dependent parameters. It is shown that, at a small exciton lifetime, the condensed phase does not appear even at large exciton densities because the system is a nonequilibrium one. It is shown that the effect of a finite value of the exciton lifetime may be important in the 2D case due to a super-radiant decay of excitons in low-dimensional structures. The Fourier transform of the density-density correlation function (structure factor) has a sharp maximum at a some value of the wave vector, which evidences for a strong spatial correlation in the system. In 2D case, the light scattering by a crystal with the condensed phase will be observed in the form of a concentric circle. The study of this scattering by a 2D crystal with the exciton condensed phase may be a powerful method of obtaining an information about the condensed phase.

1. Introduction

There is a lot of attempts to find the exciton condensed phase in low-dimensional structures [1–3]. In the present work, we study the influence of the imbalance of a system caused by the finite exciton lifetime and fluctuations of the exciton density on the exciton condensed phase formation in crystals of different dimensions. The finiteness of an exciton lifetime shows itself in two ways. First, a decrease in the exciton lifetime leads to a decrease in the exciton density. So, for the fixed exciton creation rate, we get downward conditions for the exciton condensation. For example, the observed essential role of exciton-exciton annihilation (Auger process) forced to revise many of the earlier researches [4]. Secondly, with the finite exciton lifetime, the system becomes non-equilibrium, which has a great impact on conditions of the exciton condensation, too: under certain circumstances, the condensation phase does not have time to be formed even with a high density of free

excitons. So, at a small exciton lifetime, the condensed phase may not appear even if quantum-mechanical calculations for an equilibrium state predicts it.

In spite of many articles studying the exciton condensation with the wave vector $k = 0$ (Bose–Einstein condensation), we will consider the condensation in the real space caused by an attractive interaction between excitons. For example, the authors of article [5] claimed the observation of the biexciton condensation in a 3D ZnP₂ crystal, when the temperature of the system was greater than the temperature needed for the Bose-condensation. Even for a 2D system of indirect excitons with double quantum wells and the long-range dipole-dipole repulsive interaction, the authors of [6] showed the existence of a minimum in the energy of the system for a non-zero exciton density, by taking into account the exchange and van der Waals interactions. Due to the repulsive dipole-dipole interaction between indirect excitons, such a minimum takes place only for moderate distances between quantum wells ($d < 1.1R_{\text{ex}}$). In addition, the author of article [7] showed that a dielectric electron-hole liquid is energetically more advantageous than a gas of free excitons. Furthermore, the real low-dimensional semiconductor systems include alloys like Ga_{1-x}Al_xAs, Cd_{1-x}Mn_xTe, etc. So, even with the ideal technology, fluctuations of their composition are exist, which causes the exciton scattering. At the same time, the free path of an exciton is compared with the distance between excitons. As shown in work [8], this fact strictly restricts the possibility of the Bose-condensation due to the presence of a disorder in the Kosterlitz–Thouless phase transition.

Quantum-mechanical calculations of the exciton condensed phase taking into account a creation of excitons, their destruction, and a possible spatial distribution over phases (exciton liquid and exciton gas) are extremely complicated, need the solution of

the density matrix equations, and are not carried out till now. In the present work, we use the phenomenological approach suggested in [9–11] when studying the exciton condensation in the 3D case and expand it to two-dimensional systems (quantum wells) and one-dimensional systems (quantum wires). In our approach, we use the powerful methods developed in studies of the spinodal decay dynamics [12, 13] and generalize them for the systems of particles with finite lifetime. It is assumed that the exciton free energy for a system of excitons with infinite lifetime is the known function of the exciton density (calculated with the help of quantum mechanics or postulated with some parameters). Later, we solve the balance equation for the density of excitons taking into account their creation, destruction, and fluctuation processes. Taking into consideration the finite lifetime of excitons can have a significant impact on results by changing the threshold value of the density or causing an inhomogeneity of the spatial distribution of the exciton density.

2. Model and Basic Equations

We consider a nondegenerate exciton system, in which excitons are created with a steady rate, excitons have a finite lifetime caused by radiation, radiationless processes, and exciton-exciton annihilation. Here, we study the system at a temperature, which is not low enough to obtain a Bose-condensate (with the wave vector $k = 0$), but is suitable to get the liquid phase of excitons (when the exciton condensation occurs due to exciton-exciton interactions).

