ASYMMETRIC NUCLEAR MATTER USING SKYRME POTENTIAL

H.M.M. MANSOUR, Z. METAWEI

Physics Department, Faculty of Science, Cairo University
(Giza, Egypt; e-mail: mansourhesham@yahoo.com)

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The binding energy, symmetry energy, pressure, incompressibility, and the velocity of sound are calculated for the asymmetric nuclear matter using the Skyrme interaction. The behavior of these physical quantities is studied for different values of the asymmetry parameter α_τ, the density ρ, and the temperature T. A good agreement is obtained in comparison with the previous theoretical estimates.

1. Introduction

The static properties of nuclear matter (NM), e.g., binding energy, symmetry energy, incompressibility, etc., can be determined by the equation of state (EOS). In the last decade, the study of EOS of nuclear matter has a great interest in nuclear physics and astrophysics [1—3]. The equation of state of NM is closely related to the study of nuclear fission, heavy ion reactions, and hot neutron stars in astrophysics. It is also of interest to study the thermal properties of NM, e.g., free energy, pressure, entropy, effective mass, and chemical potential, and all possible phases, in which the matter may exist. Strongly interacting nuclei, in which the number of neutrons is more than that of protons, are encountered in neutron stars. In stable nuclei, the net asymmetry $\alpha_\tau = N - Z/N + Z$ ranges up to about 0.24. In the future, the accelerator experiments with rare isotopes will extend the range of α_τ to values well in excess of 0.2. In contrast, α_τ could be as large as 0.95 in the interiors of neutron stars. The energetics associated with the $n{-}p$ asymmetry can be characterized by the so-called symmetry energy E_{sym} which is the leading coefficient of the expansion of the energy in α_τ. The energy E_{sym} is interesting in many astrophysical calculations concerning neutron stars, because it is related to the proton and neutron chemical potentials. Besides the realistic NN interaction (see, e.g., [4]), many calculations were carried out with the use of effective NN interactions. Most of the calculations of hot NM considered the symmetric case [5, 6] or the asymmetric one [7, 8]. The polarized NM has also been studied in [9—11] at the zero temperature and in [12] at a finite temperature. A lot of progress has been made to develop tools of many-body theory such as the Brueckner hole — line expansion, the coupled cluster or "exponential S" method, the self-consistent calculation of Green's function, and variational calculations which are based on cluster expansion techniques, as well as Monte Carlo methods. The different techniques yield predictions for the bulk properties of nuclear systems at normal densities which tend to agree with one another. Such calculations can be also used to obtain predictions for nuclear matter under extreme conditions like those in neutron stars, supernova explosions, or in central heavy-ion collisions. They yield the equation of state of NM at high densities and/or high temperatures. The Skyrme effective interaction potential has been widely used to investigate the properties of NM. This is because it produces simple analytical expressions to deal with. The parameters of the Skyrme force have been calculated by many authors [13, 14]. The properties of NM and neutron matter were used to put constraints on the parameters which are not well determined from the nuclear data.

In the present work, the Skyrme interaction [14] has been chosen to calculate the energy, symmetry energy, pressure, incompressibility, and velocity of sound. In the literature, there are a large number of the sets of Skyrme parameters. The best sets of Skyrme parameters are

