# LONGITUDINAL AND LATERAL VIBRATIONS OF A PLATE PIEZOCERAMIC TRANSFORMER

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Recently, the author has investigated the characteristics of a Rosen-type piezotransformer with regard for the difference in elastic compliances of input and output sections. A refined formula has been derived for the transform ratio. It was established that the transform ratio is inversely proportional to the square frequency. This fact explains why the operation of planar transformers at higher modes of vibrations is not effective. The longitudinal mechanical stresses in the excited and generating sections of a transformer are axisymmetric in respect to the separating line that is in good matching with experimental data. In this paper, the previous author's results are used to analyze the modes of vibrations, frequency behaviour of the transform ratio, and input admittance of a transverse-longitudinal transformer. The spectrum of vibrations of the piezotransformer is compared with that of a rectangular piezoelectric plate. It is shown that a piezoplate has more rigid spectrum than a piezotransformer. The couplings between the lateral and longitudinal vibrations in a piezoplate and a piezotransformer are similar.

# 1. Introduction

The idea of the double transformation of electric energy into mechanical one and back by means of a monolithic piezoceramic plate has been first proposed almost fifty years ago by Rosen [12]. His device was made from barium titanate piezoceramics as a thin rectangular plate with two sections of transversal and longitudinal polarization. In [9], a method of electric equivalent circuits was used to derive an approximate formula of the transform ratio without regard for the difference between elastic compliances in the sections which may get for 10–15 ps. Extensive experimental data for Rosen-type transformer structures of various piezoceramic compositions are presented in [11], where the first resonance was mainly considered.

A piezoelectric transformer for AC-DC converters with a multilayered construction in the thickness direction having low output impedance has been presented in [13]. A strip electrode at the middle of the output part of a piezotransformer minimizes heat generation [1]. The main difference between traditional electromagnetic and piezoceramic transformers is in their small dimensions, weight, stray fields [9, 11, 12], and their high efficiency [1-3, 5, 13].

In [6], the forced vibrations of a classical Rosen-type transverse-longitudinal piezotransformer were analyzed, and a more precise formula for the transform ratio with regard for the difference of the elastic compliances in the input and output sections was derived. It is established that a transform ratio is inversely proportional to the square frequency [7]. This fact explains why the operation of planar transformers at higher modes of vibrations is not effective. The longitudinal mechanical stresses in the excited and generating sections of a transformer were considered in [8], and it was shown that the stresses in both sections are axisymmetric in respect to the separating line that is in good matching with experimental data.

In the present paper, the results of my previous articles [6-8] are used to analyze the modes of vibrations, frequency behaviour of the transform ratio, and input admittance of a transverse-longitudinal transformer in respect to the dependence of the transformer parameters on the difference of the elastic compliances in sections. A satisfactory matching of the calculated results and experimental data is observed. Insofar as possible, the IRE standard notation [4] will be used. The difference between these analyses and my previous articles is as follows. All calculations were made in complex form. An approximate formula for resonant longitudinal frequencies is derived. The spectrum of vibrations of the piezotransformer is compared with that of a rectangular piezoelectric plate. It is shown that a piezoplate has more rigid spectrum than a piezotransformer. The reason for such a phenomena may be the difference between elastic properties in sections. The couplings between lateral and longitudinal vibrations in a piezoplate and a piezotransformer are similar.

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## 2. Longitudinal Vibrations of a Planar Piezotransformer

A classic Rosen-type planar monolithic piezotransformer consists of an excited (input) section of length  $l_1$  with transverse polarization and a generating (output) section of length  $l_2$  with longitudinal polarization (Fig. 1). A plate thickness is 2h, and a plate width is 2b. There are the very thin silver electrodes at the upper and lower surfaces of the input section and at the edge of the output section and a preliminary polarization throughout the thickness. The generating (output) section has a single very thin silver electrode at the edge and the almost uniform polarization along the length. The one-dimensional approximation is used because the width 2b is more less than the length  $l_1+l_2$ . The variables in the first and second sections are marked as "1" and "2", respectively.

