

**EXTRACTING THE WEAK PHASE  $\gamma$  FROM  $B^\mp$  MESONS  
DECAYS TO TWO VECTOR  $D^*$  AND  $K^{*\mp}$  MESONS**

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A method to measure the weak phase  $\gamma$  and  $r_{B\lambda}$ , the magnitude of the ratios of the amplitudes  $A_\lambda(B^- \rightarrow \bar{D}^{*0} K^{*-})$  to  $A_\lambda(B^- \rightarrow D^{*0} K^{*-})$  through the interference of the charged  $B$  meson decay channels  $B^- \rightarrow D^{*0} K^{*-}$  and  $B^- \rightarrow \bar{D}^{*0} K^{*-}$ , where the  $D^{*0}$  and  $\bar{D}^{*0}$  decay to  $D^0/\bar{D}^0\pi^0$  and to  $D^0/\bar{D}^0\gamma$ , has been proposed. As the common hadronic final states, the doubly Cabibbo-suppressed modes of decay  $D^0$ -mesons were chosen. We show that the  $CP$  violating asymmetries of  $B^- \rightarrow D^{*0}/\bar{D}^{*0} \rightarrow D^0/\bar{D}^0 \rightarrow K^+\pi^-\pi^0$ ,  $K^{*-}$  and  $B^- \rightarrow D^{*0}/\bar{D}^{*0} \rightarrow D^0/\bar{D}^0 \rightarrow K^+\pi^-\gamma$ ,  $K^{*-}$  decays have opposite signs.

$B^- \rightarrow D^0 K^-$  and  $B^- \rightarrow \bar{D}^0 K^-$  which is sensitive to the weak phase  $\gamma$ . As  $f$ , one may take the states with different properties (see, e.g., [4–6]). We also emphasize that, along with the basic method, there exist its various versions, which include the study of multiparticle decays of  $B$  mesons [7, 8], the use of just “marked”  $D^{**}$  states [9], and the decays of  $B$  mesons to a pair of vector mesons [10]. Since these measurements will be carried out under conditions of a small amount of useful events, it will be necessary to combine all the modes in order to improve the statistics and to minimize the total error of measurements of the weak phase  $\gamma$  [11, 12].

**1. Introduction**

A characteristic feature of the standard model (SM) is the mechanism of violation of the  $CP$  symmetry in the weak interaction which is related to the Cabibbo–Kobayashi–Maskawa mixing matrix for quarks (CKM) [1]. Therefore, for testing this mechanism, it is necessary to possess the theoretically pure (a theoretical error must be at most 0.1%) methods of measurements of all the angles ( $\alpha$ ,  $\beta$ , and  $\gamma$ ) of a unitary triangle related to the CKM matrix [2].

A theoretically pure method of measurements of the weak phase  $\gamma$  ( $\gamma = \arg(V_{ub}^*)$  [3], where  $V_{ij}$  – elements of the CKM matrix) was proposed in [4]. This method is based on the phenomenon of interference between the “tree” amplitudes of transitions of  $b$  quarks:  $b \rightarrow c\bar{u}s$  and  $b \rightarrow u\bar{c}s$ . The amplitude of the first transition underlies the mechanism which is responsible for the decay  $B^- \rightarrow D^0 K^-$ , whereas the second amplitude – for the decay  $B^- \rightarrow \bar{D}^0 K^-$ . If  $D^0$  and  $\bar{D}^0$  mesons decay to the general final state  $f$ , then there appears the interference between the amplitudes of the transitions

The most accurate direct measurement of the weak phase  $\gamma$  was performed on the study of the decays of  $B^\mp \rightarrow \tilde{D}^0 K^\mp$ ,  $B^\mp \rightarrow \tilde{D}^{*0} K^\mp$ ,  $B^\mp \rightarrow \tilde{D}^0 K^{*\mp}$ ,  $\tilde{D}^{*0} \rightarrow \tilde{D}^0\pi^0$ ,  $\tilde{D}^0\gamma$ ,  $\tilde{D}^0 \rightarrow K_S^0\pi^-\pi^+$  [13, 14]. Here and further, by  $\tilde{D}^0$  ( $\tilde{D}^{*0}$ ), we denote a superposition of  $D^0$  and  $\bar{D}^0$  mesons ( $D^{*0}$  and  $\bar{D}^{*0}$  mesons). Both experimental groups used the decays of  $D^0$  and  $D^{*0}$  mesons, and the processing of experimental data accounted the difference of phases between two decay modes of  $D^{*0}$  to  $D^0\pi^0$  and to  $D^0\gamma$ , which was indicated in [15].

In the present work, we will consider the effects of violation of the  $CP$  symmetry in the decays of  $B^-$  meson to a pair of linearly polarized vector  $\tilde{D}^{*0}$  and  $K^{*-}$  mesons, namely:

$$B^- \rightarrow \tilde{D}^{*0}(\rightarrow \tilde{D}^0(\rightarrow f)\pi^0) K^{*-}(\rightarrow K\pi), \tag{1}$$

$$B^- \rightarrow \tilde{D}^{*0}(\rightarrow \tilde{D}^0(\rightarrow f)\gamma) K^{*-}(\rightarrow K\pi), \tag{2}$$

$$B^- \rightarrow \tilde{D}^{*0}(\rightarrow \tilde{D}^0(\rightarrow \bar{f})\pi^0) K^{*-}(\rightarrow K\pi), \tag{3}$$

$$B^- \rightarrow \tilde{D}^{*0}(\rightarrow \tilde{D}^0(\rightarrow \bar{f})\gamma) K^{*-}(\rightarrow K\pi). \tag{4}$$

As the final states  $f$ , we will use the Cabibbo-allowed modes in the decays of  $\bar{D}^0$  meson. Respectively, they will be doubly Cabibbo-suppressed modes in the decays of  $D^0$  meson, for example,  $f = K^+\pi^-, K^{*+}\pi^-, K^+\pi^-\pi^0, K^+\pi^-\pi^+\pi^-,$  etc. We recall that the proposition to use the decay  $B^\mp \rightarrow D^* K^{*\mp}$  with the further transition  $D^{*0}/\bar{D}^{*0} \rightarrow D^0/\bar{D}^0 \pi^0, D^0/\bar{D}^0 \rightarrow f$  for the determination of the weak phase  $\gamma$  and other parameters was advanced in [10]. But a neutral vector  $D^{*0}/\bar{D}^{*0}$  meson possessing the decay mode to  $D^0/\bar{D}^0 \pi^0$ , which has the relative width of 62% [16] and was considered in [10], decays also to  $D^0/\bar{D}^0 \gamma$  with the relative width of 38% [16]. Therefore, as will be shown below, the combined analysis of these two modes of the decay of  $D^*$  mesons not only will increase the number of useful events but also, due to the phase difference between these two modes, will open new possibilities in the study of  $CP$ -odd effects in the decay of  $B^\mp \rightarrow D^* K^{*\mp}$  and in the determination of the weak phase  $\gamma$  and other parameters.