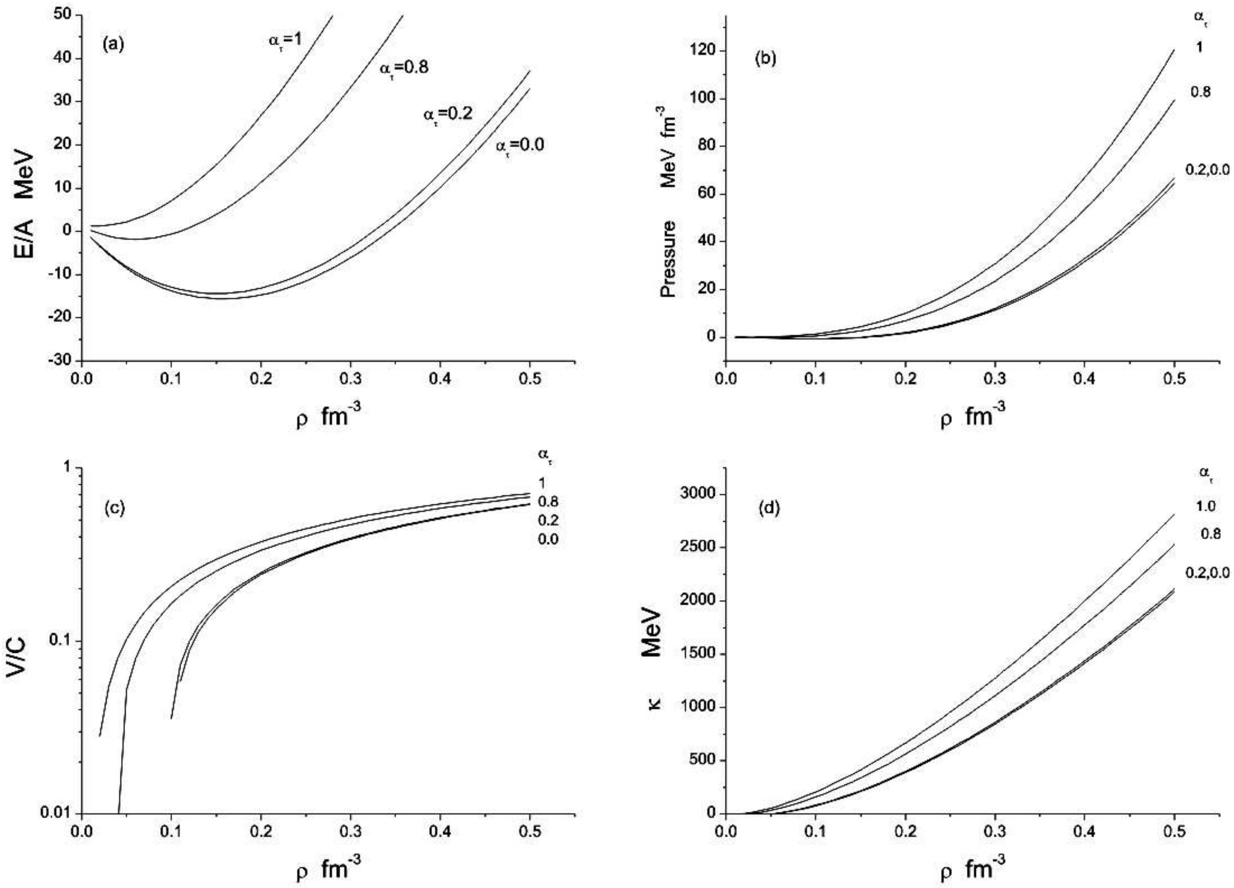

Fig. 1. Energy per nucleon (*a*), pressure (*b*), the velocity of sound (*c*), and the incompressibility (*d*) at zero temperature are shown as a function of the density for different values of α_τ

given in [14] which fits both nuclear and neutron matter properties. The calculations for the physical quantities are presented as a function of the density ρ, the temperature T, and the asymmetry parameter α_τ. The main aim of the present work is to study the behavior of the properties of NM with extreme neutron excess and those of a more realistic neutron matter with a proton mixture. The reason for this is in the last stage of the type-II supernova [15, 16], where a highly asymmetric nuclear matter is formed because of electron capture processes and the star reaches the proton/ neutron ratio $Z/N = \frac{1}{2}$. This value stays almost constant during the collapse time until the core bounces and the shock wave is formed.