The physical processes running in the plate of a planar piezotransformer are as follows. The applied voltage  $V_1$  (from an external generator) draws a current  $I_1$  in the exciting section and excites electromechanical vibrations in the device due to the inverse piezoelectric effect. In order to excite the intensive electroelastic vibrations in a plate, the frequency of an electric voltage  $V_1$  is selected to be equal to the resonant one. The mechanical deformations of the output section due to the direct piezoelectric effect induce a piezoelectric charge  $Q_2$ , an electric voltage  $V_2$ , and an electric current  $I_2$  at its electrode.

The derivation of basic relations for the forced electroelastic vibrations of a transverse-longitudinal piezotransformer was made in [6]. The variables with index "1" have transverse orientation, and the variables with index "2" have longitudinal orientation.

The components of the mechanical displacements  $U_1$ ,  $U_2$  and stresses  $\sigma_1$ ,  $\sigma_2$  in a plate are complex functions of a complex variable x and may be written on base of relations (1.11)–(1.15) in [6] as

$$U_1 = \frac{d_{31}E_1l_1}{x} \frac{[\Delta_*\sin(Lx) + tA_*\cos(x + Lx)]}{\Delta_*\cos(x)},$$
 (1)

$$\sigma_1 = \frac{d_{31}E_1}{s_{11}^E} \frac{\Delta_*[\cos(Lx) - \cos(x)] - tA_*\sin(x + Lx)}{\Delta_*\cos(x)},$$
(2)

$$U_2 = \frac{d_{31}E_1l_1}{x} \frac{[q\Delta_*\sin(Ltx) + t^2A_*\cos(tx + Ltx)]}{t\Delta_*\cos(tx)}, \quad (3)$$

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Fig. 1

 $\sigma_2 =$ 

$$= \frac{d_{31}E_1}{s_{11}^E} \frac{q\Delta_*[\cos(Ltx) - \cos(tx)] - t^2A_*\sin(tx + Ltx)}{t^2\Delta_*\cos(tx)},$$

where

$$x = x_1 - jx_2 = x_1(1 - \frac{j}{2Q_m}) \tag{5}$$

is the complex dimensionless frequency,  $k_1^2 = \rho \omega^2 s_{11}^E$ ,  $k_2^2 = \rho \omega^2 s_{33}^D$ ,  $k_1 l_1 = x$ ,  $\rho$  is the density,  $\omega$  is the angular frequency,  $d_{31}$ ,  $g_{33}$  are piezoelectric constants [4],  $s_{11}^E, s_{33}^D$  are the elastic compliance constants at a constant electric field E and a constant electric displacement D, respectively,  $E_1$  and  $D_2$  are the electric field intensity and the electric displacement (induction) in the sections, y is the longitudinal coordinate,  $k_2 l_2 = tx$ ,  $y/l_1 = L, k_1 y = Lx, k_2 y = L\delta tx, \delta = l_1/l_2$ ,

$$d_1 = 1 - \cos(x), d_2 = 1 - \cos(tx), \tag{6}$$

$$\Delta_* = \cos(x)\sin(tx) + t\delta\sin(x)\cos(tx),\tag{7}$$

$$A_* = d_1 \cos(tx) - \frac{q}{t^2} d_2 \cos(x), \tag{8}$$

and

$$q = \frac{g_{33}D_2}{d_{31}E_1} \tag{9}$$

is the unknown factor which characterizes an influence of the electric displacement  $D_2$  on the input section. The functions  $d_1, d_2, A_*$ , and  $\Delta_*$  are complex-valued too, and

$$\Delta_* = \Delta_1 + j\Delta_2,\tag{10}$$

$$\Delta_1 = \cos(x_1)\sin(tx_1) + t\delta\sin(x_1)\cos(tx_1),$$
(11)

$$\Delta_2 = (1 + t^2 \delta) x_2 \sin(x_1) \sin(tx_1) - (1 + \delta) tx_2 \cos(x_1) \cos(tx_1).$$
(12)

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(4)

In deriving (11), (12), the hyperbolic expressions of the trigonometric functions of imaginary argument were used, and it was assumed that  $\sinh(x_2) \cong x_2$ ,  $\cosh(x_2) \cong 1$ . The terms proportional to  $(x_2)^2$  as well as the products of small quantities were omitted. After these reformations,  $\Delta_1 \equiv 0$  but  $\Delta_2 \neq 0$  at the resonant frequencies, and all mechanical components are finite.

