THE GENERAL FINDING TECHNIQUE OF EFFECTIVE PARAMETERS FOR NON-UNIFORM SOLIDS[1]

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A general theoretical method of determination of the effective parameters of heterogeneous physical solids is discussed. This method is demonstrated for a simple example of the determination of the effective electrical resistance and in the more complex case of finding the effective thermal conductivity of two-layer structures in photothermal experiments. Unlike previous works devoted to this problem, the bulk light absorption is considered. It is shown that the effective thermal parameters depend in the general case on the optical parameters of layers and the effective sample. The method of standardization of the effective thermal conductivity is proposed.

1. Introduction

One of the basic tasks of physics is searching for the effective parameters of those physical bodies which are inhomogeneous. These bodies can be associations of homogeneous ones. Effective parameters can be different: effective density, effective mass, effective moment of inertia, effective heat capacity, effective electric resistance, effective thermal resistance, effective dielectric permeability, effective light refraction index, effective absorption light coefficient, effective radius, etc. Often enough, the word "effective" is dropped. Effective parameters enter physics for two reasons: first, knowing a value of the effective parameter helps to receive the integral description (presentation) of a heterogeneous physical body; secondly, this gives us the possibility by simple calculations to forecast a result of the measurement of a certain physical quantity in the experiment.

2. Effective Parameters of Heterogeneous Solids in the General Case

The universal method to introduce effective parameters is as follows. Let a physical quantity A that depends on the parameter p of a homogeneous physical body,

$$A = f_A(p), \tag{1}$$

be measured in the experiment. Consider a heterogeneous physical body with the same sizes and the form as the homogeneous one under the same physical conditions, and let the quantity A depend on the parameters $p_1, p_2, ...p_n$ of the heterogeneous physical body as follows:

$$A = g_A(p_1, p_2, ..., p_n). \tag{2}$$

Then, by definition, the effective parameter p of the heterogeneous physical body can be determined from the equation

$$f_A(p) = g_A(p_1, p_2, ..., p_n). \tag{3}$$

At the same time, it is worth to mark which exactly physical quantity has being measured in experiment. In our case, the quantity A is in this role. We call such a quantity as "the base measurable magnitude".

Equation (3) can have synonymous upshots, can have a few solutions, and can have no solutions at all. In the first case, the effective parameter is monovaluable, in the second — polyvaluable, and the third case means that there is no possibility to replace a heterogeneous physical body by a homogeneous one so that the result of measuring the quantity A in the latter case be identical to that for the heterogeneous body.

Suppose now that it is possible to measure other quantity B in the experiment rather than A. Let its dependence on the parameter p be

$$B = f_B(p) \tag{4}$$

for a homogeneous body and

$$B = g_B(p_1, p_2, ..., p_n) \tag{5}$$

for a heterogeneous one. Then, in the search for the effective parameter p, we get another equation unlike Eq. (3):

$$f_B(p) = g_B(p_1, p_2, ..., p_n). \tag{6}$$

Consequently, we will also get another value for p. So we get ambiguity for the effective parameter.

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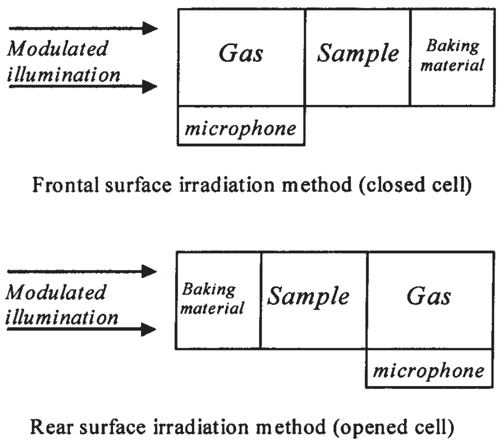

Fig. 1. Sketch of the photoacoustic experiments

Nevertheless, if one always specifies "the base measurable magnitude" by using the concept "effective parameter", then the ambiguity disappears and remains only in the case of a polyvaluable solution of Eq. (3) or (6).