2. Method of Calculations

Let NM be composed of N neutrons and Z protons with $N \neq Z$. This system is called asymmetric nuclear matter. All the nucleons are contained in a periodic box of volume Ω. The composition of the system is characterized by

$$A = N + Z. \tag{1}$$

The neutron excess parameter is defined by

$$\alpha_\tau = (N - Z)/A. \tag{2}$$

There are two Fermi momenta in this general case, namely, k_n for neutrons and k_p for protons. These Fermi momenta are related to the neutron-excess parameter by the relations

$$k_n^3 = k_f^3(1 + \alpha_\tau), \tag{3}$$

$$k_p^3 = k_f^3(1 - \alpha_\tau), \tag{4}$$

where k_f is the Fermi momentum for symmetric NM which is related to the density ρ by the relation

$$k_f^3 = \frac{3}{2}\pi^2\rho = \frac{3}{2}\pi^2\frac{A}{\Omega}. \tag{5}$$

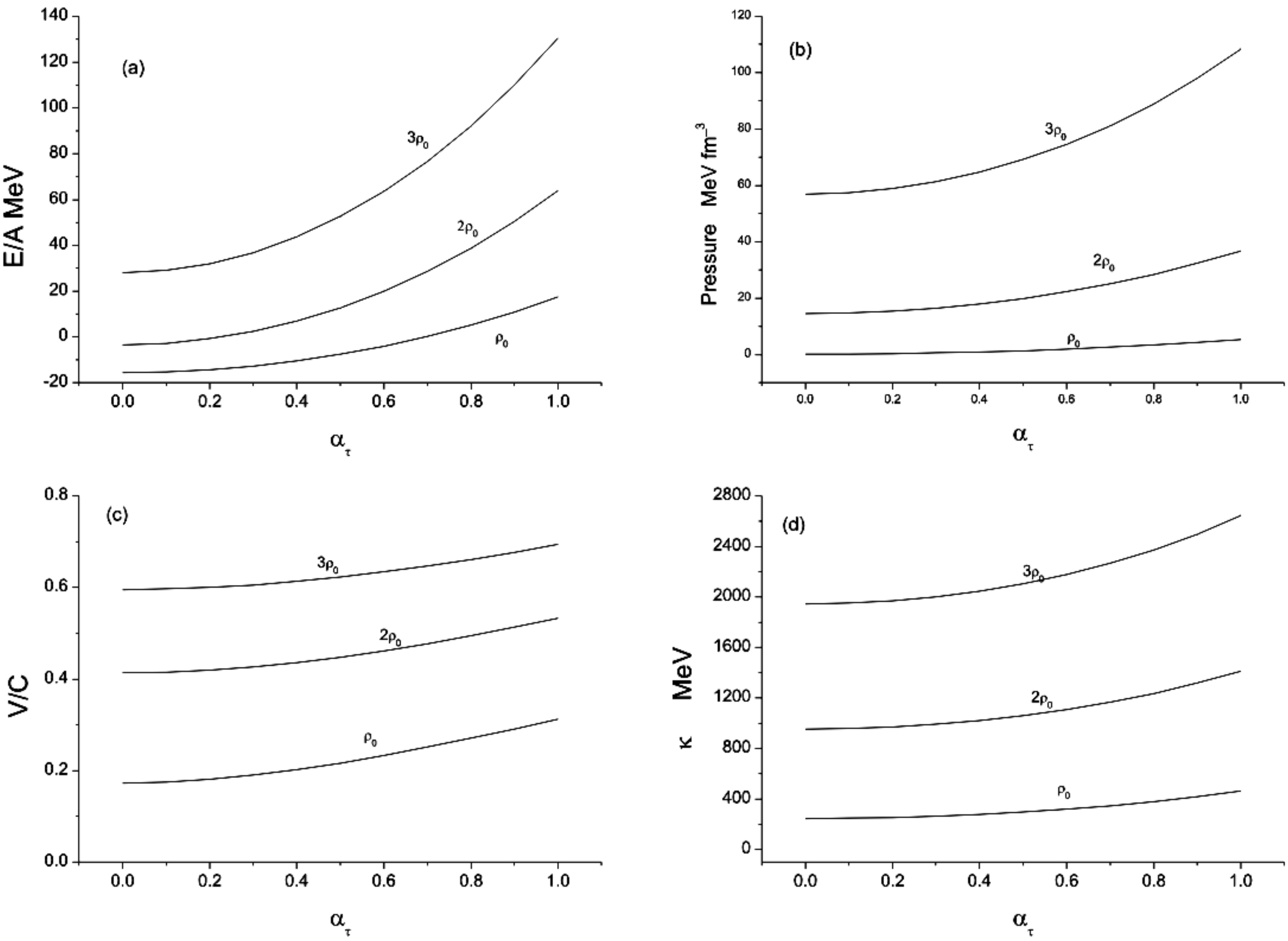

Fig. 2. Energy per nucleon (a), the pressure (b), the velocity of sound (c), and the incompressibility (d) are shown as a function of α_τ at temperature $T = 0$ for different values of the density

The binding energy of nuclear matter can be written as

$$E = \langle T\rangle + \frac{1}{2}\sum_{\mathbf{k}_1,\mathbf{k}_2}\langle \mathbf{k}_1\mathbf{k}_2|V|\mathbf{k}_1\mathbf{k}_2 - \mathbf{k}_2\mathbf{k}_1\rangle, \quad (6)$$