The electric current in the input section of a piezotransformer is determined as the first derivative of the total piezocharge  $Q_1$  with respect to time, which may be written under a harmonic law of variation in the variables as [6]

$$I_{1} = j\omega Q_{1} = j\omega \int_{s1}^{0} D_{1}ds =$$

$$= 2j\omega b\varepsilon_{33}^{T} \int_{-l1}^{0} \left( E_{1} + \frac{d_{31}\sigma_{1}}{\varepsilon_{33}^{T}} \right) dy =$$

$$= -j\omega C_{01}^{s} V_{1} - j\omega C_{01}^{T} V_{1} \frac{k_{31}^{2} \tan(x)}{x} +$$

$$+ j\omega \frac{C_{01}^{T} V_{1} k_{31}^{2} s_{33}^{D} d_{1}^{2} \cos(tx)}{l_{1}\Delta \cos(x)} - j\omega \frac{2bd_{31}d_{33}D_{2}d_{1}d_{2}}{\varepsilon_{33}^{T}\Delta}.$$
 (13)

Here,  $\varepsilon_{33}^T$  is the permittivity when the stress is constant or zero [4], and

$$C_{01}^{T} = \frac{2bl_{1}\varepsilon_{33}^{T}}{2h}, C_{01}^{S} = (1 - k_{31}^{2})C_{01}^{T},$$
  
$$k_{31}^{2} = \frac{d_{31}^{2}}{s_{11}^{E}\varepsilon_{33}^{T}}, V_{1} = -2E_{x1}h, g_{33} = \frac{d_{33}}{\varepsilon_{33}^{T}},$$
 (14)

The voltage  $V_2$  induced at the output of the generating sections is equal to [6]

$$V_{2} = -\int_{0}^{l_{2}} E_{2} dy = -\beta_{33}^{T} D_{2} l_{2} \left( 1 + k_{D}^{2} - \frac{k_{D}^{2} \tan(tx)}{tx} \right) + \frac{g_{33}^{2} s_{11}^{E} D_{2} d_{2}^{2} \cos(x)}{s_{33}^{D} \Delta \cos(tx)} + \frac{g_{33} d_{31} V_{1} d_{1} d_{2}}{2h\Delta},$$
(15)

where [8]

$$k_{33}^2 = \frac{d_{33}^2}{s_{33}^E \varepsilon_{33}^T}, k_D^2 = \frac{g_{33}^2}{s_{33}^D \beta_{33}^T}, \beta_{33}^T = \frac{1}{\varepsilon_{33}^T}.$$
 (16)

The input admittance of a piezotransformer is determined as the ratio of an input current  $I_1$  to an input voltage  $V_1$ :

$$Y_1 = \frac{I_1}{V_1} = -j\omega C_{01}^T \left[ 1 - k_{31}^2 + k_{31}^2 \frac{\tan(x)}{x} \right] +$$

$$+j\omega C_{01}^T \frac{k_{31}^2 s_{33}^D d_1^2 \cos(tx)}{l_1 \Delta \cos(x)} + j\omega \frac{2bd_{31}d_{33}D_2 d_1 d_2}{\Delta \varepsilon_{33}^T V_1}.$$
 (17)

The ratio of an output voltage  $V_2$  to an input voltage  $V_1$  is the transform ratio of a piezotransformer and may be found from Eq. (15) as

$$K_{21} = \frac{V_2}{V_1} = \frac{\beta_{33}^T D_2 l_2}{V_1} \left[ 1 + k_D^2 - k_D^2 \frac{\tan(tx)}{(tx)} \right] + \frac{g_{33}^2 s_{11}^E D_2 d_2^2 \cos(x)}{V_1 s_{33}^B \Delta \cos(tx)} + \frac{g_{33} d_{31} d_1 d_2}{2h\Delta}.$$
 (18)

Relations (13), (15), (17), and (18) are very common and may be used to analyze the interaction between the input and output sections.