We assume that a local equilibrium of the exciton system, which corresponds to a certain value of the exciton density, is established quickly after the exciton excitation. The establishment of the equilibrium between spatial regions is a slower process. At distances much larger than the exciton free path (when the coherence of a wave function gets lost), we can describe the system by the exciton density $c(\mathbf{r})$. This system's state is described by means of the distribution function $\rho(c(\mathbf{r}))$ which depends on the exciton density $c(\mathbf{r})$ as a system parameter. We take into account the fluctuations of $c(\mathbf{r})$ and consider it as an order parameter. To find $\rho(c(\mathbf{r}), t)$, we use the balance equation [9–11]

$$\frac{\partial c(\mathbf{r})}{\partial t} = G - \operatorname{div} \mathbf{j} - \frac{c(\mathbf{r})}{\tau} - \gamma c(\mathbf{r})^2 + \xi_d(\mathbf{r}, t) + \xi_\tau(\mathbf{r}, t). \quad (1)$$

Here, G is the creation rate of particles due to the constant pumping, τ is the average lifetime of an exciton, γ is the collision-annihilation rate, $\xi_d(\mathbf{r}, t)$ is

the random force induced by thermal fluctuations, $\xi_\tau(\mathbf{r}, t)$ describes the fluctuations associated with the probabilistic character of the creation and disappearance of particles, \mathbf{j} is the probability current due to diffusion processes, $\mathbf{j} = -M\nabla\mu(\mathbf{r})$, M is the mobility which is proportional to the diffusion coefficient D , and $\mu(\mathbf{r}) = \frac{\delta F}{\delta c(\mathbf{r})}$, where F is the free energy of the exciton system in the case where the exciton lifetime would be infinite. We present it in the following phenomenological form:

$$F = \int d\mathbf{r} \left[\frac{1}{2} K (\nabla c)^2 + f(c) \right]. \quad (2)$$

Here, f depends on the fluctuating field $c(\mathbf{r})$, the first term in the integrand is the heterogeneity energy. We assume that the free-energy density $f(c)$ has the Landau–Ginzburg form,

$$f(u) = \frac{a}{2} (c - c_0)^2 + \frac{d}{3} (c - c_0)^3 + \frac{b}{4} (c - c_0)^4, \quad (3)$$

where c_0 is the density at the critical point, a , b , c , and d are phenomenological parameters of the system. The function $f(u)$ in the form (3) is chosen so that the system has two phases. We will consider phase transitions of the first kind close to those of the second kind ($d \approx 0$). The phenomenological parameters in Eqs. (1)–(3) can be calculated with regard for the quantum-mechanical character of a movement or be determined from experimental data. The method of expressing the relation between the microscopic and macroscopic quantities in Eq. (1) was proposed in [14] within nonequilibrium thermodynamics. The nonlinear equation (1) together with (2), (3) for the exciton density are used in the case where the system cannot be described by the Gross–Pitaevsky equation due to the fact that the coherence of the wave function is lost at the distances under consideration.

For Gaussian fluctuations, the stochastic equation (1) is equivalent to the following Fokker–Planck kinetic equation:

$$\frac{\partial}{\partial t} \rho(c(\mathbf{r}), t) = - \int d\mathbf{r} \frac{\delta(j_d + j_\tau)}{\delta c(\mathbf{r})}. \quad (4)$$

In this equation, the term j_τ was obtained in [9–11]. It's value is determined by the exciton finite lifetime, the exciton-exciton annihilation rate, and the pumping as

$$j_\tau = \left\{ G - \frac{c(\mathbf{r})}{\tau} - \gamma [c(\mathbf{r})]^2 \right\} \rho - \frac{1}{2} \frac{\delta \{ (G + c(\mathbf{r})/\tau + \gamma [c(\mathbf{r})]^2) \rho \}}{\delta c(\mathbf{r})}. \quad (5)$$

In Eq. (4), j_d is the probability current arisen due to diffusion processes and thermal fluctuations [15]:

$$j_d = M\Delta \left[\frac{\delta F}{\delta c(\mathbf{r})} \rho + \kappa T \frac{\delta \rho}{\delta c(\mathbf{r})} \right]. \quad (6)$$