## 2. Polarization State of Vector Mesons and Effects of Violation of the $CP$ Symmetry

We denote the amplitudes of the cascade decays (1)–(4), respectively, as  $A_\lambda^{f\pi}, A_\lambda^{f\gamma}, A_\lambda^{\bar{f}\pi},$  and  $A_\lambda^{\bar{f}\gamma}$ , and that of the corresponding  $CP$ -conjugate decays as  $\bar{A}_\lambda^{f\pi}, \bar{A}_\lambda^{f\gamma}, \bar{A}_\lambda^{\bar{f}\pi},$  and  $\bar{A}_\lambda^{\bar{f}\gamma}$ . Then these amplitudes in the standard model, with neglecting the small effects of mixing in the system  $D^0 - \bar{D}^0$  [17], have the form

$$A_\lambda^{f\pi(\gamma)} = (r_{Df} \pm z_\lambda^-) A_{c\lambda}, \quad (5)$$

$$A_\lambda^{\bar{f}\pi(\gamma)} = (1 \pm r_{Df} z_\lambda^- e^{-2i\delta_{Df}}) A_{c\lambda}, \quad (6)$$

$$\bar{A}_\lambda^{\bar{f}\pi(\gamma)} = \pm \sigma_\lambda (r_{Df} \pm z_\lambda^+) A_{c\lambda}, \quad (7)$$

$$\bar{A}_\lambda^{\bar{f}\pi(\gamma)} = \pm \sigma_\lambda (1 \pm r_{Df} z_\lambda^+ e^{-2i\delta_{Df}}) A_{c\lambda}, \quad (8)$$

where  $\lambda = \{0, \parallel, \perp\}$  denotes the polarization states of vector mesons,  $\sigma_0 = \sigma_\parallel = 1, \sigma_\perp = -1, A_{c\lambda} \equiv |V_{cb}V_{us}| a_{c\lambda} e^{i\delta_{c\lambda}}, a_{c\lambda},$  and  $a_{u\lambda}$  are positive dimensionless parameters which characterize the decays  $B^- \rightarrow D^{*0} K^{*-}$  and  $B^- \rightarrow \bar{D}^{*0} K^{*-}$ , the phases  $\delta_{c\lambda}$  and  $\delta_{u\lambda}$  are conditioned by the strong interaction in the final state of these decays,  $z_\lambda^\pm \equiv r_{B\lambda} e^{i(\delta_\lambda \pm \gamma)}, \delta_\lambda \equiv \delta_{B\lambda} + \delta_{Df}, \delta_{B\lambda} \equiv \delta_{u\lambda} - \delta_{c\lambda},$

$$\delta_{Df} \equiv \arg\left(\frac{A(\bar{D}^0 \rightarrow f)}{A(D^0 \rightarrow f)}\right), r_{Df} \equiv \sqrt{\frac{Br(D^0 \rightarrow f)}{Br(\bar{D}^0 \rightarrow f)}},$$

$$r_{B\lambda} \equiv \left| \frac{A_\lambda(B^- \rightarrow \bar{D}^{*0} K^{*-})}{A_\lambda(B^- \rightarrow D^{*0} K^{*-})} \right| = \left| \frac{V_{ub}V_{cs}}{V_{cb}V_{us}} \right| \frac{a_{u\lambda}}{a_{c\lambda}}.$$

The upper sign in formulas (5)–(8) corresponds to processes (1), (3) and  $CP$ -conjugate processes, and the lower sign to processes (2), (4) and  $CP$ -conjugate processes. We recall that the contributions from two modes of the decays of  $D^{*0}$  to  $D^0\pi^0$  and to  $D^0\gamma$  to the amplitudes of the cascade decays (5)–(8) have opposite signs because  $D_\pm^* \rightarrow D_\pm\pi^0$ , whereas  $D_\pm^* \rightarrow D_\mp\gamma$  [15], which is a consequence of the conservation of the  $CP$  symmetry in these decays. Here,  $D_\pm^*$  and  $D_\mp$  ( $D_+^*$  and  $D_+$  ( $D_-^*$  and  $D_-$ )) mean the  $CP$ -even (-odd) states of the neutral  $D^*$  and  $D$  systems.

The probability of decays (1), (2) and effects of the direct violation of the  $CP$  symmetry in decays (1)–(4) depend significantly on values of the parameters  $r_{Df}$  and  $r_{B\lambda}$ . Though the decays of  $D$  mesons have been studied for a long time, the value of  $r_{Df}$  was measured with admissible accuracy only for the final state  $f = K^+\pi^-$ :  $r_{D(K^+\pi^-)} = 0.060 \pm 0.002$  [18]. Therefore, the numerical calculations in the present work will be made for this final state. It is expected [19] that the parameters

$$r_B \equiv \left| \frac{A(B^- \rightarrow \bar{D}^0 K^-)}{A(B^- \rightarrow D^0 K^-)} \right|, r_B^* \equiv \left| \frac{A(B^- \rightarrow \bar{D}^{*0} K^-)}{A(B^- \rightarrow D^{*0} K^-)} \right|,$$

and  $r_{B\lambda}$  in SM will be comparable in value. The early estimates of “color-suppressed” decays [19] led to values of  $r_B, r_B^*,$  and  $r_{B\lambda}$  to be about 0.1. However, the recent observations and measurements of the “color-suppressed” decays  $b \rightarrow c$  ( $\bar{B}^0 \rightarrow D^0 h^0, D^{*0} h^0; h^0 = \pi^0, \rho^0, \omega, \eta, \eta'$ ) [20] indicate that the “suppression by color” is not so efficient as was expected. Therefore, the parameters  $r_B, r_B^*, r_{B\lambda}$  can be about 0.2 [8]. On the other hand, by studying the decays  $B^\mp \rightarrow \tilde{D}^0 K^\mp, B^\mp \rightarrow \tilde{D}^{*0} K^\mp, \tilde{D}^{*0} \rightarrow \tilde{D}^0 \pi^0, \tilde{D}^0 \gamma, \tilde{D}^0 \rightarrow K_S^0 \pi^-\pi^+$  [13, 14],  $B^- \rightarrow \tilde{D}^0 K^-, B^- \rightarrow \tilde{D}^{*0} K^-, \tilde{D}^{*0} \rightarrow \tilde{D}^0 \pi^0, \tilde{D}^0 \gamma, \tilde{D}^0 \rightarrow K^+\pi^-$  [21], the groups BaBar and Belle obtained close results for the weak phase  $\gamma$ . But their results for the parameter  $r_B$  differ significantly:  $r_B = 0.21 \pm 0.08 \pm 0.03 \pm 0.04$  [13],  $r_B = 0.12 \pm 0.08 \pm 0.03 \pm 0.04$  [14]. For the parameter  $r_B^*$ , they got  $r_B^* = 0.12 \pm_{-0.11}^{+0.16} \pm 0.02 \pm 0.04$  [13],  $r_B^* = 0.17 \pm 0.10 \pm 0.03 \pm 0.03$  [14]. In addition, the average weighted values of the parameters  $r_B$  and  $r_B^*$  for different methods of measurements are as follows:  $r_B = 0.10 \pm 0.04, r_B^* =$

$0.09 \pm 0.04$  [22]. That is, these values do not differ, in fact, from the naive expectation [19]. Thus, the determination of the parameters  $r_B$  and  $r_B^*$  with admissible accuracy remains to be the important problem. As for values of the parameters  $r_{B\lambda}$  for different polarization states of vector mesons, no experimental limitations for these quantities are available up to now. Therefore, on the numerical calculations of the CP asymmetries and other observables in processes (1)–(4), we will consider not only the expected value  $r_{B\lambda}=0.2$  [8] for each polarization state of vector mesons, but also other values of the parameters  $r_{B\lambda}$ .