Under no-load case, which characterizes the possibilities of a piezotransformer, it is customary to consider that the electrostatic induction at the output section is equal to zero  $D_2 = 0$ . Hence, the influence factor q must be equal to zero as well. At the resonant frequencies,  $\Delta_1 \equiv 0$  and  $\Delta_* = j\Delta_2$ . Therefore,

$$K_{21\text{res}} = -\frac{jl_1}{2h} \frac{d_{33}d_{31}}{s_{11}^E \varepsilon_{33}^T} \frac{2Q_m}{(\delta x_1)(tx_1)} \frac{d_{10}d_{20}}{\Delta_{20}},\tag{19}$$

where

$$d_{10} = 1 - \cos(x_1), d_{20} = 1 - \cos(tx_1), \Delta_{20} =$$
$$= (1 + t^2 \delta) \sin(x_1) \sin(tx_1) - (1 + \delta) t \cos(x_1) \cos(tx_1). (20)$$

Formula (19) shows that the resonant transform ratio is directly proportional to the product  $d_{10}d_{20}$  and inversely proportional to the squared frequency  $(x_1)^2$ . The absolute value of the resonant transform ratio (19) can be written as

$$K_{21}|_{\rm res} = \frac{l_1}{2h} \frac{d_{33}|d_{31}|}{s_{11}^E \varepsilon_{33}^T} \frac{2Q_m}{\delta t(x_1)^2} \frac{|d_{10}d_{20}|}{|\Delta_{20}|}.$$
 (21)

This formula explains why the calculated data for the first and second modes are approximately equal. The fact is that the product  $(d_{10}d_{20})$  for TsTStBS-2 ceramics is equal to 0.92 at the first resonance and 3.85 at the second one, while  $(x_1)_2/(x_1)_1 = 2$ . Hence, the relation  $d_{10}d_{20}/(x_1)^2$  approximately remains for the first two modes. For the third mode, the product  $(d_{10}d_{20})$ decreases to 0.93, while  $(x_1)_3/(x_1)_1 = 3$ , and the calculated transform ratio decreases more than 9 times. The relation  $d_{10}d_{20}/(x_1)_4^2$  tends to zero for the fourth

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mode. The transform ratio for higher overtones of longitudinal vibrations became less than 1/9 of that for the first mode, and the device operation is not good enough.

Relations (11) and (12) can be simplified in the following way. The resonant frequencies are the roots of the equation  $\Delta_1 = 0$  which can be replaced by the next approximate formula

$$\Delta_1 \cong \cos(x_1)\sin(tx) + \sin(x_1)\cos(tx_1) =$$
  
=  $\sin((1+t)x_1) = 0,$  (22)

and

$$x_{1res} \cong \frac{n\pi}{(1+t)}$$
  $(n = 1, 2, 3, ...).$  (23)

Relation (20) can be also simplified:

$$\Delta_{20} = (1 + t^2 \delta) [\sin(x_1) \sin(tx_1) - \mu \cos(x_1) \cos(tx_1)] \cong$$
  
$$\cong 2t \cos[(1 + t)x_1], \tag{24}$$

$$\mu = \frac{(1+\delta)t}{(1+t^2\delta)} \cong \frac{2t}{(1+t^2)} \cong 1.$$
(25)

The real piezotransformers have such dimensions that the lengths of their sections are equal,  $l_1 \cong l_2$ ,  $\delta \cong 1$ , and, for  $1 \ge t \ge 0.7$ , the ratio  $\mu$  differs from 1 by at most 6%. Figure 2 illustrates the frequency dependences of the functions  $z1 = \Delta_1$ ,  $b1 = 2t \cos[(1 + t)x_1]$ ,  $b2 = \sin[(1+t)x_1]$  for t = 0.8, which corresponds to TsTStBS-2 ceramics [10]. It is easy to see the zeros of functions z1and b2 almost coincide. On the other hand, functions b1and b2 have a phase shift of about  $\pi/4$ , so that the zeros of one function coincide with the extremes of the second function.