Instead of the density $c(\mathbf{r})$, we introduce the quantity $u(\mathbf{r})$ describing a deviation of the order parameter from the average value: $u(\mathbf{r}) = c(\mathbf{r}) - c_0$, where c_0 is determined by the condition that the average value of the first term in j_τ equals zero, i.e.,

$$G - \frac{c_0}{\tau} - \gamma c_0^2 - \gamma \langle u^2 \rangle = 0, \quad (7)$$

where $\langle \dots \rangle$ stands for the averaging over ρ . Then we introduce the two-point correlation function of the density,

$$\tilde{S}(\mathbf{r} - \mathbf{r}', t) \equiv \langle u(\mathbf{r})u(\mathbf{r}') \rangle, \quad (8)$$

which has the following Fourier transform (also called the dynamical structure factor):

$$S(\mathbf{k}) = \int d\mathbf{r} \tilde{S}(\mathbf{r}) \exp(-i\mathbf{k} \cdot \mathbf{r}). \quad (9)$$

Note that the function $\tilde{S}(\mathbf{r} - \mathbf{r}', t)$ is needed to express the light scattering.

Eq. (4) contains functional derivatives. To evaluate such functional derivatives, instead of the continuous order parameter $c(\mathbf{r})$, a discretized one is usually employed as follows. The crystal should be divided into many small cells. The size of a cell is larger than the period of the crystal lattice and smaller than the characteristic length of the inhomogeneous order parameter. We denote the radius vector of a cell as \mathbf{n} , and $c_{\mathbf{n}}$ is the order parameter in the \mathbf{n} -th cell. The distribution function $\rho(c(\mathbf{r}), t)$ depends on the order parameter in every cell as $\rho(c(\mathbf{r}), t) = \rho(\dots, c_{\mathbf{n}}, \dots, t) \equiv \rho$. In the discrete approximation, $u_{\mathbf{n}} = c_{\mathbf{n}} - c_0$. Under such designations, we obtain a discrete counterpart of the Laplacian operator

$$\Delta\psi(\mathbf{r}) \Big|_{\mathbf{r}=\mathbf{n}} = \sum_{\mathbf{n}'} \Delta_{\mathbf{nn}'} \psi_{\mathbf{n}'}, \quad (10)$$

where

$$\Delta_{\mathbf{nn}'} \equiv -\frac{1}{\mathcal{N}} \sum_{\mathbf{k}} k^2 \exp[i\mathbf{k} \cdot (\mathbf{n} - \mathbf{n}')] \quad (11)$$

with the total number of cells \mathcal{N} . The summation over \mathbf{k} in Eq. (11) runs over the Wigner–Seitz cell. Here, we note that $\sum_{\mathbf{n}'} \Delta_{\mathbf{nn}'} = 0$.

Taking into account Eqs. (10), (11), the master equation (4) can be rewritten as

$$\frac{\partial \rho}{\partial t} = - \sum_{\mathbf{n}} \frac{\partial j_{\mathbf{n}}}{\partial c_{\mathbf{n}}}. \quad (12)$$

Here,

$$\begin{aligned} j_{\mathbf{n}} = & -MK\Delta_{\mathbf{nn}'}\Delta_{\mathbf{n}'\mathbf{n}''}c_{\mathbf{n}''}\rho + M\Delta_{\mathbf{nn}'}\frac{\partial f}{\partial c_{\mathbf{n}'}}\rho + \\ & + \left(G - \frac{c_{\mathbf{n}}}{\tau} - \gamma c_{\mathbf{n}}^2\right)\rho + \\ & + \frac{1}{v_0}M\kappa T\Delta_{\mathbf{nn}'}\frac{\partial \rho}{\partial c_{\mathbf{n}'}} - \frac{1}{2v_0}\frac{\partial(G + c_{\mathbf{n}}/\tau + \gamma c_{\mathbf{n}}^2)\rho}{\partial c_{\mathbf{n}}} \end{aligned} \quad (13)$$

(a repeated index implies the summation).

Let us introduce the l -point distribution function as

$$\rho_l(u_{\mathbf{n}_1}, u_{\mathbf{n}_2}, \dots, u_{\mathbf{n}_l}) = \int \prod_{\mathbf{n} \neq \mathbf{n}_1, \dots, \mathbf{n}_l} du_{\mathbf{n}} \rho(\dots, u_{\mathbf{n}}, \dots). \quad (14)$$