As follows from Eqs. (6) and (8), the violation of the CP symmetry in decays (3) and (4) is slight, because  $r_{Df} r_{B\lambda} \approx 0.01$ . Therefore, we may assume with high accuracy that  $|A_{\lambda}^{\bar{f}\pi(\gamma)}| = |\bar{A}_{\lambda}^{f\pi(\gamma)}|$  and can neglect the small effects of violation of the CP symmetry in these decays. Thus, only the transition  $b \rightarrow c \bar{u} s$  will contribute, in this approximation, to the probability of decays (3) and (4). Therefore, the measurement of the angular distributions of these decays will allow one to determine the parameters  $a_{c\lambda}$  and  $\delta_{c\lambda}$ . We recall that  $R_{c\lambda} \equiv a_{c\lambda}^2 / \sum_{\lambda=0,||,\perp} a_{c\lambda}^2$  are shares of the polarization (a longitudinal polarization and two transverse ones) of a vector meson in decays (3) and (4) and in CP-conjugate decays and only the share of the longitudinal polarization of vector mesons  $R_{c0} = 0.86 \pm 0.06 \pm 0.03$  [23] is measured up to now. Thus, taking the relation  $R_{c0} + R_{c||} + R_{c\perp} \equiv 1$  and the experimental data into account, we may draw conclusion that the vector mesons in the decay  $B^- \rightarrow D^{*0} K^{*-}$  are mainly created in the state with longitudinal polarization. In the subsequent consideration of processes (1)–(4), we will neglect the small effects of violation of the CP symmetry in decays (3) and (4).

The angular distributions of products of the decay of  $B$  mesons to two helically or linearly polarized vector mesons with the subsequent transition of both vector mesons into two pseudoscalar mesons or the transition of one vector meson to two pseudoscalar mesons and the other vector meson to a pseudoscalar meson and photon were determined in [24]. We are sure that it is more expedient to consider the ratio of the differential probability of decays (1), (2) to the probability of decays (3) and (4), namely:

$$\frac{d^3 \mathcal{R}_{f, D\pi(\gamma)}}{d \cos \theta_1 d \cos \theta_2 d\Phi} \equiv \frac{1}{\Gamma_{\bar{f}, D\pi(\gamma)}} \frac{d^3 \Gamma_{f, D\pi(\gamma)}}{d \cos \theta_1 d \cos \theta_2 d\Phi}, \quad (9)$$

$$\frac{d^3 \bar{\mathcal{R}}_{\bar{f}, D\pi(\gamma)}}{d \cos \theta_1 d \cos \theta_2 d\Phi} \equiv \frac{1}{\bar{\Gamma}_{\bar{f}, D\pi(\gamma)}} \frac{d^3 \bar{\Gamma}_{\bar{f}, D\pi(\gamma)}}{d \cos \theta_1 d \cos \theta_2 d\Phi}. \quad (10)$$

Ratios (9), (10) are not only free from many theoretical and experimental uncertainties. Moreover, their study allows one also to separate, as will be shown below, the contributions of the mechanism of the transition  $b \rightarrow c$  and that of the transition  $b \rightarrow u$  to the decay amplitudes. The ratio of the differential probability of decay (1) to the probability of decay (3) in a helical coordinate system is

$$\begin{aligned} \frac{d^3 \mathcal{R}_{f, D\pi}}{d \cos \theta_1 d \cos \theta_2 d\Phi} = & \frac{9}{16\pi} \left( 2R_0^\pi \cos^2 \theta_1 \cos^2 \theta_2 + \right. \\ & + \left( R_{||}^\pi \cos^2 \Phi + R_{\perp}^\pi \sin^2 \Phi - \xi_{||}^\pi \sin 2\Phi \right) \sin^2 \theta_1 \sin^2 \theta_2 + \\ & \left. + \left( \zeta^\pi \cos \Phi - \xi_0^\pi \sin \Phi \right) \sin 2\theta_1 \sin 2\theta_2 / \sqrt{2} \right), \quad (11) \end{aligned}$$

and the ratio of the differential probability of decay (2) to the probability of decay (4) looks as

$$\begin{aligned} \frac{d^3 \mathcal{R}_{f, D\gamma}}{d \cos \theta_1 d \cos \theta_2 d\Phi} = & \frac{9}{32\pi} \left( 2R_0^\gamma \sin^2 \theta_1 \cos^2 \theta_2 - \right. \\ & - \left( R_{||}^\gamma \cos^2 \Phi + R_{\perp}^\gamma \sin^2 \Phi - \xi_{||}^\gamma \sin 2\Phi \right) \sin^2 \theta_1 \sin^2 \theta_2 + \\ & \left. + \left( R_{||}^\gamma + R_{\perp}^\gamma \right) \sin^2 \theta_2 - \frac{\zeta^\gamma \cos \Phi - \xi_0^\gamma \sin \Phi}{\sqrt{2}} \sin 2\theta_1 \sin 2\theta_2 \right), \quad (12) \end{aligned}$$

where  $\theta_1$  is the angle between the direction of motion of a  $D$ -meson from the decay  $D^* \rightarrow D\pi$  or  $D^* \rightarrow D\gamma$  and the direction opposite to the direction of motion of a  $B$ -meson in the rest system of the  $D^*$ -meson,  $\theta_2$  is the angle between the direction of motion of a  $K$ -meson from the decay  $K^* \rightarrow K\pi$  and the direction opposite to the direction of motion of a  $B$ -meson in the rest system of the  $K^*$ -meson, and  $\Phi$  is the angle between planes of the decay  $D^* \rightarrow D\pi$  (або  $D^* \rightarrow D\gamma$ ) and  $K^* \rightarrow K\pi$  in the rest system of the  $B$ -meson. The quantities  $R_{\lambda}^{\pi(\gamma)}$ ,  $\xi_{0,||}^{\pi(\gamma)}$ , and  $\zeta^{\pi(\gamma)}$  have the form

$$R_{\lambda}^{\pi(\gamma)} = R_{c\lambda} \left( (r_{Df} \pm x_{\lambda}^-)^2 + (y_{\lambda}^-)^2 \right), \quad (13)$$

$$\xi_i^{\pi(\gamma)} \equiv \frac{\Im \left( A_{\perp}^{f\pi(\gamma)} \left( A_i^{f\pi(\gamma)} \right)^* \right)}{\sum_{\lambda=0,\parallel,\perp} \left| A_{\lambda}^{f\pi(\gamma)} \right|^2},$$

$$\zeta^{\pi(\gamma)} \equiv \frac{\Re \left( A_{\parallel}^{f\pi(\gamma)} \left( A_0^{f\pi(\gamma)} \right)^* \right)}{\sum_{\lambda=0,\parallel,\perp} \left| A_{\lambda}^{f\pi(\gamma)} \right|^2}, \quad (14)$$

