2D SU(2) PRINCIPAL CHIRAL MODEL: DUAL REPRESENTATION IN THE CLASSICAL LIMIT AND LOW-TEMPERATURE ASYMPTOTICS OF CORRELATION FUNCTIONS

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We develop an analytical approach to the investigation of the low-temperature limit of the two-dimensional (2D) lattice SU(2) principal chiral model. The basic idea is to formulate the model on a dual lattice. A dual representation is derived in the classical limit, i.e. in the region of large angular momenta. The derivation involves one approximation which is a certain asymptotic relation between the $6j$ symbols and the Clebsch—Gordan coefficients. The leading terms of two-point and link-link correlation functions are evaluated in the dual formulation, and it is shown that both correlations have power-like decay. This claim is a result of the second approximation which consists in substituting the SU(2) generalized characters by their asymptotics which hold uniformly in the vicinity of the identity element of the SU(2) link matrix. Moreover, if certain local defects are bound into multipoles and cannot produce a mass gap, the low-temperature region seems to be completely dominated by the spin-wave contribution.

1. Introduction

Dual transformations recommended themselves as a very powerful analytical method in many branches of physics. Among the most famous examples are Abelian spin and gauge models on a lattice [1]. It is especially worth emphasizing a dual of the 2D XY model which has been used to prove the existence of a soft phase at low temperatures with power-like decay of the correlation function [2]. In this case, the dual of the XY model is a local theory for certain discrete variables. No similar representation is known for any non-Abelian model. On the other hand, there exists the representation of 2D models in terms of link variables [3], and this representation can be formulated directly on the dual lattice. In the present paper, we use this representation to derive the approximate dual form of the 2D SU(2) principal chiral model in the low-temperature limit. The low-temperature properties of this model are crucial for the construction of its continuum limit. It is widely expected that the model possesses no phase transition, the correlation function has exponential decay at any coupling, and the model is asymptotically free. Despite being more than twenty years old, this expectation has not been proven rigorously. On the contrary, certain percolation theory arguments suggest that all non-Abelian models have soft low-temperature phase with power-like decay of the correlation function [4]. Calculations we performed here in the framework of a dual representation support this scenario.

Let us explain our strategy and main ideas. In some of our previous papers, we developed a weak coupling expansion for SU(N) spin models using the link representation for the partition and correlation functions [5]. In paper [6], we have constructed a dual representation of the SU(2) model on the basis of the link formulation and have investigated some of its low-temperature properties. Let us briefly summarize our approach. The partition function reads

$$Z = \int \prod_l dV_l \exp\left[\beta \sum_l \mathrm{Tr} V_l\right] \prod_p \left[\sum_r d_r \chi_r \left(\prod_{l \in p} V_l\right)\right], \tag{1}$$

where $V_l \in \mathrm{SU}(N)$, and dV_l is the invariant measure on the group. $\prod_p$ is a product over all plaquettes of the 2D lattice, the sum over r runs over all representations of SU(N), and $d_r = \chi_r(I)$ is the dimension of the r-th representation. The SU(N) character χ_r depends on a product of link matrices $V_l = V_n(x)$ around a plaquette

$$\prod_{l \in p} V_l = V_n(x) V_m(x+n) V_n^\dagger(x+m) V_m^\dagger(x) \ . \tag{2}$$

Periodic boundary conditions are imposed on V_l. From now on, we concentrate on the SU(2) model. As follows from (1), the partition function on the dual lattice may be written as

$$Z = \sum_{r_x = 0, \frac{1}{2}, 1, \ldots}^{\infty} \prod_x \left[(2r_x + 1) \sum_{m_i(x) = -r_x}^{r_x}\right] \prod_l \Xi_l \ . \tag{3}$$

We used here the definition of SU(2) characters to express them as the sums over magnetic numbers m_i. Due to the trace, there are 4 independent variables m_i at each dual site, thus $i = 1, 2, 3, 4$. Ξ_l is given by

$$\Xi_l = \int dV e^{\beta \mathrm{Tr} V} \; V_{r_x}^{m_1 n_1} V_{r_{x+n}}^{\dagger m_2 n_2} \; , \tag{4}$$

where V_r^{mn} is a matrix element of the r-th representation.

A similar form for the two-point correlation function in the representation j reads

$$\Gamma_j(x,y) = \frac{1}{2j+1} \sum_{s_1=-j}^{j} ... \sum_{s_R=-j}^{j} \langle \prod_{l \in C_{xy}} \frac{\Xi_l^{s_i s_{i+1}}(j)}{\Xi_l(0)} \rangle \; , \tag{5}$$

where $s_{R+1} = s_1$ and the link integral on $l \in C_{xy}$ is

$$\Xi_l^{s_i s_{i+1}}(j) = \int dV e^{\beta \mathrm{Tr} V} \; V_{r_x}^{m_1 n_1} V_{r_{x+n}}^{\dagger m_2 n_2} V_j^{s_i s_{i+1}} \; . \tag{6}$$

Here, C_{xy} is some path connecting points x and y and consisting of links dual to links of the original lattice. As a matter of fact, this form of the partition function can also be viewed as a dual representation of the 2D SU(2) model which has, pretty similar to the XY model, a local form. However, there is also a difference. While the dual of the XY model is a local theory for integers which label the representation of $U(1)$ group, this is not the case for a non-Abelian model. It is clear from the equations above that the summation over magnetic numbers makes the effective theory for r_x highly non-local, and this non-locality persists at any temperatures. This fact prevents the direct extension of methods of [2] to the partition function (3). On the other hand, it is not obvious *a priori* that this non-locality has anything to do with expected non-perturbative phenomena like the mass gap generation. As an example, consider the partition function of a three-component Gaussian field written in the spherical coordinates. The integration over angle variables produces a complicated non-local theory for the radial component of the Gaussian field. However, such a non-locality cannot change the Gaussian nature of the field and by itself has no non-perturbative origin. Being guided by this simple observation, we have been looking for such a dual representation, where the Gaussian distribution of fluctuations would be manifestly seen in the low-temperature regime. In the next section, we will prove that the partition function (3) can be approximated by the following expression in the region of large angular momenta (large r_x):

$$Z \approx \int_0^{2\pi} \prod_x d\phi_x \int_0^{\pi} \prod_x \sin\theta_x d\theta_x \times$$

$$\times \sum_{r_x = 0, \frac{1}{2}, ...}^{\infty} \prod_x (2r_x+1)^2 \prod_{x,n} Q(\cos\Delta_l, r_x, r_{x+n}; \beta) \; . \tag{7}$$

Here, the link angle Δ_l is defined as

$$\cos\Delta_l = \cos\theta_x \cos\theta_{x+n} +$$

$$+ \sin\theta_x \sin\theta_{x+n} \cos(\phi_x - \phi_{x+n}). \tag{8}$$

The function $Q = Q(l)$ plays the role of a dual Boltzmann factor and will be given below. The derivation of Eq.(7), as well as a similar expression for the correlation function, uses only one approximation, namely a certain asymptotic relation between the $6j$-symbols and the Clebsch—Gordan coefficients.