After the integration of Eq. (12) over all variables $\{u_{\mathbf{n}}\}$ except for l variables, $u_{\mathbf{n}_1}, u_{\mathbf{n}_2}, \dots, u_{\mathbf{n}_l}$, we obtain the infinite set of coupled equations for the distribution functions of different orders. In order to solve this set of equations, we use the LBM-approximation:

$$\rho_2(u_{\mathbf{n}}, u_{\mathbf{n}'}) = \rho_1(u_{\mathbf{n}})\rho_1(u_{\mathbf{n}'}) \left[1 + \frac{\tilde{S}(\mathbf{n} - \mathbf{n}')}{\langle u^2 \rangle^2} u_{\mathbf{n}} u_{\mathbf{n}'} \right]. \quad (15)$$

This approximation was suggested in [13] and is of great use till now [16, 17]. With the LBM-approximation, we split Eq. (12) into the set of equations for single-point distribution functions.

After some manipulation similar to the calculation presented in [10] in the 3D case, we obtain the following solution for the single-point distribution function:

$$\rho_1(u) = \rho_0 \exp[-\Phi(u)], \quad (16)$$

where

$$\Phi(u) = \frac{X(u)}{T + \frac{G}{D_f}}, \quad (17)$$

$$\begin{aligned} X(u) = & f - \left\langle \frac{\partial f}{\partial u} \right\rangle u - \frac{\gamma}{D_f} \left(\frac{u^3}{3} - \langle u^2 \rangle u \right) + \\ & + \frac{u^2}{2\langle u^2 \rangle} \left(\frac{G}{D_f} + T + \frac{\gamma \langle u^3 \rangle}{D_f} - \left\langle u \frac{\partial f}{\partial u} \right\rangle \right), \end{aligned}$$

and $D_f = -\Delta_{\text{nn}}$. The average value of an arbitrary quantity $O(u)$ is evaluated as

$$\langle O(u) \rangle = \int du O(u) \rho_1(u) \quad (18)$$

with $\rho_1(u)$ given in Eq. (16).

The two-point correlation function (Eq. (8)) equals

$$S(\mathbf{r}) = \frac{1}{(2\pi)^d} \int \frac{d\mathbf{k} (Tk^2 + G) \exp(i\mathbf{k} \cdot \mathbf{r})}{k^4 + \left(\frac{\langle u \frac{\partial f}{\partial u} \rangle}{\langle u^2 \rangle} \right) k^2 + \frac{1}{\tilde{\tau}} + 2\gamma c + \gamma \frac{\langle u^3 \rangle}{\langle u^2 \rangle}}. \quad (19)$$

Here, we have introduced the dimensionless variables $\tilde{u} = u(-a/b)^{-1/2}$, $\tilde{\mathbf{r}} = \mathbf{r}/\xi$, $\tilde{T} = T/T_0$, $\tilde{\tau} = \tau/\tau_0$, $\tilde{\gamma} = \gamma\xi^2/[M(-a)l^d]$, and $\tilde{G} = G\xi^2b/(Ma^2l^d)$, where $T_0 = a^2l^d/(\kappa b)$, $\tau_0 = [l^{d-2}Ma^2[b(-a)]^{-1/2}]^{-1}$, d is the spatial dimension (in our case, $d = 1, 2$, or 3), $\xi = [K/(-a)]^{1/2}$ is the correlation length, and l is the cell size; $\tilde{\mathbf{k}} = \mathbf{k}\xi$, $\tilde{k} = |\tilde{\mathbf{k}}|$. Equations (16)–(19) are presented in dimensionless units, but the mark “tilde” is omitted.

Using formula (11) for systems of different dimensions gives us such results for D_f :

$$D_f = \begin{cases} \frac{4\pi^2}{3} & \text{(for 1D case),} \\ 2\pi & \text{(for 2D case),} \\ \frac{3}{5}(6\pi^2)^{2/3} & \text{(for 3D case).} \end{cases} \quad (20)$$

In the preceding equation, we assume that the cell size l is of the order of the correlation length ξ [9–12].

Taking Eq. (18) into account, we can express $\langle \frac{\partial f}{\partial u} \rangle$, $\langle u^2 \rangle$, $\langle u^3 \rangle$, and $\langle u \frac{\partial f}{\partial u} \rangle$ as functions of $\rho_1(u)$, which can be expressed, in turn, as a function of these four average values. So we obtain the system of transcendental equations. Below, this system will be solved numerically.