where  $i = \{0, \parallel\}$ ,  $x_{\lambda}^{\pm}$  and  $y_{\lambda}^{\pm}$  are the real and imaginary parts of the complex-valued quantities  $z_{\lambda}^{\pm}$ , respectively. Ratio (10) has the same form as (11) or (12) with the change of  $\Phi \rightarrow -\Phi$ ,  $R_{\lambda}^{\pi(\gamma)}$  by  $\bar{R}_{\lambda}^{\pi(\gamma)}$ ,  $\xi_i^{\pi(\gamma)}$  by  $\bar{\xi}_i^{\pi(\gamma)}$ , and  $\zeta^{\pi(\gamma)}$  by  $\bar{\zeta}^{\pi(\gamma)}$ . Moreover,

$$\bar{R}_{\lambda}^{\pi(\gamma)} = R_{c\lambda} \left( (r_{Df} \pm x_{\lambda}^+)^2 + (y_{\lambda}^+)^2 \right), \quad (15)$$

and the formulas for  $\bar{\xi}_i^{\pi(\gamma)}$  and  $\bar{\zeta}^{\pi(\gamma)}$  have the same form as (14) with the change of  $A_{\lambda}^{f\pi(\gamma)}$  by  $\bar{A}_{\lambda}^{f\pi(\gamma)}$  and  $A_{\lambda}^{f\pi(\gamma)}$  by  $\bar{A}_{\lambda}^{f\pi(\gamma)}$ .

If six coefficients of the angular distributions (11)–(12) and the corresponding distributions for the  $CP$ -conjugate decays are measured, we can construct a lot of observables which violate the  $CP$  symmetry, namely:  $R_{\lambda}^{\pi(\gamma)} - \bar{R}_{\lambda}^{\pi(\gamma)}$ ,  $\xi_i^{\pi(\gamma)} + \bar{\xi}_i^{\pi(\gamma)}$ , and  $\zeta^{\pi(\gamma)} - \bar{\zeta}^{\pi(\gamma)}$ . Moreover, the observables  $\xi_i^{\pi(\gamma)} + \bar{\xi}_i^{\pi(\gamma)}$  characterize also the magnitude of the effects of violation of the  $T$  symmetry.

Let us assume that only the quantities  $R_{\lambda}^{\pi(\gamma)}$  and  $\bar{R}_{\lambda}^{\pi(\gamma)}$ , even for one polarization state of vector mesons, for example,  $R_0^{\pi(\gamma)}$  and  $\bar{R}_0^{\pi(\gamma)}$ , were measured on the first stage of experiments. This will allow one, nevertheless, to determine a value of the weak phase  $\gamma$ . Indeed, relations (13) and (15) imply that the charge mean ratios  $\mathcal{R}_{\lambda}^{\pi(\gamma)}$  and the parameters  $\mathcal{A}_{\lambda}^{\pi(\gamma)}$ , which describe the effects of violation of the  $CP$  symmetry for a given linearly polarized state of vector mesons, have the form

$$\mathcal{R}_{\lambda}^{\pi(\gamma)} \equiv \frac{R_{\lambda}^{\pi(\gamma)} + \bar{R}_{\lambda}^{\pi(\gamma)}}{2} =$$

$$= R_{c\lambda} (r_{Df}^2 + r_{B\lambda}^2 \pm r_{Df} (x_{\lambda}^- + x_{\lambda}^+)), \quad (16)$$

$$\mathcal{A}_{\lambda}^{\pi(\gamma)} \equiv \frac{R_{\lambda}^{\pi(\gamma)} - \bar{R}_{\lambda}^{\pi(\gamma)}}{2} = \pm r_{Df} R_{c\lambda} (x_{\lambda}^- - x_{\lambda}^+). \quad (17)$$

It is significant that the effects of violation of the  $CP$  symmetry in decays (1) and (2) for a given linearly polarized state of vector mesons have equal magnitudes and opposite signs, as follows from (17). Therefore, the common study of these decays will allow us to join these quantities and to obtain that the summary effect of violation of the  $CP$  symmetry in decays (1) and (2) for a given linearly polarized state of vector mesons is described by the parameter

$$\Delta \mathcal{A}_{\lambda} \equiv \mathcal{A}_{\lambda}^{\pi} - \mathcal{A}_{\lambda}^{\gamma} = 2 r_{Df} R_{c\lambda} (x_{\lambda}^- - x_{\lambda}^+). \quad (18)$$

Moreover, the common study of processes (1) and (2) for a given linearly polarized state of vector mesons allows us to separate the contribution of the transitions  $b \rightarrow c \bar{u} s$  and  $b \rightarrow u \bar{c} s$  to this polarization state of vector mesons. Indeed, Eq. (16) yields

$$\mathcal{R}_{\lambda}^{\pi} + \mathcal{R}_{\lambda}^{\gamma} = 2 R_{c\lambda} (r_{Df}^2 + r_{B\lambda}^2), \quad (19)$$

$$\Delta \mathcal{R}_{\lambda} \equiv \mathcal{R}_{\lambda}^{\pi} - \mathcal{R}_{\lambda}^{\gamma} = 2 r_{Df} R_{c\lambda} (x_{\lambda}^- + x_{\lambda}^+). \quad (20)$$

Then the contribution of the mechanism of  $b \rightarrow u \bar{c} s$  to a given linearly polarized state of vector mesons in decays (1) and (2) is

$$R_{c\lambda} r_{B\lambda}^2 = \frac{\mathcal{R}_{\lambda}^{\pi} + \mathcal{R}_{\lambda}^{\gamma}}{2} - R_{c\lambda} r_{Df}^2. \quad (21)$$

Thus, the measurement of the quantities  $\mathcal{R}_{\lambda}^{\pi}$ ,  $\mathcal{R}_{\lambda}^{\gamma}$ , and  $R_{c\lambda}$  will allow one to find a value of this contribution and to determine the parameter  $r_{B\lambda}^2$  for all polarization states of vector mesons without any assumption as for the quantity  $\gamma$  and the strong interaction phases. For example, it is expected for longitudinally polarized vector mesons in decays (1) and (2) in the final state  $f = K^+ \pi^-$  and at  $r_{B0}=0.2$  that the sum  $\mathcal{R}_0^{\pi} + \mathcal{R}_0^{\gamma}$  will be about 7%. It is also important that, by using Eq. (20), we can obtain the useful lower bound for the parameter  $r_{B\lambda}$ ,  $4 r_{Df} R_{c\lambda} r_{B\lambda} \geq |\Delta \mathcal{R}_{\lambda}|$ .