In Section 3, we use this dual representation to compute the leading terms of two-point and link-link correlation functions. The function $Q(l)$ in (7) is still too complicated for a rigorous analysis. Therefore, we need some reliable approximation for it. The underlying philosophy behind our approximation was explained in [5]. In short, it can be paraphrazed as follows. As one can rigorously prove by contour estimates [7], the Gibbs measure of 2D models with continuous symmetry and at low temperatures is strongly concentrated around configurations, on which the link matrix V_l is close to unity, and this statement does not depend on the quantity actually being computed, i.e. fixed or long-distance one. In fact, in the parameterization $\mathrm{Tr}V = 2\cos\omega$, the only essential region of configurations saturating the path integral is the one where $\omega \leq \mathcal{O}(\beta^{-1/2})$. Outside this region, the configurations give exponentially small contributions. Our idea is then to replace the function $Q(l)$ by its asymptotics at large β calculated in the region $\omega \sim \mathcal{O}(\beta^{-1/2})$ and uniformly valid in r_x, i.e. for $\omega r \sim \mathcal{O}(1)$. Of course, this is also the region of the conventional perturbation theory (PT), where one could compute fixed-distance quantities. Nevertheless, there is one essential difference. The conventional PT is the expansion around an ordered state. In the case of the long-distance quantities, one usually says that spins are slowly rotated and eventually cover the whole group space, therefore PT cannot be applied. This objection does not work in our case: however, the distance between spins is the link matrix which is always close to unity if β is sufficiently large. Therefore, there is no any obvious objection for computations of the long-distance quantities in this region. Within this approximation, by supposing that local defects cannot produce a mass gap, we will prove that the two-point correlation function (5) decays as

$$\Gamma_j(x,y) \approx e^{-\frac{(2j+1)^2}{2\beta}D(x-y)} \ , \ D(x-y) \asymp \frac{1}{2\pi}\ln \mid x-y \mid \ . \quad (9)$$

2. Dual Representation in the Classical Limit

In this section, we prove Eq.(7) and compute the link function $Q(l)$. We present here one of the possible proofs which is based on the exact calculation of traces of SU(2) matrices. To calculate the traces, we use the representation for SU(2) matrix elements, in which the dependence on the magnetic numbers enters only through the Clebsch—Gordan coefficients $C^{a\alpha}_{b\beta\ c\gamma}$ [1]

$$V^{mn}_r(\omega,\theta,\phi) = \sum_{\lambda k} C^{rn}_{rm\ \lambda k}\ U^{\lambda}_r(\omega,\theta,\phi) \ , \quad (10)$$

where $U^{\lambda}_r(\omega,\theta,\phi)$ is expressed through the spherical harmonics $\mathrm{Y}_{\lambda k}(\theta,\phi)$ and the generalized characters $\chi^r_\lambda(\omega)$ of SU(2)

$$U^{\lambda}_r(\omega,\theta,\phi) = (-i)^{\lambda}\frac{2\lambda+1}{2r+1}\sqrt{\frac{4\pi}{2\lambda+1}}\ \mathrm{Y}_{\lambda k}(\theta,\phi)\ \chi^r_\lambda(\omega) \ . \quad (11)$$

The generalized character of rank λ in representation r can be defined through the relation

$$\chi^r_\lambda(\omega) = i^{\lambda}\sum_{m=-r}^{r} e^{-im\omega}C^{rm}_{rm\ \lambda 0} \ . \quad (12)$$

We recall that the invariant measure in this parameterization is

$$\int dV = \frac{1}{4\pi^2}\int_0^{2\pi}\sin^2\frac{\omega}{2}d\omega\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\phi \quad (13)$$

and the fundamental trace becomes

$$\mathrm{Tr}V_{1/2} = 2\cos\frac{\omega}{2} \ . \quad (14)$$

We substitute all these expressions into Eq.(4) and integrate over θ and ϕ angles using the orthogonality relation for spherical harmonics. Taking into account that $V^{\dagger\ mn}_r(\omega,\theta,\phi) = V^{mn}_r(-\omega,\theta,\phi)$ and $\chi^r_\lambda(-\omega) = (-1)^{\lambda}\chi^r_\lambda(\omega)$, the result can be given as

$$\Xi_l = \sum_{\lambda k}(-1)^k\frac{2\lambda+1}{(2r_x+1)(2r_{x+n}+1)}\times$$

$$\times C^{r_x n_x}_{r_x m_x\ \lambda k}\ C^{r_{x+n}n_{x+n}}_{r_{x+n}m_{x+n}\ \lambda -k}\ \Pi_l \ , \quad (15)$$

where $\Pi_l = \Pi_l(r_x, r_{x+n}, \lambda; \beta)$ is

$$\Pi_l = \frac{2}{\pi}\int_0^{\pi}\sin^2\omega\ e^{2\beta\cos\omega}\ \chi^{r_x}_\lambda(2\omega)\ \chi^{r_{x+n}}_\lambda(2\omega)\ d\omega \ . \quad (16)$$

Substituting (15) into (3), we write down the result as

$$Z = \sum_{r_x=0,\frac{1}{2},1,\ldots}^{\infty}\ \sum_{\lambda_l k_l}\prod_l\left[(-1)^{k_l}\frac{2\lambda_l+1}{(2r_x+1)(2r_{x+n}+1)}\Pi_l\right]\times$$

$$\times\prod_x\left[(2r_x+1)\mathrm{TR}_x\right] \ , \quad (17)$$

where TR_x denotes the sum over magnetic numbers

$$\mathrm{TR}_x = \sum_{m_1,m_2,m_3,m_4} C^{r_x m_2}_{r_x m_1\ \lambda_1 k_1}C^{r_x m_3}_{r_x m_2\ \lambda_2 k_2}\times$$

$$\times C^{r_x m_4}_{r_x m_3\ \lambda_3 -k_3}C^{r_x m_1}_{r_x m_4\ \lambda_4 -k_4} \ . \quad (18)$$