# 3. The Lateral Vibrations of a Planar Piezotransformer

The spectrum of a Rosen-type piezotransformer has longitudinal and lateral modes of vibrations. This device was designed for the operation on the fundamental extensional mode, but it can operate effectively on the second extensional mode as well. As a rectangular plate, it can be excited at many strong and weak resonances.

In [2], the interesting construction of a ceramic piezotransformer was described. It differs from that of usually used devices. A thin piezoceramic rectangular plate was separated into two sections of equal areas, but the separating line was oriented along the length of a plate. The input electrodes were fabricated on both the



top and bottom surfaces, and the poling of the input part was in the direction of depth. The output electrode was fabricated on the narrow side surface, and the output part was polarized in the direction of width. The authors of [2] regard that the vibration mode adopted there was the fundamental length extensional mode.

It was established in my experiments with models of a Rosen-type piesotransformer, that the lateral vibrations are more intense than the longitudinal vibrations, but their output voltages are small. I think that the construction of [2] can operate very effectively at such intense lateral mode. Relations (1)-(25) can be used for the approximate description of the lateral vibrations of such devices. But this manner is not applicable for a usual Rosen-type device, because the lateral vibrations of the input and output sections correspond to various elastic properties. The elastic compliances of the input and output sections are averaged upon the plate area.

## 4. Numerical Results

In this paper, all the calculations were made in complex form, and the parameters  $d_{31} = -160 \times 10^{-12}$  C/N;  $d_{33} =$  $330 \times 10^{-12}$  C/N;  $s_{11}^E = 12.5 \times 10^{-12}$  m<sup>2</sup>/N;  $\varepsilon_{33}^T = 2100\varepsilon_0$ of TsTStBS-2 ceramics were used for a plate of  $80 \times 18 \times 2$ mm in size, i.e.  $l_1/2h = 20$ ,  $(l_1 + l_2)/2b = 4.44$ . After substituting these parameters in Eq. (1), the results were plotted in Fig. 3 for t = 0.8,  $V_1 = 10$  V,  $Q_m = 100$ , as the distributions of the absolute values of elastic displacements for the first three longitudinal modes of the input section.

The notations U11, U21, and U31 are the displacements in the first section at the first, second, and third longitudinal modes, respectively,  $V_1$  is the input voltage, and  $E_1 = V_1/2h$ . The displacement amplitudes

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are very small and decrease as the frequency increases. The displacement distributions in the output section are same, but they have opposite sign.

The numerical results for elastic stresses are demonstrated in Fig. 4. The notations T1 and T2 are the stresses in the first section of the plate at the first and second longitudinal modes of vibrations, respectively. The amplitude of mechanical stresses at the third longitudinal mode is very small and cannot be plotted on the same figure.

Figure 5 illustrates the dependence of the transform ratio modulus (curves 1, 2) and the frequency equation (curves 3, 4) on the frequency for t=1 and t=0.8, respectively. I used such small value of  $Q_m$  in my calculations for simplicity, because graphs 1 and 2 in Fig. 5 are very narrow for the real piezotransformer mechanical quality. It is easy to see that the



transformation ratios are almost equal for the first and second longitudinal modes for both hypothetic types of ceramics. The third and sixth modes have equal transform ratios as well. The fourth and eighth modes are absent.

### 5. Comparison with Experimental Data

The experimental investigations were carried out on several models of piezotransformers. Copper silvered wires with a diameter of 0.1 mm were soldered to the surface plate electrodes and used to connect to the circuit. Vibrations were excited by a variable voltage from a generator in the range from 20 to 200 kHz. The output section of a piezotransformer was connected to an electronic digital voltmeter and an electronic digital frequency meter controlling the operation frequency. Table represents the characteristic frequencies  $f_m$  and  $f_n$  in kHz and the corresponding maximum and minimum admittances Y in mSm for the first eight registered modes of vibrations of a planar piezotransformer with dimensions  $80 \times 18 \times 2$ mm made of PKD ceramics. The following values for maxima of the output potential were obtained (the frequency in kHz and the transformation ratio are given in the numerator and the denominator, respectively): 21.23/256, 42.01/342, 61.81/118, 86.35/14.9, 93.05/18, 96.78/15.1, 99.33/18, and 136.51/113. The measured value of  $Q_m$  at the first and second modes was 170. After substituting  $Q_m$  and the ceramics data in (21), we obtain  $|K_{21}|_1 = 345$  and  $|K_{21}|_2 = 352$ .