Equations (18) and (20) can be also represented as

$$\Delta \tilde{\mathcal{R}}_{\lambda} \equiv \frac{\mathcal{R}_{\lambda}^{\pi} - \mathcal{R}_{\lambda}^{\gamma}}{\mathcal{R}_{\lambda}^{\pi} + \mathcal{R}_{\lambda}^{\gamma}} = \rho_{f\lambda} \cos \delta_{\lambda} \cos \gamma, \quad (22)$$

$$\Delta \tilde{\mathcal{A}}_{\lambda} \equiv \frac{\mathcal{A}_{\lambda}^{\pi} - \mathcal{A}_{\lambda}^{\gamma}}{\mathcal{R}_{\lambda}^{\pi} + \mathcal{R}_{\lambda}^{\gamma}} = \rho_{f\lambda} \sin \delta_{\lambda} \sin \gamma, \quad (23)$$

where  $\rho_{f\lambda} \equiv 2 r_{Df} r_{B\lambda} / (r_{Df}^2 + r_{B\lambda}^2)$ ,  $0 < \rho_{f\lambda} \leq 1$ . Then, by adding and subtracting these equations, we obtain the equivalent system

$$\rho_{f\lambda} \cos(\delta_{\lambda} - \gamma) = \Delta \tilde{\mathcal{R}}_{\lambda} + \Delta \tilde{\mathcal{A}}_{\lambda}, \quad (24)$$

$$\rho_{f\lambda} \cos(\delta_\lambda + \gamma) = \Delta\tilde{R}_\lambda - \Delta\tilde{A}_\lambda, \quad (25)$$

from which we determine

$$\gamma = \frac{\sigma}{2} \arccos \frac{\Delta\tilde{R}_\lambda - \Delta\tilde{A}_\lambda}{\rho_{f\lambda}} - \frac{\tau}{2} \arccos \frac{\Delta\tilde{R}_\lambda + \Delta\tilde{A}_\lambda}{\rho_{f\lambda}} + (k-n)\pi, \quad (26)$$

$$\delta_\lambda = \frac{\sigma}{2} \arccos \frac{\Delta\tilde{R}_\lambda - \Delta\tilde{A}_\lambda}{\rho_{f\lambda}} + \frac{\tau}{2} \arccos \frac{\Delta\tilde{R}_\lambda + \Delta\tilde{A}_\lambda}{\rho_{f\lambda}} + (k+n)\pi, \quad (27)$$

where  $\sigma, \tau = \pm 1$  and  $k, n = 0, \pm 1, \dots$ . In order to determine the parameters  $\delta_\lambda$  and  $\gamma$ , we can use, instead of the system of equations (24), (25), the equivalent system of equations

$$\rho_{f\lambda} \cos(\delta_\lambda - \gamma) = \frac{R_\lambda^\pi - R_\lambda^\gamma}{R_\lambda^\pi + R_\lambda^\gamma},$$

$$\rho_{f\lambda} \cos(\delta_\lambda + \gamma) = \frac{\bar{R}_\lambda^\pi - \bar{R}_\lambda^\gamma}{\bar{R}_\lambda^\pi + \bar{R}_\lambda^\gamma}.$$

Thus, (26) and (27) imply that, for any polarization state of vector mesons, there exist eight physically different values of  $\delta_\lambda$  and  $\gamma$  which yield the same values of  $\Delta\tilde{R}_\lambda$  and  $\Delta\tilde{A}_\lambda$ . We note that the weak phase  $\gamma$  is independent of the state of polarization of vector mesons, whereas the strong interaction phases  $\delta_\lambda$  can depend on the state of polarization of vector mesons. Therefore, the common analysis of all the polarization states of vector mesons would allow one to decrease the uncertainty on the determination of the weak phase  $\gamma$  from Eqs. (24)–(25). It is worth emphasizing that the use of only one mode of the decay of  $D^{*0}$  mesons to  $D^0\pi^0$  does not allow one (without the extraction of the values characterizing

the interference of different polarization states of vector mesons from experimental data) to determine the weak phase  $\gamma$  [10].

In addition, Eqs. (22) and (23) yield a number of restrictions on the parameters  $\rho_{f\lambda}$  and  $\gamma$ , for example,

$$|\cos \gamma| \geq \left| \frac{\Delta\tilde{R}_\lambda}{\rho_{f\lambda}} \right|. \quad (28)$$

For the sake of illustration, the Table presents the upper bounds for the angle  $\gamma$  (in degrees) for the final state  $f = K^+\pi^-$  which are obtained from (22) and (28) for  $r_{B\lambda} = 0.06, 0.12, \text{ and } 0.18$ . While calculating the bounds for the quantity  $\gamma$  in the second, third, and fourth columns, we assumed that  $|\delta_\lambda| \leq 30^\circ$ . The bounds in the fifth and sixth columns correspond to  $\delta_\lambda = 0$  and  $\delta_\lambda = 60^\circ$ , respectively. In the parentheses, we give values of  $\Delta\tilde{R}_\lambda$  on the initial values of the parameters  $\rho_{f\lambda}$ ,  $\gamma$ , and  $\delta_\lambda$ . As shown in the second column of the Table, the greatest values of  $\Delta\tilde{R}_\lambda$  and, respectively, the most strict restrictions on the quantity  $\gamma$  occur for  $r_{B\lambda} = 0.06$ , because, in this case, the quantity  $\rho_{(K^+\pi^-)\lambda} = 1$ . With increase in  $r_{B\lambda}$ , which corresponds to a decrease in the quantities  $\rho_{f\lambda}$  and  $\Delta\tilde{R}_\lambda$ , the bounds become less strict. In addition, the more strict restrictions on the quantity  $\gamma$  hold for  $\delta_\lambda = 0^\circ$ , as shown in the fifth column. On the other hand, with increase in  $\delta_\lambda$ , the bounds become less strict, as it is demonstrated for  $\delta_\lambda = 60^\circ$  in the sixth column.

The measurement of the quantities  $\mathcal{R}_\lambda^\pi$  and  $\mathcal{R}_\lambda^\gamma$ ,  $r_{Df}$ , and  $R_{c\lambda}$  can also allow one to get a restriction on the quantity  $\sin^2 \gamma$ . Indeed, with the use of the identity

$$r_{Df}^2 + r_{B\lambda}^2 \pm r_{Df} (x_\lambda^- + x_\lambda^+) \equiv r_{Df}^2 \sin^2 \delta_\lambda \cos^2 \gamma + r_{Df}^2 \sin^2 \gamma + (r_{B\lambda} \pm r_{Df} \cos \delta_\lambda \cos \gamma)^2,$$

Eq. (16) yields the following restriction on  $\sin^2 \gamma$ :

$$r_{Df}^2 R_{c\lambda} \sin^2 \gamma \leq \mathcal{R}_\lambda^{\pi(\gamma)}. \quad (29)$$

**Upper bound of  $\gamma$  (in degrees) obtained from Eq. (22) and inequality (28) (the numbers in the parentheses correspond to values of  $\Delta\tilde{R}_\lambda$  on the initial values of the parameters  $\rho_{f\lambda}$ ,  $\gamma$ , and  $\delta_\lambda$ )**

| Initial value of $\gamma$ | $ \delta_\lambda  \leq 30^\circ$ |                       |                       | $\delta_\lambda = 0$  | $\delta_\lambda = 60^\circ$ |
|---------------------------|----------------------------------|-----------------------|-----------------------|-----------------------|-----------------------------|
|                           | $r_{B\lambda} = 0.06$            | $r_{B\lambda} = 0.12$ | $r_{B\lambda} = 0.18$ | $r_{B\lambda} = 0.18$ | $r_{B\lambda} = 0.18$       |
| 50                        | 56 (0.56)                        | 64 (0.45)             | 71 (0.33)             | 67 (0.39)             | 80 (0.19)                   |
| 60                        | 64 (0.43)                        | 70 (0.35)             | 75 (0.26)             | 73 (0.30)             | 81 (0.15)                   |
| 70                        | 73 (0.30)                        | 76 (0.24)             | 80 (0.18)             | 78 (0.21)             | 84 (0.10)                   |
| 80                        | 82 (0.15)                        | 83 (0.12)             | 85 (0.09)             | 84 (0.10)             | 87 (0.05)                   |