This sum can be brought to the following form [8]:

$$\mathrm{TR}_x = (2r_x+1)^2(-1)^{k_3+k_4}\sum_{Jm}C^{Jm}_{\lambda_2 k_2\ \lambda_1 k_1}C^{Jm}_{\lambda_3 k_3\ \lambda_4 k_4}\times$$

$$\times\left\{\begin{array}{ccc}\lambda_1 & \lambda_2 & J\\ r_x & r_x & r_x\end{array}\right\}\left\{\begin{array}{ccc}\lambda_3 & \lambda_4 & J\\ r_x & r_x & r_x\end{array}\right\} \ . \quad (19)$$

Two last symbols in this expression are $6j$-symbols. Using the orthogonality relation for spherical harmonics, we rewrite the right-hand side of Eq.(19) as

$$\mathrm{TR}_x = (2r_x+1)^2(-1)^{k_3+k_4}\int_0^{\pi}\sin\theta d\theta\int_0^{2\pi}d\phi\times$$

$$\times\sum_{J_1 m_1}C^{J_1 m_1}_{\lambda_2 k_2\ \lambda_1 k_1}\left\{\begin{array}{ccc}\lambda_1 & \lambda_2 & J_1\\ r_x & r_x & r_x\end{array}\right\}\mathrm{Y}_{J_1 m_1}(\theta,\phi)\times$$

$$\times\sum_{J_2 m_2}C^{J_2 m_2}_{\lambda_3 k_3\ \lambda_4 k_4}\left\{\begin{array}{ccc}\lambda_3 & \lambda_4 & J_2\\ r_x & r_x & r_x\end{array}\right\}\mathrm{Y}^{\star}_{J_2 m_2}(\theta,\phi) \ . \quad (20)$$

In the Appendix, we construct the asymptotic representation for $6j$-symbols of the kind entering SU(2) traces in (19). It follows from expansion (72) that the $6j$-symbol can be substituted in the classical, i.e. large-r_x, limit by

$$\left\{\begin{array}{ccc}a & b & c\\ r & r & r\end{array}\right\} \asymp \frac{(-1)^{2r+c}}{\sqrt{(2r+1)(2c+1)}}\ C^{c0}_{a0\ b0} \ . \quad (21)$$

[1]Throughout this paper, we use the notations and conventions of [8] for the Clebsch—Gordan coefficients, the $6j$-symbols, etc.

To proceed further, we note that the sum in (20)

$$\sum_{Jm} \sqrt{\frac{(2a+1)(2b+1)}{4\pi(2J+1)}}\, C^{J0}_{a0\ b0} C^{Jm}_{ak\ bn} \mathrm{Y}_{Jm}(\theta,\phi)$$

is an expansion of the spherical harmonics into the Clebsch—Gordan series. Thus, up to an unessential sign factor which cancels on every link, TR_x has the following asymptotic form valid at large values of the angular momenta r_x:

$$\mathrm{TR}_x \asymp (2r_x+1)\times$$
$$\times \frac{4\pi}{\sqrt{(2\lambda_1+1)(2\lambda_2+1)(2\lambda_3+1)(2\lambda_4+1)}}\times$$
$$\times \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \times$$
$$\times \mathrm{Y}_{\lambda_1 k_1}(\theta,\phi)\mathrm{Y}_{\lambda_2 k_2}(\theta,\phi)\mathrm{Y}^{\star}_{\lambda_3 k_3}(\theta,\phi)\mathrm{Y}^{\star}_{\lambda_4 k_4}(\theta,\phi)\ . \quad (22)$$

The sums over magnetic numbers k_i are factorized in every link and can be calculated by using the addition theorem for spherical harmonics

$$\sum_k \mathrm{Y}_{\lambda k}(\theta_1,\phi_1)\mathrm{Y}^{\star}_{\lambda k}(\theta_2,\phi_2) = \frac{2\lambda+1}{4\pi}\ P_\lambda(\cos\Delta)\ , \quad (23)$$

where $P_\lambda(x)$ is the Legendre polynomial. The partition function gets the form

$$Z \approx \int_0^{2\pi} \prod_x d\phi_x \int_0^{\pi} \prod_x \sin\theta_x d\theta_x \times$$

$$\times \sum_{r_x=0,1/2,\dots}^{\infty} \prod_x (2r_x+1)^2 \prod_{x,n} Q(\cos\Delta_l, r_x, r_{x+n};\beta)\ , \quad (24)$$

where Δ_l is defined in (8), and the link function $Q = Q(l)$ is given by

$$Q(l) = \sum_\lambda \frac{2\lambda+1}{(2r_x+1)(2r_{x+n}+1)}\ P_\lambda(\cos\Delta_l)\ \Pi_l\ . \quad (25)$$

The extension of this result to the correlation function is as follows. The first step is to calculate $\Xi_l(j)$ given by (6). Substituting expressions (11)–(14) into (6), we integrate over θ and ϕ angles expanding the result into the Clebsch—Gordan series

$$\Xi_l^{s_1 s_2}(j) = \sum_{\lambda k}\sum_{\lambda_1 k_1}\sum_{\lambda_2 k_2} (-1)^{k_2} \frac{(2\lambda+1)(2\lambda_1+1)}{(2j+1)(2r_1+1)(2r_2+1)} \times$$

$$\times C^{\lambda_2 0}_{\lambda_1 0\ \lambda 0} C^{\lambda_2 -k_2}_{\lambda_1 k_1\ \lambda k} C^{r_1 n_1}_{r_1 m_1\ \lambda_1 k_1} C^{r_2 n_2}_{r_2 m_2\ \lambda_2 k_2} C^{j s_2}_{J s_1\ \lambda k}\ \Pi_l(j)\ , \quad (26)$$

where

$$\Pi_l(j) = (-1)^{\frac{1}{2}(\lambda+\lambda_1-\lambda_2)}\times$$

$$\times \int_{-\pi}^{\pi} \sin^2\omega\ e^{2\beta\cos\omega}\ \chi^{r_1}_{\lambda_1}(2\omega)\ \chi^{r_2}_{\lambda_2}(2\omega)\chi^{j}_{\lambda}(2\omega)\ \frac{d\omega}{\pi}\ . \quad (27)$$