where the single particle wave functions $|\mathbf{k}\rangle$ are given by

$$|\mathbf{k}\rangle = |k\rangle\xi_\sigma\xi_\tau. \quad (7)$$

where $|k\rangle$ is the ordinary plane wave, ξ_σ and ξ_τ are, respectively, the spin and isotopic spin wave functions, and $\langle T\rangle$ is the expectation value of the kinetic energy. In our calculations, we consider the Skyrme interaction which is given by [17]

$$V_{\text{Skyrme}} = t_0(1+\varkappa_0 P_\sigma)\delta + \frac{1}{2}\, t_1(1+\varkappa_1 P_\sigma)(\overline{k}'^2\delta + \delta\overline{k}^2) +$$

$$+ t_2(1+\varkappa_2 P_\sigma)\overline{k}'\cdot\delta\overline{k} + \frac{1}{6}t_3(1+\varkappa_3 P_\sigma)\rho^\alpha(\mathbf{R})\delta, \quad (8)$$

where $\delta = \delta(\mathbf{r}_i - \mathbf{r}_j)$, $\overline{k} = \frac{1}{2i}(\nabla_i - \nabla_j)$ is the relative momentum operator acting on the wave function to the right, $\overline{k}'$ is the adjoint of $\overline{k}$, P_σ is the spin-exchange operator, and $\mathbf{R} = (\mathbf{r}_i + \mathbf{r}_j)/2$. Calculating the binding energy per nucleon, we can determine the pressure, incompressibility, and the velocity of sound which are given by [17]