These inequalities will yield the restrictions on the angle  $\gamma$  if  $\mathcal{R}_\lambda^\pi < r_{Df}^2 R_{c\lambda}$  or  $\mathcal{R}_\lambda^\gamma < r_{Df}^2 R_{c\lambda}$ . However, since  $R_{c\lambda} (r_{Df} - r_{B\lambda})^2 \leq \mathcal{R}_\lambda^{\pi(\gamma)} \leq R_{c\lambda} (r_{Df} + r_{B\lambda})^2$ , it is necessary that, at least,  $|r_{Df} - r_{B\lambda}| < r_{Df}$ , i.e.  $r_{B\lambda} < 2r_{Df}$ , in order that  $\mathcal{R}_\lambda^{\pi(\gamma)} < r_{Df}^2 R_{c\lambda}$ . For the final state  $f = K^+\pi^-$ , this means that  $r_{B\lambda} < 0.12$ . Thus, for the expected values of  $r_{B\lambda} \simeq 0.2$ , inequality (29) cannot yield a restriction on the angle  $\gamma$  for the final state  $f = K^+\pi^-$ . But if the experiment will indicate that  $\mathcal{R}_\lambda^{\pi(\gamma)} < r_{Df}^2 R_{c\lambda}$ , this will testify that  $r_{B\lambda} < 2r_{Df}$  and  $\pm r_{Df} (x_\lambda^- + x_\lambda^+) > r_{B\lambda}^2$ .

To study the effects of violation of the CP symmetry in decays (1) and (2) for a given linearly polarized state of vector mesons, we consider the CP asymmetries  $A_\lambda^\pi$  and  $A_\lambda^\gamma$ . Then it follows from Eqs. (13), (15), (22), and (23) that these CP asymmetries are

$$A_\lambda^{\pi(\gamma)} \equiv \frac{R_\lambda^{\pi(\gamma)} - \overline{R}_\lambda^{\pi(\gamma)}}{R_\lambda^{\pi(\gamma)} + \overline{R}_\lambda^{\pi(\gamma)}} = \pm \frac{\rho_{f\lambda} \sin \delta_\lambda \sin \gamma}{1 \pm \rho_{f\lambda} \cos \delta_\lambda \cos \gamma}. \quad (30)$$

As seen from (30), the asymmetries  $A_\lambda^\pi$  and  $A_\lambda^\gamma$  have opposite signs, and their difference

$$\Delta A_\lambda \equiv A_\lambda^\pi - A_\lambda^\gamma = \frac{2 \rho_{f\lambda} \sin \delta_\lambda \sin \gamma}{1 - \rho_{f\lambda}^2 \cos^2 \delta_\lambda \cos^2 \gamma}. \quad (31)$$

Then, at fixed values of the parameters  $\Delta \tilde{\mathcal{A}}_\lambda$  and  $\rho_{f\lambda}$ , function (31) can be represented as

$$\Delta A_\lambda = \frac{2 \Delta \tilde{\mathcal{A}}_\lambda}{1 - \rho_{f\lambda}^2 \cos^2 \gamma + (\Delta \tilde{\mathcal{A}}_\lambda)^2 \cot^2 \gamma}, \quad (32)$$

whose domain of definition is the segment  $\gamma_0 \leq \gamma \leq \pi - \gamma_0$ , where  $\gamma_0 \equiv \arcsin \frac{|\Delta \tilde{\mathcal{A}}_\lambda|}{\rho_{f\lambda}}$ . As follows from (32), the plot of the function  $\Delta A_\lambda$  will be symmetric relative to the axis  $\gamma = \pi/2$ . Therefore, it is sufficient to consider the behaviour of function (32) on the segment  $\gamma_0 \leq \gamma \leq \pi/2$ . Let  $\Delta \tilde{\mathcal{A}}_\lambda > 0$  ( $\Delta \tilde{\mathcal{A}}_\lambda < 0$ ), and let  $\gamma$  vary from  $\gamma_0$  to  $\gamma_1 \equiv \arcsin \sqrt{\frac{|\Delta \tilde{\mathcal{A}}_\lambda|}{\rho_{f\lambda}}}$ . Then the function  $\Delta A_\lambda$  will increase (decrease) monotonously from  $2 \Delta \tilde{\mathcal{A}}_\lambda$  to  $\frac{2 \Delta \tilde{\mathcal{A}}_\lambda}{1 - (\rho_{f\lambda} - |\Delta \tilde{\mathcal{A}}_\lambda|)^2}$ . If  $\gamma$  varies from  $\gamma_1$  to  $\pi/2$ , the function  $\Delta A_\lambda$  will decrease (increase) monotonously to  $2 \Delta \tilde{\mathcal{A}}_\lambda$ . In order to extract the weak phase  $\gamma$  from the observable  $\Delta A_\lambda$ , it is necessary, at least, that function (32) be changed essentially on the segment  $\gamma_0 \leq \gamma \leq$

$\pi/2$ . For example, if the ratio of the difference between the maximum and minimum values of the function  $\Delta A_\lambda$  to the minimum value of this function will be greater than 30%, then  $\rho_{f\lambda} - |\Delta \tilde{\mathcal{A}}_\lambda| > 0.48$ . Then, at the values of the observable  $|\Delta \tilde{\mathcal{A}}_\lambda|$  to be at least 10%, we get that the parameter  $\rho_{f\lambda}$  must be greater than 0.58. Thus, under these conditions, the extraction of  $\gamma$  from the observable  $\Delta A_\lambda$  will be possible iff the quantity  $r_{B\lambda}$  will satisfy the inequality  $0.32 r_{Df} < r_{B\lambda} < 3.13 r_{Df}$ .

For the final state  $f = K^+\pi^-$  and for the parameter  $r_{B\lambda}$  equal to 0.18, i.e.  $\rho_{(K^+\pi^-)\lambda} = 0.6$ , and for  $\Delta \tilde{\mathcal{A}}_\lambda$  equal to 0.5, 0.3, and 0.1, we get that the quantity  $\Delta A_\lambda$  will vary from 1.0, 0.6, and 0.2, respectively, to 1.01, 0.66, and 0.27. Moreover, the maximum value of  $\Delta A_\lambda$  will be reached at  $\gamma$  equal to 66, 45, and 24° or at  $\gamma$  equal to 114, 135, and 156°, respectively. In this case, the minimum value of the quantity  $\Delta A_\lambda$  will be reached at  $\gamma$  equal to 56, 30, and 10° or at  $\gamma$  equal to 124, 150, and 170°, respectively, and also at  $\gamma$  equal to 90° for any values of  $\Delta \tilde{\mathcal{A}}_\lambda$ . Thus, for the final state  $f = K^+\pi^-$  and at the expected value of  $r_{B\lambda} \sim 0.2$  [8], the difference of the CP asymmetries  $A_\lambda^\pi$  and  $A_\lambda^\gamma$  will take significant values in the wide range of variations in  $\Delta \tilde{\mathcal{A}}_\lambda$ . However, at  $|\Delta \tilde{\mathcal{A}}_\lambda| \geq 0.2$ , the quantity  $\Delta A_\lambda$  is weakly changed in the region of admissible values of  $\gamma$ . Therefore, the extraction of the weak phase  $\gamma$  from the observable quantity  $\Delta A_\lambda$  can turn out to be a difficult problem. Consider  $|\Delta \tilde{\mathcal{A}}_\lambda| \sim 0.1$ . Though values of the quantity  $\Delta A_\lambda$  decrease, this quantity is changed, however, in the region of admissible values of  $\gamma$  by at least 30%, which allows one to extract the weak phase  $\gamma$  from the observable quantity  $\Delta A_\lambda$ .