With this result, we can write the correlation function (5) in the form

$$\Gamma_j(x,y) = \frac{1}{Z}\sum_{\{r_x\}}^{\infty} \prod_{l\notin C_{xy}} \left[\sum_{\lambda k} (-1)^k \frac{2\lambda+1}{(2r_x+1)(2r_{x+n}+1)} \Pi_l(0)\right] \prod_x [(2r_x+1)\mathrm{TR}_x] \times$$

$$\times \prod_{l\in C_{xy}} \left[\sum_{\lambda k}\sum_{\lambda_1 k_1}\sum_{\lambda_2 k_2} (-1)^{k_2} \frac{(2\lambda+1)(2\lambda_1+1)}{(2j+1)(2r_x+1)(2r_{x+n}+1)} C^{\lambda_2 0}_{\lambda_1 0\ \lambda 0} C^{\lambda_2 -k_2}_{\lambda_1 k_1\ \lambda k}\ \Pi_l(j)\right] \mathrm{TR}_\Gamma, \quad (28)$$

where TR_Γ denotes the trace

$$\mathrm{TR}_\Gamma = \frac{1}{(2j+1)}\times$$

$$\times \sum_{s_1 s_2,\dots,s_R} C^{j s_2}_{j s_1\ \lambda_1 k_1} C^{j s_3}_{j s_2\ \lambda_2 k_2} \dots C^{j s_1}_{j s_R\ \lambda_R k_R}\ . \quad (29)$$

To calculate this trace, we re-express the Clebsch—Gordan coefficients in terms of $3j$ symbols and use the recoupling formula

$$\sum_k (-1)^{j-k} \begin{pmatrix} a & b & j \\ \alpha & \beta & -k \end{pmatrix} \begin{pmatrix} j & d & c \\ k & \delta & \gamma \end{pmatrix} =$$

$$= (-1)^{2a} \sum_{x\xi} (-1)^{x-\xi}(2x+1)\times$$

$$\times \begin{pmatrix} a & c & x \\ \alpha & \gamma & -\xi \end{pmatrix} \begin{pmatrix} x & d & b \\ \xi & \delta & \beta \end{pmatrix} \begin{Bmatrix} b & d & x \\ c & a & j \end{Bmatrix}\ . \quad (30)$$

Making successive use of this formula, one can shift the dependence on j to the $6j$-symbols only. Suppose now that j is large enough. Taking the asymptotics of $6j$'s (21) and comparing the result with the Clebsch–Gordan expansion of the product of the spherical harmonics $\prod_{i=1}^{R} \mathrm{Y}_{\lambda_i k_i}(\theta,\phi)$, we end up with the following asymptotic form at large values of j:

$$\mathrm{TR}_\Gamma \asymp \prod_{i=1}^{R} (2\lambda_i+1)^{-1/2} \times$$

$$\times \int_0^{\pi} \sin\theta d\theta \int_0^{2\pi} d\phi \prod_{i=1}^{R} \mathrm{Y}_{\lambda_i k_i}(\theta,\phi) \ . \tag{31}$$

Substituting this expression into Eq.(28), we arrive at the following approximate dual representation for the two-point correlation function:

$$\Gamma_j(x,y) = \int_0^{\pi} \sin\alpha d\alpha \int_0^{2\pi} d\varphi \ \langle \prod_{l\in C_{xy}} \frac{Q(l;j)}{Q(l;0)} \ \rangle \ . \tag{32}$$

Here, $Q(l;0)$ is given in (25), and $Q(l;j)$ reads

$$Q(l;j) =$$

$$= \sum_{\lambda\lambda_1\lambda_2} \frac{\sqrt{(2\lambda+1)(2\lambda_1+1)(2\lambda_2+1)}}{(2r_x+1)(2r_{x+n}+1)(2j+1)} \begin{pmatrix} \lambda_1 & \lambda & \lambda_2 \\ 0 & 0 & 0 \end{pmatrix} \times$$

$$\times T_{\lambda_1\lambda\lambda_2}(\theta_x,\phi_x;\alpha,\varphi;\theta_{x+n},\phi_{x+n}) \ \Pi_l(j) \ . \tag{33}$$

We introduced here a tripolar scalar harmonic as follows:

$$T_{\lambda_1\lambda\lambda_2}(\theta_x,\phi_x;\alpha,\varphi;\theta_{x+n},\phi_{x+n}) = \sum_{kk_1k_2} \begin{pmatrix} \lambda_1 & \lambda & \lambda_2 \\ k_1 & k & k_2 \end{pmatrix} \times$$

$$\times \mathrm{Y}_{\lambda_1 k_1}(\theta_x,\phi_x)\mathrm{Y}_{\lambda k}(\alpha,\varphi)\mathrm{Y}_{\lambda_2 k_2}(\theta_{x+n},\phi_{x+n}) \ . \tag{34}$$

The expectation value in (32) refers to the partition function (24). Equations (24), (25) and (32)—(34) define our effective dual model.

3. Asymptotic Behaviour of Correlation Functions

3.1. Asymptotic expansion of $Q(l)$

We start this section from the calculation of the low-temperature asymptotics of the link function $Q(l)$. This actually requires to find the asymptotic expansion of the function Π_l, Eq. (16). The first step is to express the generalized characters entering Π_l through the associated Legendre functions of the first kind taken on the cut $x \in [-1,1]$:

$$\chi^r_\lambda(2\omega) = \sqrt{\frac{\pi}{2}} \left[\frac{(2r+1)(2r+1+\lambda)!}{(2r-\lambda)! \sin\omega} \right]^{1/2} \mathrm{P}^{-\lambda-1/2}_{2r+1/2}(\cos\omega). \tag{35}$$

The dominant contribution to the integral in (16) at $\beta \to \infty$ comes from the vicinity of the point $\omega = 0$. Therefore, we need an expansion for the associated Legendre functions at large r_x which is uniformly valid in the neighbourhood of the point $\omega = 0$. Such an expansion is given by MacDonald's formula [9][2]

$$\mathrm{P}^{-\mu}_{\nu}(\cos\omega) = \left[(\nu+\frac{1}{2}) \cos\frac{\omega}{2} \right]^{-\mu} \times$$

$$\times \left\{ J_\mu(\alpha) + \sin^2\frac{\omega}{2} \left[\frac{\alpha}{6} J_{\mu+3}(\alpha) - \right.\right.$$

$$\left.\left. - J_{\mu+2}(\alpha) + \frac{1}{2\alpha} J_{\mu+1}(\alpha) \right] + \mathcal{O}(\sin^4\frac{\omega}{2}) \right\}, \tag{36}$$

where $\alpha = (2\nu+1)\sin\frac{\omega}{2}$. Higher-order terms can also be expressed through Bessel functions of the same argument $J_b(\alpha)$. To calculate the ratio of factorials in (35), we use expansion (67) with $x = 2r+1$:

$$\left[\frac{(2r+1+\lambda)!}{(2r-\lambda)!} \right]^{1/2} = (2r+1)^{\lambda+1/2} \times$$

$$\times \left[1 - \frac{\lambda(\lambda^2+\frac{3}{2}\lambda+\frac{1}{2})}{6(2r+1)^2} + \mathcal{O}[(2r+1)^{-4}] \right] \ . \tag{37}$$