$$P = \rho^2\frac{\partial(E/A)}{\partial\rho} \quad (9)$$

and

$$V = \sqrt{\frac{\partial P}{\partial e}}, \quad (10)$$

where

$$e = \rho\times(m_N C^2 + \frac{E}{A}), \quad (11)$$

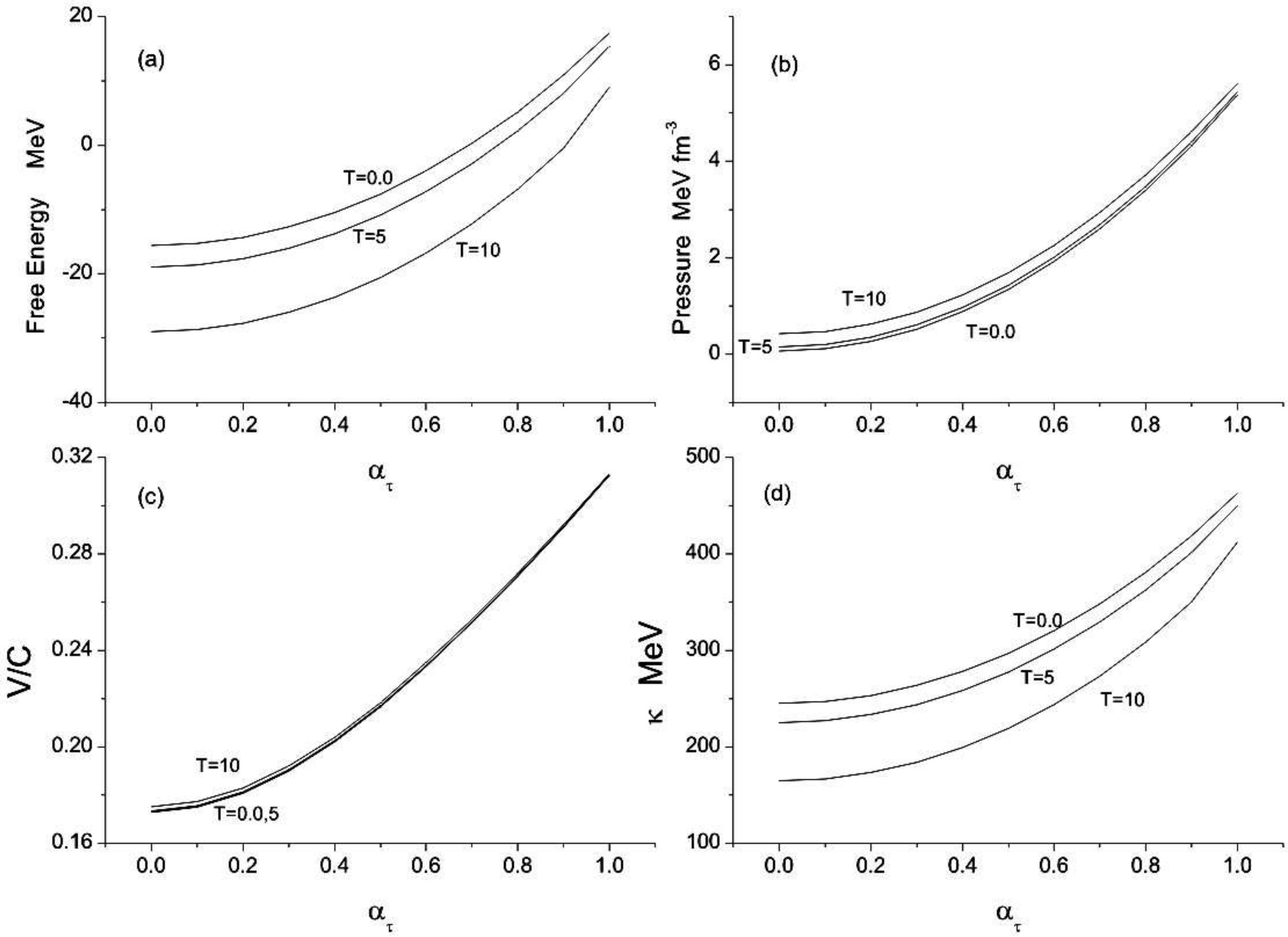

Fig. 3. Free energy (a), the pressure (b), the velocity of sound (c), and the incompressibility (d) at ρ_0 are shown as a function of α_τ for different values of temperature

$$\mathcal{K} = 9\rho^2 \frac{\partial^2(E/A)}{\partial\rho^2}. \quad (12)$$

The thermal properties of NM are completely determined, if the free energy $F(\rho, T)$ per nucleon is determined [18]. We have

$$F(\rho, T) = E(\rho, T = 0) - TS(\rho, T), \quad (13)$$

where $E(\rho, T = 0)$ is the energy per nucleon at the temperature $T = 0$ and S is the entropy of the system [19]

$$S(\rho, T) = \frac{\pi^2}{2} \frac{m^*}{\hbar^2 k_f^2} k^2 T[(1 + \alpha_\tau)^{1/3} + (1 - \alpha_\tau)^{1/3}], \quad (14)$$

where m^* is the effective mass, and k is the Boltzmann constant. The pressure, velocity of sound, and incompressibility at finite temperatures are determined using Eqs. (9)—(14).