3. Numerical Analysis of the Phase Separation

Let's analyze the numerical results which were obtained using the algorithm presented above. The dependence of the distribution function on the concentration for the 3D case was calculated in [10], and the dependences for the 2D and 1D cases are presented here for the first time. Figure 1 shows the dependence of the distribution function on the dimensionless density for systems with different exciton lifetimes and for crystals with different dimensions (1D, 2D, 3D) at the same average density. Here, we assume that, by using a pumping, we can create the same exciton density in

systems with different exciton lifetimes. With increase in the exciton lifetime in two- and three-dimensional systems, the density distribution function becomes a function with two maxima, and the distance between maxima grows with the further increase in the lifetime. At $\tau \rightarrow \infty$, the locations of maxima correspond to the equilibrium densities of a gas and a liquid calculated from the condition for the free energy (3) to be minimum. The appearance of two maxima in the 2D and 3D cases indicates the possibility of a phase stratification. The first and second maxima correspond, respectively, to the gas and condensed phases. It is seen that the stratification occurs when the exciton lifetime is greater than a certain critical value (at $\tilde{\tau} < \tilde{\tau}_c \approx 10$): the disability to create the second phase is explained by the nonequilibrium nature of the system, because the possibility of phase transitions in a nonequilibrium system depends not only on the density, but also on the exciton kinetic parameters (*e.g.*, lifetime). It is seen that the splitting of the distribution function does not occur in 1D systems. It is connected with the general fact that the possibility of the phase stratification is destroyed by fluctuations [18]. We emphasize once more that all results are given for a fixed concentration which we maintain by means of the pumping.

All curves in Fig. 1 are given for a fixed exciton density. Let us analyze the situation with a variable pumping in the system with a fixed exciton lifetime. With increase in the pumping (or in the average exciton concentration), we have firstly the density distribution function with one maximum, which corresponds to the gas phase. With a further increase in the pumping, the second maximum which describes the exciton condensed phase appears. Figure 2,*a* shows the dependence of the density distribution function in a 2D system with small pumping at the time moment when the condensed phase appears. According to Eq. (7), we can determine the critical average density c_{0c} from this value of the pumping, at which the condensed phase appears. If the pumping is less than a certain critical value, this second maximum does not arise. Figure 2,*b* shows the dependence of the critical exciton average density (in dimensionless units), at which the condensed phase appears in the two-dimensional case at the lifetime τ . At $\tilde{\tau} = \tau/\tau_0 \rightarrow \infty$, this density corresponds to the critical density for stable particles. It is seen that, at values $\tau/\tau_0 < 100$, this critical density can significantly differ from the critical density of stable particles.

Let us do some estimates in the two-dimensional case. τ_0 has an order of the system's equilibrium establishment time and is determined from the system's

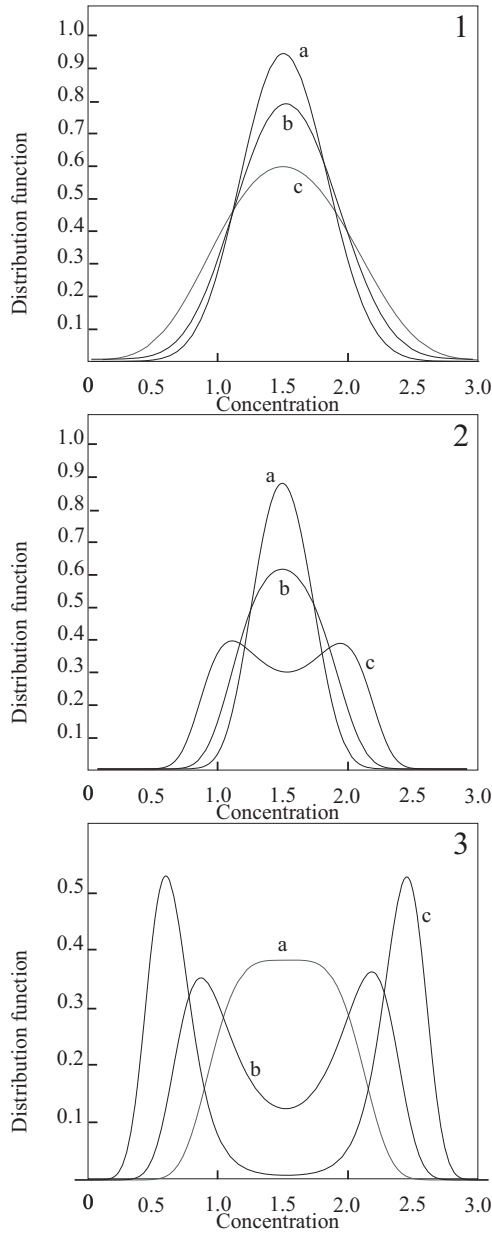


Fig. 1. Distribution function for different values of the dimension parameter d : 1D (1), 2D (2), 3D (3). In Figs. 1–3: $\tilde{\tau} = 1$ (a), $\tilde{\tau} = 10$ (b), $\tilde{\tau} = 100$ (c)