With these results, by using the recursion relations for Bessel functions to reduce the order to $\lambda+\frac{1}{2}$, we obtain the representation for the generalized character of the form

$$\chi^r_\lambda(2\omega) = \sqrt{\frac{\pi}{2}} \left[\frac{2r+1}{2\sin\frac{\omega}{2}} \right]^{1/2} B(r,\lambda;\omega,\partial_h) \mid_{h=1} \ J_{\lambda+\frac{1}{2}}(\alpha h). \tag{38}$$

In the region $\omega \to 0$, $\omega r \sim \mathcal{O}(1)$, the function $B(r,\lambda;\omega)$ has the asymptotic expansion

$$B(r,\lambda;\omega,\partial_h) = 1 + \frac{\sin^2\frac{\omega}{2}}{3} \left[\frac{7}{4} - \left(\frac{2\lambda}{\alpha^2}(\lambda+1) - \frac{1}{2} \right) \frac{\partial}{\partial h} \right] +$$

[2] Equivalently, one could express generalized characters through the Jacobi polynomials and use the Hilb-type formula for the asymptotic expansion of Jacobi polynomials [10].

$$+\mathcal{O}\left(\sin^4\frac{\omega}{2}\right) \ , \tag{39}$$

where $\alpha = 2(2r+1)\sin\frac{\omega}{2}$. Substituting two last equations into (16), we get, after the change of variables $\cos\omega = 1 - y^2/4\beta$, that

$$\Pi_l = \frac{1}{2\beta}e^{2\beta}\left[(2r_x+1)(2r_{x+n}+1)\right]^{\frac{1}{2}}\int\limits_0^{\sqrt{8\beta}} dy\ ye^{-\frac{1}{2}y^2}\times$$

$$\times J_{\lambda+1/2}\left[(2r_x+1)\frac{y}{\sqrt{2\beta}}\right]\times$$

$$\times J_{\lambda+1/2}\left[(2r_{x+n}+1)\frac{y}{\sqrt{2\beta}}\right]\left[1+\mathcal{O}(\beta^{-1})\right] \ . \tag{40}$$

It is seen now that the extension of the integration region to infinity is a quite harmless procedure since it only introduces corrections of the order $\mathcal{O}(e^{-4\beta})$. Hence, as we have discussed in Introduction, the probability of large fluctuations of the link matrix is exponentially small. We think that, using our representation, one can derive more precise bounds on the large fluctuations than those obtained in [7]. The integration over the whole positive axis in (40) yields the leading term of the asymptotic expansion of Π_l

$$\Pi_l = \frac{1}{2\beta}e^{2\beta}\left[(2r_x+1)(2r_{x+n}+1)\right]^{\frac{1}{2}}\times$$

$$\times\exp\left\{-\frac{1}{4\beta}\left[(2r_x+1)^2+(2r_{x+n}+1)^2\right]\right\}$$

$$\times\left[I_{\lambda+\frac{1}{2}}\left(\frac{(2r_x+1)(2r_{x+n}+1)}{2\beta}\right)+\mathcal{O}(\beta^{-1})\right] \ . \tag{41}$$

The summation over λ in (25) can be performed with help of the formula

$$\sum_{\lambda=0}^{\infty}(2\lambda+1)I_{\lambda+\frac{1}{2}}(t)P_\lambda(x) = \left(\frac{2t}{\pi}\right)^{1/2}e^{tx} \ . \tag{42}$$

We thus obtain

$$Q(l) = C(\beta)\exp\left(-\frac{1}{4\beta}\rho_n^2(x)\right)\left[1+\mathcal{O}(\beta^{-1})\right] \ , \tag{43}$$

where $C(\beta) = (2/\pi)^{1/2}e^{2\beta}/(2\beta)^{3/2}$ and

$$\rho_n^2(x) = (2r_x+1)^2+(2r_{x+n}+1)^2-$$

$$-2(2r_x+1)(2r_{x+n}+1)\cos\Delta_l \ . \tag{44}$$

One sees that $Q(l)$ is a familiar Gaussian measure written in the spherical coordinates. Since $2r_x+1$ runs over all positive integers, we change the summation variable $r_x \to 2r_x+1$. Using the Poisson resummation formula, we get then the expression for the partition function (24) after $r_x \to (2\beta)^{1/2}r_x$ as

$$Z \approx \sum_{m_x=-\infty}^{\infty}\int\limits_0^{2\pi}\prod_x d\phi_x\int\limits_0^{\pi}\prod_x\sin\theta_x d\theta_x\int\limits_0^{\infty}\prod_x r_x^2dr_x\times$$

$$\times\exp\left\{-\frac{1}{2}\sum_{x,n}(r_x^2+r_{x+n}^2-2r_xr_{x+n}\cos\Delta_l)+\right.$$

$$\left.+2\pi i\sqrt{2\beta}\ r_xm_x\right\} \ . \tag{45}$$

3.2. $\boldsymbol{\Gamma_j(x,y)}$

The calculation of the two-point function proceeds in a similar way. We only slightly change the strategy to avoid the problems with treating complicated tripolar harmonics in (33). Using the equality

$$\sqrt{(2\lambda+1)(2\lambda_1+1)(2\lambda_2+1)}\times$$

$$\times\begin{pmatrix}\lambda_1 & \lambda & \lambda_2\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\lambda_1 & \lambda & \lambda_2\\ k_1 & k & k_2\end{pmatrix} =$$

$$= \sqrt{4\pi}\int\limits_0^{2\pi}d\phi\int\limits_0^{\pi}\sin\theta d\theta\Upsilon^\star_{\lambda k}(\theta,\phi)\Upsilon^\star_{\lambda_1k_1}(\theta,\phi)\Upsilon^\star_{\lambda_2k_2}(\theta,\phi) \tag{46}$$

and summing over all k, we can rewrite (33) as

$$Q(l;J) = 2\frac{\sqrt{4\pi}}{(4\pi)^3}\int\limits_0^{2\pi}d\phi\int\limits_0^{\pi}\sin\theta d\theta\int\limits_0^{\pi}\frac{d\omega}{\pi}\sin^2\omega\ e^{2\beta\cos\omega}\times$$

$$\times T_j(\omega,\cos\Delta)T_{r_x}(\omega,\cos\Delta_x)T_{r_{x+n}}(-\omega,\cos\Delta_{x+n}) \ , \tag{47}$$

where

$$T_r(\omega,\cos\Delta) \equiv T_r =$$

$$= \frac{1}{2r+1}\sum_\lambda(-1)^{\frac{1}{2}\lambda}(2\lambda+1)P_\lambda(\cos\Delta)\chi^r_\lambda(2\omega) \ . \tag{48}$$