$f_m$ , kHz	21.31	42.29	61.90	81.66	92.38	96.04	99.37	136.3
$f_n$ , kHz	21.69	42.88	62.07	81.85	93.19	98.19	100.2	137.2
$Y_m$ , mSm	19	20.8	7.41	14.5	41.1	95.8	28.4	34.1
$Y_n$ , mSm	0.095	0.24	1.76	3.23	3.53	1.18	0.56	1.02

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The first two modes are similar in the maximum admittance and transformation ratio, the third and fourth modes are much weaker, and the 96.04-kHz mode is of the greatest intensity and its admittance exceeds that of the first mode by five times. On the other hand, the intense 93.05-, 96.78-, and 99.33-kHz modes induce a much smaller voltage at the output electrodes than the first two modes do. The 81.66-kHz resonance is not detected from the output voltage, but a new 86.35kHz mode, which is absent in Table and nothing can be said about its deformation, is observed. The graph of Fig. 6 was obtained, when the input section of a piezotransformer was connected with a sweep generator IChX-300 between its input and output sockets as a rejecting circuit. A sweep generator IChX-300 had the input and output impedances equal to 135 Ohm. The greater the input admittance of a piezoelectric plate, the deeper the rejecting downfall in a graph. Labels L1-L6 on a graph correspond to the longitudinal modes, T1-T3 denote the intense lateral modes, and E1 is an edge mode. The length-width ratio for the testing transformer is 80:18=4.44. Many years ago, I investigated the planar vibrations of a thin rectangular piezoceramic plate with various length-width ratios in the interval 1 < a/b < 8. Now I have a grate pleasure to compare the characteristics of a piezotransformer with those for a rectangular plate. Figure 7 demonstrates the amplitude-frequency dependence for such a plate with a/b=4.5. Both the amplitude and frequency in Figs. 6, 7 are presented in relative units.

It is easy to see that the amplitude-frequency characteristics are similar for a rectangular piezoceramic plate and a plate piezoceramic transformer. Their main difference is as follows. The piezoplate spectrum is more rigid than that of a piezotransformer with the same geometric dimensions. The even numbers of longitudinal

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modes are under a ban for rectangular plates, so that modes L2, L4, and L6 are absent. A number of registered resonances for a piezoplate is six in a frequency range, where a piezotransformer has eight modes. I think that the reason lies in different properties of the sections. These different elastic properties are the reason for the splintering of a lateral mode T2 on two modes T2 and T3 too. On the other hand, the strong lateral mode T2can increase its weak neighbours.

#### 6. Conclusions

A refined formula of the transform ratio for a planar piezotransformer of the transverse-longitudinal type is reformed and discussed. It is shown that the transform ratio near resonant frequencies is directly proportional to the piezoelectric modulus and inversely proportional to the squared frequency. The transform ratios at the first and second modes of vibrations for all piezoceramics are almost the same.

The operation of a planar piezotransformer at high frequencies is not good enough.

The one-dimensional rod model describes well the first six longitudinal modes of a piezotransformer.

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#### ПОЗДОВЖНІ ТА ПОПЕРЕЧНІ КОЛИВАННЯ ПЛАСТИНЧАТОГО П'ЄЗОКЕРАМІЧНОГО ТРАНСФОРМАТОРА

В.Л. Карлаш

Резюме

Аналізується уточнена формула коефіцієнта трансформації плоского п'єзотрансформатора поперечно-поздовжнього типу. Показано, що поблизу резонансних частот коефіцієнт трансформації прямо пропорційний п'єзоелектричному модулю та обернено пропорційний квадрату частоти. Одновимірна стержнева модель добре описує перші шість поздовжніх мод.