phenomenological parameters. Its typical value for $\epsilon = 10^{-3}$ eV, $D = 10$ cm²/s, $T = (1 \div 4)$ K is $10^{-12} \div 10^{-11}$ s. It is seen from Fig. 2, *b* that, for $\tau = 10^{-10}$ s and $\tau_0 = 10^{-12}(10^{-11})$ s, the critical density changes by 60% (25%) with respect to the critical density of stable particles.

This result may be important in the two-dimensional case where the exciton lifetime can be small due to the

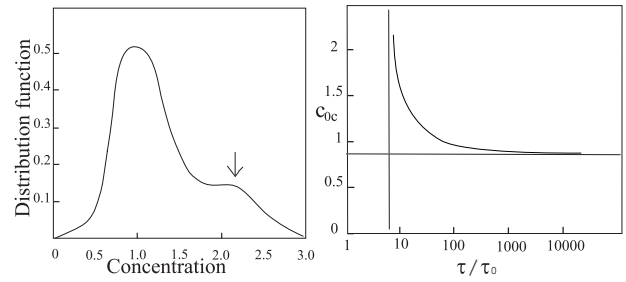


Fig. 2. *a* — the time moment of the appearance of the second maximum; *b* — the dependence of the critical value of the mean density, at which the condensed phase arises, on the lifetime (in dimensionless units)

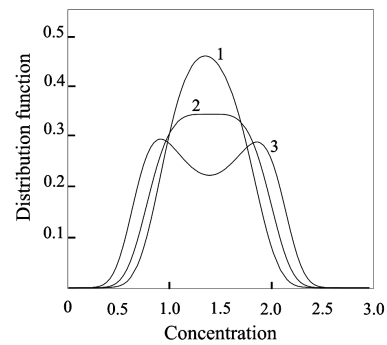


Fig. 3. Distribution function in the 2D case for different values of the annihilation parameter $\tilde{\gamma}$ with the same mean concentration: $\tilde{\gamma} = 0.1$ (1), 0.01 (2), 0.001 (3)

super-radiant decay of excitons [19, 20]. Due to these processes, the radiative lifetime can have an order of several ps. For example, in anthracene, the lifetime of a surface exciton is equal to 0.65×10^{-12} (theory, [22]) and $(0.2 \div 2) \times 10^{-12}$ s (experiment, [23]). In such systems, the condensation can never occur. The lifetime in semiconductor quantum wells can be also very small (25 ps, [24]), and this can have significant impact on the exciton condensation processes.

It can be seen from Eq. (7) that similar effects should take place with varying the annihilation parameter γ . The influence of the annihilation intensity in a 2D system is presented in Fig. 3. From the inspection of the figure, we can state that the single-point distribution function has one maximum for large γ , and the phase stratification does not take place (similar results for the 3D case were obtained in [10]). We would like to underline that we mean not a trivial decrease in the exciton number due to a finite lifetime, which can be compensated by increase in the pumping, but the manifestation of the disbalance of the system. For a

small lifetime, the system has no time to create a new phase even under a long-term stationary irradiation and a high pumping.

4. Structure Factor and Light Scattering

The calculation of the dynamical structure factor was carried using Eqs. (9) and (19). The dynamical structure factor versus the wave vector for different system dimensions is presented in Fig. 4. The dependences of $S(k)$ have maximum at some value of k , which is the evidence for a strong spatial correlation in the system. It is seen that the broadening of the dynamical structure factor with approaching the critical value takes place (the critical value of k , k_{crit} , is shown by the arrow). We note that the dynamical structure factor becomes broader with decrease in the dimension.

As is well known, the structure factor determines the angle dependence of a light scattering by a system. Let us consider the two-dimensional case and use the structure factor $S(k)$ to analyze the angle dependence of the probe light scattered by a crystal with a quantum well, in which the exciton condensed phase exists. The spatial change of a polarizability caused by the condensed phase formation leads to a change in the light scattering as compared with the non-irradiated case.