We have used (23), and

$$\cos\Delta = \cos\theta\cos\alpha+\sin\theta\sin\alpha\cos(\varphi-\phi) \ ,$$

$$\cos\Delta_x = \cos\theta\cos\theta_x+\sin\theta\sin\theta_x\cos(\phi-\phi_x). \tag{49}$$

Representing the leading order term in the expansion of the generalized character (38) in terms of spherical Bessel functions

$$\chi^r_\lambda(2\omega) \approx (2r+1) j_\lambda \left[2(2r+1)\sin\frac{\omega}{2}\right] \ , \quad (50)$$

we calculate

$$T_r \approx \sum_\lambda (2\lambda+1) e^{i\frac{\pi}{2}\lambda} P_\lambda(\cos\Delta) j_\lambda \left[2(2r+1)\sin\frac{\omega}{2}\right] =$$

$$= \exp\left[2i(2r+1)\sin\frac{\omega}{2}\cos\Delta\right] \ . \quad (51)$$

With this asymptotics, we obtain, after all integrations, that the ratio in Eq.(32) reads

$$\frac{Q(l;j)}{Q(l;0)} =$$

$$= \exp\left\{-\frac{1}{4\beta}\sum_k \left[J_k^2 + 2J_k(R_k(x) - R_k(x+n))\right]\right\} \ , \quad (52)$$

where

$$J_1 = J\sin\alpha\cos\varphi \ , \ J_2 = J\sin\alpha\sin\varphi \ , \ J_3 = J\cos\alpha \quad (53)$$

$$R_1(x) = (2r_x+1)\sin\theta_x\cos\phi_x \ ,$$

$$R_2(x) = (2r_x+1)\sin\theta_x\sin\phi_x \ ,$$

$$R_3(x) = (2r_x+1)\cos\theta_x \ . \quad (54)$$

We have denoted here $J = 2j+1$. Using again the Poisson resummation formula, we get

$$\Gamma_j(x,y) = \frac{1}{Z}\sum_{m_x=-\infty}^{\infty}\int_0^\pi \sin\alpha d\alpha \times$$

$$\times \int_0^{2\pi} d\varphi \ e^{-\frac{|x-y|}{4\beta}J^2} \int \prod_{x,k} dR_k(x) \ e^{-R_k(x)G^{-1}_{xx'}R_k(x')} \times$$

$$\times \prod_{l\in C_{xy}} \exp\left[-\frac{1}{\sqrt{2\beta}} J_k(R_k(x) - R_k(x+n))\right] \times$$

$$\times \exp\left[2\pi i\sqrt{2\beta}\sum_x m_x \sqrt{\sum_k R_k^2(x)}\right] \ . \quad (55)$$

The summation over m_x corresponds to that over local defects which are, in their turn, analogous to the vortices of the XY model. Their physical meaning can be seen from the constraint on link matrices $\prod_{l\in p} V_l = 1$ in the partition function (1). This constraint expressed in terms of elements of SU(2) algebra reads (see [11] for more details)

$$\left(\sum_k \omega_k^2(p)\right)^{1/2} = 2\pi m_x \ , \quad (56)$$

where $\omega_k(p)$ is the plaquette angle. We conclude that, at low temperatures, there are two contributions to the correlation function: spin waves (Gaussian term in (55)) and local defects. At this point, the methods of [2] can be applied to (55) in order to bound the entropy of multipole configurations. The crucial point is that this entropy is exactly the same as in the XY model [2], i.e. it is bounded from above by a β-independent function. The Boltzmann factor of the lowest multipole configurations is suppressed as $\exp[-\beta\ln \mid x-y \mid]$. This can be proved by using the inverse Laplace transform as in [11], where we have computed the contribution of the lowest dipole configuration to the free energy. We cannot prove that this suppression holds for arbitrary multipole configurations due to the complexity of the coupling between defects and spin waves, but this seems to be the case as seen from the dipole configuration. If it is indeed so, we can neglect all local defects for β sufficiently large. Performing the Gaussian integration, we finally obtain, after some manipulations with Green functions,

$$\Gamma_j(x,y) \approx \exp\left\{-\frac{(2j+1)^2}{2\beta}D(x-y)\right\} \ . \quad (57)$$

3.3. Link-link correlation function

Now we propose to consider a certain four-point correlation function which is a direct analog of the plaquette-plaquette correlation function widely used in lattice gauge models for a computation of glueball masses. In our case, this is the link-link correlation function defined as

$$\Gamma_4(R) = \frac{1}{4}\langle \mathrm{Tr}V_{l_1}\mathrm{Tr}V_{l_2}\rangle - (\frac{1}{2}\langle \mathrm{Tr}V_l\rangle)^2 \ , \quad (58)$$

where $V_{l_i} = U_{x_i}U^+_{x_i+n}$ and $\mid x_1 - x_2 \mid = R$. For the sake of simplicity, we consider V_l in the fundamental representation. Then, after the change of variables $\cos\omega = 1 - y^2/4\beta$, $\Gamma_4(R)$ becomes

$$\Gamma_4(R) = \frac{1}{16\beta^2}\left[\langle y^2(l_1)y^2(l_2)\rangle - \langle y^2(l)\rangle^2\right] \ . \quad (59)$$

All calculations from Section 3.1 can easily be repeated for this expectation value. Neglecting again all local defects, we end up with

$$\Gamma_4(R) = \frac{3}{32\beta^2}\left[G^2_{l_1 l_2} + \mathcal{O}(\beta^{-1})\right] \ , \qquad (60)$$

where $G_{l_1 l_2}$ is the link Green function [5]. If l_1 and l_2 point the same direction, then we get

$$G_{l_1 l_2} \approx -\frac{1}{\pi R^2}\left(1 + \frac{1}{R^2}\right) \qquad (61)$$

for $R \gg 1$ and, hence,

$$\Gamma_4(R) \asymp \frac{c}{\beta^2 R^4} \ . \qquad (62)$$

4. Discussion

We are not aware of any analytical calculations of the correlation function in 2D SU(N) and $O(N)$ lattice models at any fixed N beyond perturbation theory from the first principles which would show the exponential decay and the existence of a mass gap at arbitrarily low temperatures. We think therefore that any efforts which could lead to the advances of analytical methods are of great importance. The computations presented above would seem at the first glance too naive since the partition function is eventually reduced to the Gaussian form. Thereby, one could ask how reliable they are and, if not, what is the way to impove them?