3. Results and Discussion

In the present work, the properties of NM as a function of the density ρ, the neutron excess parameter α_τ, and the temperature T are calculated. The range of α_τ varies between 0 and 1, i.e. from the symmetric nuclear matter to a highly asymmetric case which has a neutron excess, then to the other extreme case of the pure neutron matter. Figure (1) gives the values of the energy per nucleon E/A, pressure P, velocity of sound V/C and the incompressibility $\mathcal{K}$ at the zero temperature for different values of ρ and α_τ. The energy has a pronounced minimum of -15.59 MeV at $\rho_o = 0.158$ fm^{-3} for $\alpha_\tau = 0$ with similar trends for larger values of α_τ, except for $\alpha_\tau = 1$ where it shows no bound states, and, in this case, the energy lies above that of the symmetric nuclear matter. For three other physical quantities $P, V/C$, and

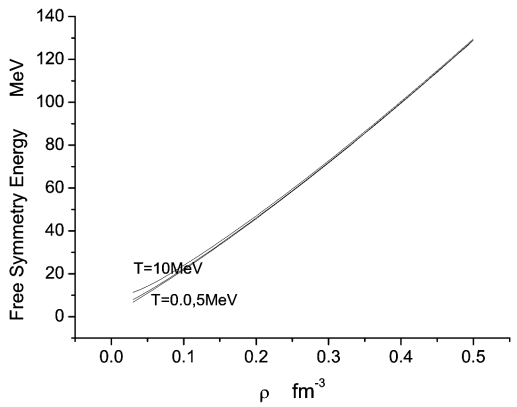

Fig. 4. Symmetry free energy as a function of the density at temperatures $T = 0$, 5, and 10 MeV

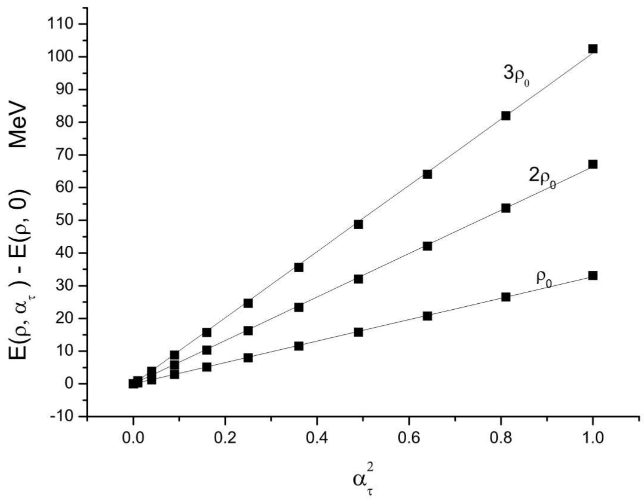

Fig. 5. Difference between $E(\rho, \alpha_\tau)$, and $E(\rho, 0)$ as a function of α_τ^2 for different densities

$\mathcal{K}$, they are presented in Fig. 1,*b, c,* and *d*. They increase with ρ as well as with α_τ. Figure 2 gives the behaviour of $E/A, P, V/C$, and $\mathcal{K}$ as a function of α_τ for different values of the density at $T = 0$. They slightly increase with α_τ, and, at higher densities, the physical quantities are larger than those for small ρ. Figure 3 gives a change of four physical quantities as a function of α_τ and T at ρ_0. The free energy and incompressibility increase with α_τ, but they decrease with increase in temperature. For the pressure and velocity of sound, it is clear that they increase with α_τ and temperature. The symmetry energy which is related to the difference between the energies at $\alpha_\tau = 0$ and α_τ by [20],

$$E(\rho, \alpha_\tau) = E(\rho, 0) + E_{\text{sym}}(\rho)\alpha_\tau^2, \tag{15}$$