Then we will find the scattered light for the normal incidence of the probe light. We denote the scattering angle as θ . In the two-dimensional case, the wave vector can be expressed as

$$k^2 = k_{\parallel}^2 + k_z^2 = \frac{\omega^2}{c^2} n^2, \quad (21)$$

where ω is the incident light wavelength, and n is the refraction factor of the crystal. It can be shown that, in this case, the scattered light intensity is proportional to

$$S(|k_{\parallel}|) = S\left(\frac{\omega}{c} n \sin \theta\right). \quad (22)$$

So, $\theta = \arcsin k_m c / \omega n$ is the maximum scattering angle. It is seen that, for $\frac{\omega}{c} n < k$, the scattering maximum cannot be observed at all.

As our structure factor has a sharp maximum at some value of k , the light scattering function has a sharp maximum at some scattering angle. So, the angular dependence of the scattered light intensity on k for different values of parameters will have a look of concentric circles (Fig. 5).

The dependence of the scattering angle on τ for different values of the pumping is shown in Fig. 6. It is seen that the angle of scattering is decreased with increase in the exciton lifetime. The same effect is

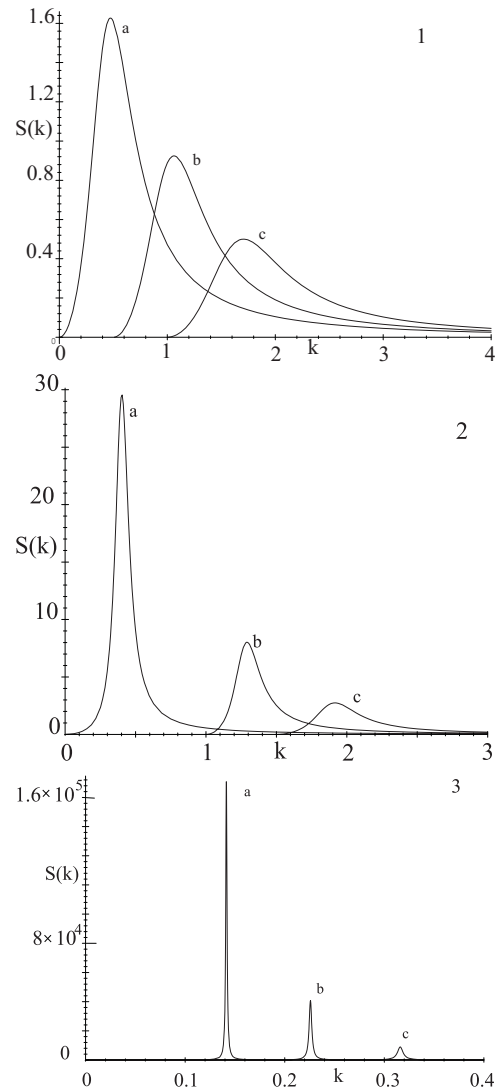


Fig. 4. Dynamical structure factor for different values of the dimension parameter d : 1D (1), 2D (2), 3D (3). In Figs. 1–3: $\tilde{\tau} = 100$ (a), 10 (b), 1 (c)

observed with decrease in the exciton creation rate (the pumping, G).

As the parameters of the condensed phase are unknown, we cannot find the angle of maximum scattering numerically. Nevertheless, we can make estimation. At $\tilde{\tau} = 1000$ for $\xi = (10^{-6} \div 10^{-5})$ cm, $n = 3$, $\tilde{k} = 10^{-1} \div 10^0$, we obtain $k = \tilde{k} / \xi = 10^4 \div 10^6$, $\omega = \frac{k c}{n} = (10^{14} \div 10^{16})$ s $^{-1}$. So with $\xi = (10^{-6} \div 10^{-5})$ cm, the scattering can be observed for the probe light frequency equal to $(10^{14} \div 10^{16})$ s $^{-1}$. Such an investigation of the scattering can be applied to study of the exciton condensed phase.