Let us imagine, for the sake of argument, that there is a phase transition to a massless phase at some finite β in our approximate dual model. Then the fact that spin waves dominate the partition function which becomes pure Gaussian is not suprising at all. The real question which our calculations reveal is what is more essential in the model than spin waves and produces the mass gap? The remainder to Π_l in (41) is $\mathcal{O}(\beta^{-1})$. The method of calculation is such that this bound holds uniformly in r_x and r_{x+n} on all links. This means that however large the lattice is, whatever quantity, fixed or long-distance one, is being calculated, all corrections are bounded like $\mathcal{O}(\beta^{-1})$ and, most importantly, these are pure perturbative corrections. In other words, in the region $\omega \sim \mathcal{O}(\beta^{-1/2})$, we do not see anything which could be termed "nonperturbative contribution" on smooth configurations. Of course, the mass gap is expected to be exponentially small in β. Though logically not excluded, it is hard to imagine that smooth perturbative corrections would eventually result in the exponential decay of a correlation function. So far we have seen only two types of exponentially small contributions in our calculations: large fluctuations of the link matrix, i.e. the region $\omega > \mathcal{O}(\beta^{-1/2})$, and local defects (of course, the latter also arise from the large fluctuations of the links). We think the arguments which we have already presented make the possibility of the mass gap generation due to these fluctuations highly implausible. We thus conclude that most of all the correlation functions in the dual model (24) decay according to the power law and, therefore the model is massless if β is sufficiently large.

Let us turn now to the full model (17). We believe that the approximation made to get the dual representation cannot change the universality class of the model and keeps all the most essential features of the full theory. First of all, the asymptotics for $6j$-symbols used here corresponds to the classical limit where one expects the naive continuum limit of the model to emerge. All corrections to the leading term are suppressed with β and, again they present only smooth perturbative contributions. In our opinion, there is only one point which could be debated. In order to get the asymptotics of the $6j$-symbol (21), one should take the asymptotic expansion of the ratio of gamma-functions of the form $\Gamma(x+a)/\Gamma(x+b)$ when $x \to \infty$. Even the first term of the standard asymptotics is known to give the accurate approximation for this ratio when $x \to \infty$ and a and b are fixed. In our case, a and b (which are proportional to λ_i) are in general not small quantities. Therefore, one cannot expect that, when r is large enough but $\lambda \sim \mathcal{O}(r)$, our asymptotics gives a reasonable approximation for a $6j$-symbol. However, it is important that, for all configurations r_i, n_i, m_i in the exact link function (15), the dominant contribution comes from the term $\lambda = 0$, and the series over λ in (15) converges very fast. It follows from the asymptotic expansion for Π_l (38)—(41) (which obviously is the same both in full and approximate dual models) and the bounds for modified Bessel functions that the series over λ converges faster than any inverse power of λ. Our approximation does not change the term $\lambda = 0$ in (15), as is seen when comparing (15) and (25). These properties give a certain justification for using our asymptotics in the whole region $\lambda \in [0, 2r]$, but the question deserves a further investigation. Of course, when λ is large enough, it would be more reasonable to use the so-called classical [12] or uniform [13] asymptotic expansion for the $6j$-symbol, and we have attempted to do this. Unfortunately, even a simpler version of this asymptotics obtained in [12] is so involved that we cannot achieve any reliable conclusion so far. What seems to be the case, however, is that this asymptotics is rather far from the perturbative region. Moreover, very strong fluctuations of the $6j$-symbol in this region can result, in principle, in a faster decay of the correlation function than that found here and produce a non-vanishing mass gap. Though we do not see why this region of configurations is important for long-distance quantities, we consider this as the only possibility to generate the non-zero mass gap.

APENDIX

Asymptotic Expansion of the 6j-symbol

Among many $6j$-symbols, those of the form $\begin{matrix} a & b & c \\ r & r & r \end{matrix}$ are of special value since all traces of $SU(2)$ matrices can be expressed through such $6j$-symbols. Therefore, the asymptotic expansion we give here might be of independent interest. The first term of the expansion at large r can be found, e.g. in [12]. In our calculations, we follow the strategy of [12]. The difference is that we expand in $(2r+1)^{-1}$ rather than in r^{-1}. Next, the result in [12] is applicable only in the case where $a+b+c$ is even. When $a+b+c$ is odd, the leading term calculated in [12] vanishes, while the corresponding $6j$-symbol differs from zero.

The starting point is the series representation for the $6j$-symbol, Eq.(9.2.1.3) in [8],

$$\begin{matrix} a & b & c \\ r & r & r \end{matrix} = (-1)^{2r+a+c} \frac{\Delta(arr)\Delta(brr)}{\Delta(abc)\Delta(crr)} S , \qquad (63)$$

where

$$\Delta(xrr) = x! \left(\frac{(2r-x)!}{(2r+x+1)!} \right)^{1/2} , \qquad (64)$$

$$S = \sum_n \frac{(-1)^n}{n!} \frac{(-a+b+c+n)!(c+n)!}{(a-n)!(-a+b+n)!(b+c+1+n)!} F_n(abc) , \qquad (65)$$

$$F_n(abc) = \frac{(2r+a-c-n)!}{(2r-b-n)!} . \qquad (66)$$

We are interested in the asymptotic expansion of (63) when $r \to \infty$ and a, b, c are fixed. To construct the expansion at large r in (64) and (66), we use the asymptotic expansion for the ratio of gamma functions

$$\frac{\Gamma(x+a)}{\Gamma(x+b)} =$$

$$= x^{a-b} \sum_{s=0}^{N-1} \frac{(-1)^s}{s! x^s} B_s^{(a-b+1)}(a) \; (b-a)_s + \mathcal{O}(x^{-N}) , \qquad (67)$$

when $x \to \infty$. Here, $B_s^{(y)}(x)$ is the generalized Bernoulli polynomial. Taking $x = 2r+1$, we find, e.g.