If the parameter  $r_{B\lambda}$  is equal to 0.12, i.e.  $\rho_{(K^+\pi^-)\lambda} = 0.8$ , and if  $\Delta \tilde{\mathcal{A}}_\lambda$  is equal to 0.5, 0.3, and 0.1, we get that the difference of the CP asymmetries,  $\Delta A_\lambda$ , will vary from 1.0, 0.6, and 0.2 to 1.10, 0.80, and 0.39, respectively. Moreover, the maximum values of the quantities  $\Delta A_\lambda$  will be reached at  $\gamma$  equal to 52, 38, and 21° or at 128, 142, and 159°, respectively, and the minimum values at  $\gamma$  equal to 39, 22, and 7° or at 141, 158, and 173°, respectively, and also at  $\gamma$  equal to 90° for any  $\Delta \tilde{\mathcal{A}}_\lambda$ . These estimates imply that, for  $|\Delta \tilde{\mathcal{A}}_\lambda| < 0.4$ ,  $\Delta A_\lambda$  varies essentially, especially at  $|\Delta \tilde{\mathcal{A}}_\lambda| \sim 0.1$ , in the region of admissible values of  $\gamma$ , which allows one to use this observable for the extraction of the weak phase  $\gamma$ . We give attention to the fact that function (32) is practically constant in the region of admissible values of  $\gamma$ , if  $r_{B\lambda} > 5 r_{Df}$  or  $r_{B\lambda} < 0.2 r_{Df}$ . But it is changed significantly if  $0.3 r_{Df} < r_{B\lambda} < 3 r_{Df}$  and most strongly if  $r_{B\lambda} \sim r_{Df}$ . For the sake of illustration, we present the

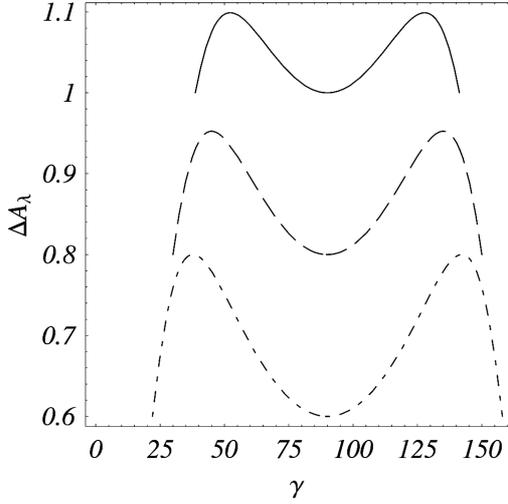


Fig. 1. Dependences of the difference of the CP asymmetries  $A_{\lambda}^{\pi}$  and  $A_{\lambda}^{\gamma}$  on  $\gamma$  (in degrees) at a fixed value of  $\Delta\tilde{A}_{\lambda}$  for  $f = K^+\pi^-$  and  $r_{B\lambda} = 0.12$ . The continuous, dashed, and dash-dotted lines correspond to  $\Delta\tilde{A}_{\lambda}$  equal to 0.5, 0.4, and 0.3, respectively

dependence of function (32) on  $\gamma$  at three values of  $\Delta\tilde{A}_{\lambda}$  and for  $\rho_{f\lambda} = 0.8$  in Fig. 1.

At fixed values of the parameters  $\Delta\tilde{A}_{\lambda}$ ,  $\rho_{f\lambda}$ , and  $\cos\delta_{\lambda}\cos\gamma > 0$ , function (22) can be represented as

$$\Delta\tilde{R}_{\lambda} = \sqrt{\rho_{f\lambda}^2 \cos^2 \gamma - (\Delta\tilde{A}_{\lambda})^2 \cot^2 \gamma}, \quad (33)$$

whose domain of definition is the segment  $\gamma_0 \leq \gamma \leq \pi - \gamma_0$ . As follows from (33), the plot of the function  $\Delta\tilde{R}_{\lambda}$  is symmetric relative to the axis  $\gamma = \pi/2$ . Therefore, it is sufficient to consider the behaviour of function (33) on the segment  $\gamma_0 \leq \gamma \leq \pi/2$ . As  $\gamma$  varies from  $\gamma_0$  to  $\gamma_1$ , function (33) will increase monotonously from zero to  $\rho_{f\lambda} - |\Delta\tilde{A}_{\lambda}|$ . But if  $\gamma$  varies from  $\gamma_1$  to  $\pi/2$ , the function  $\Delta\tilde{R}_{\lambda}$  will decrease monotonously to zero.

For  $f = K^+\pi^-$  and  $r_{B\lambda} = 0.18$ , i.e.  $\rho_{(K^+\pi^-)\lambda} = 0.6$ , and for  $\Delta\tilde{A}_{\lambda} = 0.5, 0.3$ , and  $0.1$ ,  $\Delta\tilde{R}_{\lambda}$  varies from zero at any  $\Delta\tilde{A}_{\lambda}$  to 0.1, 0.3, and 0.5, respectively. Moreover, both minimum and maximum values of relation (33) will be attained at the same  $\gamma$  as for  $\Delta A_{\lambda}$ . Thus, it is possible to extract a value of the weak phase  $\gamma$  from the observable  $\Delta\tilde{R}_{\lambda}$ , because function (33) is significantly changed in the region of admissible values of  $\gamma$  for  $f = K^+\pi^-$  and at the expected values of  $r_{B\lambda} \sim 0.2$  [8].