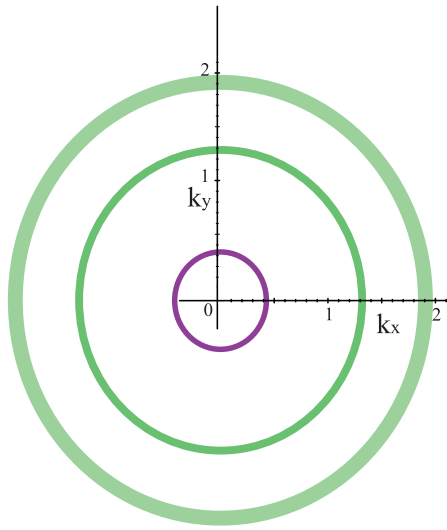


Fig. 5. Angular dependence of the scattered light intensity on k for different values of the exciton lifetime τ : $\tilde{\tau} = 1$ (a), 10 (b), 100 (c)

5. Conclusions

We have analyzed the influence of such parameters as the exciton lifetime, collision-annihilation rate, and pumping intensity on the exciton condensed phase formation in crystals of different dimensions (3D, 2D, 1D). It is shown that the disbalance of the system characterized by a finite exciton lifetime and the exciton-exciton annihilation can have a significant impact on the exciton condensation. For the exciton lifetimes τ less than a certain critical value and for the annihilation parameter γ greater than a certain critical value, the phase stratification is not possible. Here, we mean not a trivial decrease in the number of excitons due to a finite lifetime (which can be compensated by the increase in the pumping), but the nonequilibrium state of the system. With the small exciton lifetime, the system has no time to create a new phase even at a high pumping and a large exciton density. The influence of the exciton lifetime on the condensation processes in 2D systems (quantum wells) is important due to the processes of super-radiant decay of excitons. The phase stratification in 1D systems (quantum wires) does not take place due to fluctuations. The broadening of the dynamical structure factor takes place with approaching the critical value. The dynamical structure factor has a sharp maximum at some value of the wave vector, which evidences for a strong spatial correlation in the system. This should lead to the peculiar features of the light scattering by the condensed phase. Under the normal

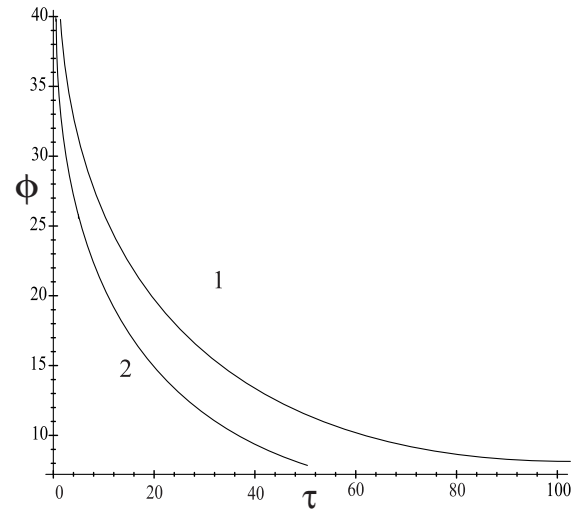


Fig. 6. Dependence of the scattering angle on τ for different values of the pumping: $\tilde{G} = 1.5$ (a), $\tilde{G} = 3$ (b)

incidence of the incoming light, the scattered light can be observed in the form of concentric circles. This can be used to find the parameters of the condensed phase (e.g., the length of coherence).

This work was supported by grant INTAS No. 03-51-5266.

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Received 23.03.06

ФЕНОМЕНОЛОГІЧНА ТЕОРІЯ КОНДЕНСАЦІЇ ЕКСИТОНІВ У КРИСТАЛАХ РІЗНОЇ ВИМІРНОСТІ

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Резюме

Для одно-, дво- та тривимірних систем знайдемо функцію розподілу густини та структурний фактор конденсованої екситонної фази в залежності від швидкості утворення екситонів у системі, часу життя вільних екситонів, інтенсивності екситон-екситонної анігіляції, флуктуацій екситонної щільності та інших параметрів системи. Для розв'язання задачі використовується феноменологічне нелінійне рівняння для щільності екситонів з урахуванням взаємодії екситонів та їхнього скінченного часу життя. Внаслідок нерівноважності системи при малих часах життя екситонів конденсована фаза не утворюється навіть за великої щільності вільних екситонів. У роботі показано, що у двовимірному випадку вплив скінченності часу життя екситонів є дуже суттєвим внаслідок надвипромінювання екситонів у низьковимірних структурах. Структурний фактор системи має різкий максимум при деякому значенні хвильового вектора, що свідчить про наявність у системі сильної кореляції. Розсіяння світла на кристалі, в якому відбуваються досліджувані процеси, має вигляд концентричного кола.