$$F_n(abc) = (2r+1)^z \times$$

$$\times \sum_s \frac{1}{s!(2r+1)^s} B_s^{(z+1)}(a-c-n) \; z(z-1)...(z-s+1) , \qquad (68)$$

where $z = a+b-c$. The substitution of this expansion into (65) leads to the sum

$$S_{as} = \sum_k \frac{(-1)^{a+k}}{k!} \times$$

$$\times \frac{(b+c-k)!(a+c-k)!}{(a-k)!(b-k)!(a+b+c+1-k)!} \; B_s^{(z+1)}(k-c) , \qquad (69)$$

where we have changed the summation variable $k = a-n$. Since $B_s^{(z+1)}(k-c)$ is a polynomial of the order s, the last sum can be reduced to the calculation of a basic sum of the form $(m = 0, 1, ..., s)$

$$S_m = \sum_k \frac{(-1)^{a+k-m}}{k!} \times$$

$$\times \frac{(b+c-m-k)!(a+c-m-k)!}{(a-m-k)!(b-m-k)!(a+b+c+1-m-k)!} =$$

$$= (-1)^{a-m} \frac{c!}{\sqrt{2c+1}} \times$$

$$\times \frac{\Delta(a-\frac{m}{2} \; c \; b-\frac{m}{2})}{[a!(a-m)!b!(b-m)!]^{1/2}} C^{c0}_{a-\frac{m}{2} \; \frac{m}{2} \; b-\frac{m}{2} \; -\frac{m}{2}} . \qquad (70)$$

The following calculations are straightforward though very cumbersome and consist mainly in using the recursion relations for Clebsch—Gordan coefficients to reduce them to one kind. Let us consider the case where $a+b+c$ is even. After a long algebra, we find

$$\begin{matrix} a & b & c \\ r & r & r \end{matrix} = \frac{(-1)^{2r+c}}{\sqrt{(2r+1)(2c+1)}} C^{c0}_{a0 \; b0} \sum_{s=0} \frac{A_s(a,b,c)}{(2r+1)^s} , \qquad (71)$$

where $A_1 = A_3 = 0$. We cannot prove that all odd coefficients of the expansion vanish, but this seems to be the case. If it is so, then it follows from the recursion relations for $6j$-symbols that when $a+b+c$ is odd, all even coefficients must be zero. To get the asymptotics in this case, the most straightforward way is to use the recursion relation for $6j$-symbols, Eq.(9.6.2.5) in [8]. This reduces the problem to the previous case where $a+b+c$ is even. In this way, we finally obtain

$$\begin{matrix} a & b & c \\ r & r & r \end{matrix} = \frac{(-1)^{2r+c}}{\sqrt{(2r+1)(2c+1)}} \times$$

$$\times \left[C^{c0}_{a0 \; b0} \sum_{s=0} \frac{A_{2s}(a,b,c)}{(2r+1)^{2s}} - C^{c0}_{a-10 \; b0} \sum_{s=0} \frac{A_{2s+1}(a,b,c)}{(2r+1)^{2s+1}} \right] . \qquad (72)$$

The first coefficients are

$$A_0 = 1 ,$$

$$A_1 = \frac{1}{2}[(a+b+c+1)(a+b-c) \times$$

$$\times (a-b+c)(-a+b+c+1)]^{1/2} ,$$

$$A_2 = \frac{1}{8} \; [\; a^2(a+1)^2 + b^2(b+1)^2 + c^2(c+1)^2 -$$

$$-2a(a+1)b(b+1) - 2a(a+1)c(c+1) - 2b(b+1)c(c+1) \;] . \qquad (73)$$

One can easily check that all terms of the expansion are symmetric under permutations of a, b, c as required by this kind of the $6j$-symbol.

1. *Savit R.* // Phys. Rev. Lett. **39** (1977) 55; Rev. Mod. Phys. **52** (1980) 453.

2. *Fröhlich J., Spencer T.* // Communs. Math. Phys. **81** (1981) 527.

3. *Batrouni G., Halpern M.B.* // Phys.Rev. D. **30** (1984) 1775.

4. *Patrascioiu A., Seiler E.* // J. Stat. Phys. **69** (1992) 573; Nucl. Phys. B. **30** (Proc. Suppl.) (1993) 184.

5. *Borisenko O., Kushnir V., Velytsky A.* // Phys. Rev. D. **62** (2000) 025013.

6. *Borisenko O., Voloshin S., Kushnir V.* // Ukr. Fiz. Zh. **48** (2003), N4, P. 300.

7. *Bricmont J., Fontaine J.-R.* // J. Stat. Phys. **26** (1981) 745.

8. *Varshalovich D.A., Moskalev A.N., Khersonskii V.K.* Quantum Theory of Angular Momentum. — Singapore: World Scientific, 1988.

9. *Bateman H., Erdelyi A.* Higher Transcendental Functions, Vol. 1. — New York: Springer, 1953.

10. *Szego G.* Orthogonal Polynomials. — Rhode Island: Amer. Math. Soc., 1975.

11. *Borisenko O., Kushnir V.* // Nucl.Phys. B. **570** (2000) 644.

12. *Ponzano G., Regge T.*// Spectroscopic and Group Theoretical Methods in Physics / Ed. by F. Bloch et al. — Amsterdam: North-Holland, 1968.

13. *Schulten K., Gordon R.G.* // J. Math. Phys. **16** (1975) 1971.

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2D SU(2) ГОЛОВНА КІРАЛЬНА МОДЕЛЬ: ДУАЛЬНЕ ПРЕДСТАВЛЕННЯ У КЛАСИЧНІЙ ГРАНИЦІ ТА НИЗЬКОТЕМПЕРАТУРНА АСИМПТОТИКА КОРЕЛЯЦІЙНИХ ФУНКЦІЙ

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Р е з ю м е

Запропоновано аналітичний підхід до вивчення низькотемпературної області двовимірної SU(2) головної кіральної моделі на гратці, що базується на формулюванні моделі на дуальній гратці. Побудовано дуальне представлення у класичній границі, тобто в області великих значень кутових моментів. Єдиним наближенням, що було використано при побудові, є деяке асимптотичне співвідношення між $6j$ символами та коефіцієнтами Клебша—Гордана. Знайдено ведучі члени двоточкової та лінк-лінкової кореляційних функцій у дуальному представленні та показано, що обидві кореляційні функції спадають за степеневим законом. Такий результат знайдено з використанням ще одного наближення, а саме заміни узагальнених характерів групи SU(2) на їх асимптотику, рівномірну в оточенні одиничного елемента SU(2) — лінкової матриці. Більш того, якщо деякі локальні дефекти зв'язані у мультиполі та не можуть генерувати масову щілину, то найбільш суттєвими конфігураціями у низькотемпературній області є спін-хвильові конфігурації.