For  $r_{B\lambda} = 0.12$ , i.e.  $\rho_{(K^+\pi^-)\lambda} = 0.8$ , and  $\Delta\tilde{A}_{\lambda} = 0.5, 0.3$ , and  $0.1$ ,  $\Delta\tilde{R}_{\lambda}$  is changed from zero at any  $\Delta\tilde{A}_{\lambda}$  to 0.3, 0.5, and 0.9, respectively. These estimates imply that function (33) is significantly changed in the wide

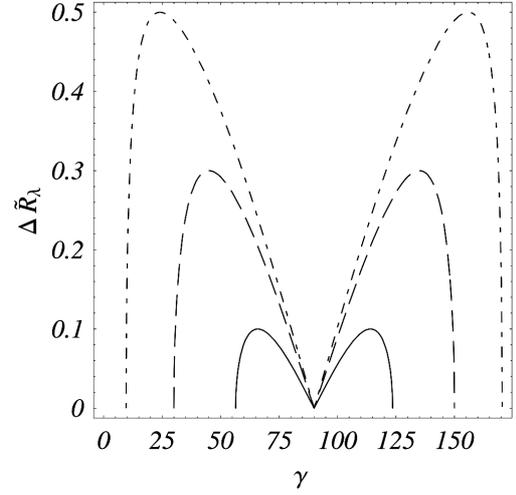


Fig. 2. Dependences of the  $\Delta\tilde{R}_{\lambda}$  on  $\gamma$  (in degrees) at a fixed value of  $\Delta\tilde{A}_{\lambda}$  for  $f = K^+\pi^-$  and  $r_{B\lambda} = 0.18$ . The continuous, dashed, and dash-dotted lines correspond to  $\Delta\tilde{A}_{\lambda}$  equal to 0.5, 0.3, and 0.1, respectively

region of variations of  $|\Delta\tilde{A}_{\lambda}|$ , which allows one to use this observable for the extraction of the weak phase  $\gamma$ . For the sake of illustration, we present the dependence of function (33) on  $\gamma$  at three values of  $\Delta\tilde{A}_{\lambda}$  and for  $\rho_{f\lambda} = 0.6$  in Fig. 2.

Thus, the analysis of the dependence of the functions  $\Delta A_{\lambda}$  and  $\Delta\tilde{R}_{\lambda}$  on the angle  $\gamma$  implies that relation (33) is preferable, as seen, for the extraction of values of the weak phase  $\gamma$  especially at great values of  $|\Delta\tilde{A}_{\lambda}|$ .

As was mentioned above, we neglect the small effects of violation of the  $CP$  symmetry in decays (3) and (4). Therefore, Eqs. (13) and (15) yield the following sum rule:

$$R_{\lambda}^{\pi} + R_{\lambda}^{\gamma} = \bar{R}_{\lambda}^{\pi} + \bar{R}_{\lambda}^{\gamma}. \quad (34)$$

The violation of this sum rule will testify to the manifestation of  $CP$ -odd effects in decays (3) and (4) for a given linearly polarized state of vector mesons. As a characteristic of the degree of the violation of (34), we will use the asymmetry

$$A_{\lambda} \equiv \frac{R_{\lambda}^{\pi} + R_{\lambda}^{\gamma} - \bar{R}_{\lambda}^{\pi} - \bar{R}_{\lambda}^{\gamma}}{R_{\lambda}^{\pi} + R_{\lambda}^{\gamma} + \bar{R}_{\lambda}^{\pi} + \bar{R}_{\lambda}^{\gamma}} = -r_{Df} \rho_{f\lambda} \sin 2\gamma \times \\ \times \sum_{\lambda'=0,||,\perp} R_{c\lambda'} r_{B\lambda'} \sin(\delta_{B\lambda} + \delta_{B\lambda'}).$$

To estimate  $A_{\lambda}$ , we assume that  $\delta_{B0} \simeq 90^{\circ}$ ,  $\delta_{B||} \simeq \delta_{B\perp} \simeq 0$ , and  $r_{B0} \simeq r_{B||} \simeq r_{B\perp}$ . Then, for  $f = K^+\pi^-$ ,

$R_{c0} = 0.86$  [23],  $\gamma = 70^\circ$  [14], and  $r_{B\lambda} \sim 0.2$  [8], we will find that the CP asymmetries for different polarization states of vector mesons are less than one per cent, namely:  $A_0 \simeq -0.06\%$ ,  $A_{\parallel} \simeq A_{\perp} \simeq -0.4\%$ . Thus, it is expected in the frame of the SM that the sum rule (34) will be satisfied with 99% accuracy. Therefore, its experimental verification is useful for the search for effects of “the new physics”.

### 3. Conclusions

We have considered the effects of violation of the CP symmetry in the decays of  $B^\mp$  mesons to a pair of linearly polarized vector  $D^*$  and  $K^{*\mp}$  mesons. While studying these decays, we propose to use two modes of decays in order to establish a polarization state of vector  $D^{*0}$  and  $\overline{D}^{*0}$  mesons, namely:  $D^{*0} \rightarrow D^0 \pi^0$  and  $D^{*0} \rightarrow D^0 \gamma$  with the subsequent transition of a  $D^0$  meson into the doubly Cabibbo-suppressed states. This allows one to determine the ratios of the transition amplitude  $B^- \rightarrow \overline{D}^{*0} K^{*-}$  to that of  $B^- \rightarrow D^{*0} K^{*-}$  for different polarization states of vector mesons without any assumptions as for the weak phase  $\gamma$  and the strong interaction phases. Moreover, the CP asymmetries for the modes  $D^{*0} \rightarrow D^0 \pi^0$  and  $D^{*0} \rightarrow D^0 \gamma$  have opposite signs, which can be verified in experiments. Moreover, the absolute value of their difference for the final state  $f = K^+ \pi^-$ ,  $r_B = 0.18$ , and  $\gamma = 70^\circ$ , if  $|\delta_\lambda|$  will be at least  $10^\circ \pmod{180^\circ}$ , can take value from 0.2 to 1.1, which allows one to hope for its experimental measurement. In addition, the use of two modes of the decay of vector  $D^{*0}$  and  $\overline{D}^{*0}$  mesons gives the possibility to determine the angle  $\gamma$  of the unitary triangle, even if only one polarization state of vector mesons will be determined. The determination of all polarization states will decrease the uncertainty of the extraction of  $\gamma$ .

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ОДЕРЖАННЯ СЛАБКОЇ ФАЗИ  $\gamma$  З РОЗПАДІВ  
 $B^\mp$ -МЕЗОНІВ НА ПАРУ ВЕКТОРНИХ  
 $D^*$ - ТА  $K^{*\mp}$ -МЕЗОНІВ

В.А. Ковальчук

Резюме

Запропоновано метод для вимірювання слабкої фази  $\gamma$  та величини  $r_{B\lambda}$  (відношення амплітуд  $A_\lambda(B^- \rightarrow \overline{D}^{*0} K^{*-})$  до

$A_\lambda(B^- \rightarrow D^{*0} K^{*-})$  за допомогою інтерференції між каналами розпаду зарядженого  $B$  мезона на  $B^- \rightarrow D^{*0} K^{*-}$  та  $B^- \rightarrow \bar{D}^{*0} K^{*-}$ , коли  $D^{*0}$  та  $\bar{D}^{*0}$  розпадаються на  $D^0/\bar{D}^0 \pi^0$  та на  $D^0/\bar{D}^0 \gamma$ . Як загальні адронні кінцеві стани розпадів  $D^0$ - та  $\bar{D}^0$ -мезонів були вибрані подвійно пригнічені

моди Кабіббо розпаду  $D^0$ -мезонів. Показано, що CP-асиметрії розпадів  $B^- \rightarrow D^{*0}/\bar{D}^{*0} \rightarrow D^0/\bar{D}^0 \rightarrow K^+ \pi^- \pi^0 K^{*-}$  та  $B^- \rightarrow D^{*0}/\bar{D}^{*0} \rightarrow D^0/\bar{D}^0 \rightarrow K^+ \pi^- \gamma K^{*-}$  мають протилежні знаки.