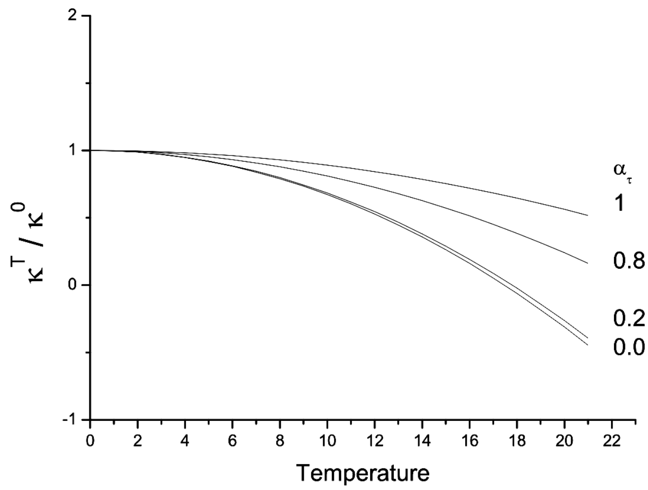

Fig. 6. Ratio of the incompressibility k^T at a finite temperature to that at the zero temperature against the temperature T for different α_τ

is shown in Fig. 4. Figure 4 gives the free symmetry energy as a function of the density for different temperatures. It is clear that the symmetry energy increases with the density and temperature.

In our calculations, the attempt is made to prove the empirical parabolic law [20] given by Eq. (15).

Figure 5 shows $E(\rho, \alpha_\tau) - E(\rho, 0)$ as a function of α_τ^2 at $T = 0.0$ MeV for three values of the density ρ_0, $2\rho_0$, and $3\rho_0$. The numerical results lie on a linear fit. This proves that the parabolic law (15) taken from the nuclear mass table can be extended up to the highest asymmetry of NM [21]. A deviation from the parabolic law could be expected at densities higher than those considered in the present work [21].

The symmetry energy is defined as

$$E_{\text{sym}}(\rho) = \frac{1}{2}\left[\frac{\partial^2 E(\rho, \alpha_\tau)}{\partial \alpha_\tau^2}\right]_{\alpha_\tau = 0}.$$

The value obtained for the symmetry energy at $T = 0$ is 31.40 MeV which is comparable with 27 MeV [18] obtained by using the Skyrme potential and 26.5 MeV [22] obtained in [10, 11]. It has been found that the observed maximum neutron star masses and surface magnetic field are best explained with $E_{\text{sym}} = 33.4$ MeV [23].

Figure 6 shows the behaviour of the ratio of the incompressibility at a finite temperature T to that at the zero temperature against the temperature for different values of α_τ. The ratio decreases as a function of T but shows an increase as a function of the isospin asymmetry α_τ. The value of $\mathcal{K}_0$ at ρ_0 is 238.1 MeV.

The calculated fall-off of the incompressibility with temperature can be fitted with a parabolic equation given by [24]

$$\mathcal{K}^T(\alpha_\tau, T) = \mathcal{K}^o(\alpha_\tau, 0)[1 - A^T(\alpha_\tau)T^2]. \quad (16)$$

At $\alpha_\tau = 0.33$ where $Z/N = 1/2$, we find $A^T(\alpha_\tau) = 2.965 \times 13^{-3}$ MeV^{-2} which is in agreement with the value 4.38×10^{-3} MeV^{-2} [24].

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РОЗРАХУНОК ПАРАМЕТРІВ АСИМЕТРИЧНОЇ ЯДЕРНОЇ МАТЕРІЇ З ВИКОРИСТАННЯМ ПОТЕНЦІАЛУ СКІРМА

Х.М.М. Мансур, З. Метавей

Р е з ю м е

За допомогою потенціалу Скірма розраховано енергію зв'зку, симетричну енергію, тиск, стисливість та швидкість звуку в асиметричній ядерній матерії. Вивчено поведінку цих величин в залежності від параметра асиметрії , густини та температури. Результати обчислень добре узгоджуються з попередніми теоретичними оцінками.