As for the effective electric resistance, we can search it from the condition that, at the identical voltage through the homogeneous and composite areas, the values of the electrical currents are identical in both cases. Let the current I_h through the homogeneous area of a circle depend on its resistance R (it is the effective resistance of a heterogeneous area) and on the voltage U as

$$I_h = \frac{U}{R}. \quad (7)$$

For a heterogeneous area, the current I_c depends on the voltage U applied to this area and on the resistances R_1, R_2, ..., R_n of homogeneous areas, which compose it, as follows:

$$I_c = \frac{U}{f\,(R_1,\, R_2,\, ...,\, R_n)}. \quad (8)$$

The effective resistance R of the composed area can be found by equating the value of currents passing through the homogeneous area of the circle I_h (7) and through the composed one I_c (8):

$$\frac{U}{R} = \frac{U}{f\,(R_1,\, R_2,\, ...,\, R_n)} \Rightarrow \quad (9)$$

$$R = f\,(R_1,\, R_2,\, ...,\, R_n)\,. \quad (10)$$

We see that the effective resistance is a monovaluable quantity. It is clear that effective parameters in physics are searched not only in electrodynamics but also in other fields.

3. Effective Heat Conductivity of Two-layer Structures in Photothermal Experiments

The determination of the effective thermal conductivity is one of the main problems in the physics of heat propagation through multilayer solid structures. The photoacoustic technique is widely applied to the experimental study of these parameters [1, 2] due to its simplicity and high sensitivity. In order to obtain a photoacoustic signal, a closed photoacoustic cell (front-surface illumination) and an open photoacoustic cell (rear-surface illumination) [3] (see the sketch of these experiments in Fig. 1) are generally used.

In [4—6], a general theoretical approach to the calculation of the effective thermal conductivity and effective thermal diffusivity especially for two-layer samples, which seems to us to be adequate for photothermal experiments, was suggested. The calculations are based on the idea that the photoacoustic signal or the temperature response is measured only at one surface of a two-layer sample. In this case, this structure can be described by means of an imaginary homogeneous sample, whose volume is similar to a "black box" for an instrument. For this reason, it can be defined as an effective one-layer sample. So, the main requirement for the calculation of the effective thermal conductivity is the equality of the temperature at the front *or* rear surface of the real two-layer structure and that at the front *or* rear surface of the effective one-layer sample. We postulate that, in this case, the calculated thermal parameters of the effective one-layer sample are the effective thermal parameters of the real two-layer structure. Such an approach is considered to be correct since the temperature is measured on the absolute Kelvin scale. At the same time, the "electrical and thermal analogy" [7, 8] often used for the calculation of effective thermal parameters is limited, because it is based on the calculations only of the temperature difference.

In the mentioned papers [4—6], it was supposed that the laser radiation was absorbed at the sample surface. Here, we generalize this approach to the search for effective thermal parameters in the case of bulk light absorption. In this case, the problem becomes complicated, in principle, due to the necessity to calculate both the effective thermal parameters and the effective optical parameters self-consistently.

We propose to obtain the effective optical parameters by means of the comparison of the measured optical parameters of a real many-layer structure and the optical parameters of the one-layer effective sample at its surface. The compared optical parameters can be, for example, the light reflection coefficient or the light propagation coefficient depending on the experimental procedure. Thus, in contrast to the case of surface light absorption, the effective thermal parameters depend on the optical parameters of the layers. Moreover, this dependence is different depending on the experimental situation. It is very important to note that, in the case of bulk light absorption, the "electrical and thermal analogy" is not applicable.

We take into account the thermal properties of the interface between the layers and show that it has a drastic role in the heat transfer through the structure. In fact, it has some thickness and a thermal conductivity that differs from the bulk thermal conductivities of the layers. For simplicity, we use the model of the surface interface, by supposing that its thickness tends to zero. In this case, it can be described by the surface thermal conductivity η [9], and the thermal parameters of the different layers are changed sharply at the interface. The necessity to account for this interface is demonstrated by a lot of experimental investigations [10—13] and theoretically [14, 15].

4. Physical Model of Heat Exchange in a Two-layer Structure and Effective One-layer Sample

In photothermal experiments, the common mechanism to produce a thermal perturbation of a solid is the absorption of an incident laser beam. Let the intensity of this beam be I_0. Assume that the emission falls normally on the surface $x = 0$ of the first layer of a two-layer structure (Fig. 2,*a*). This first layer is characterized by length d_1, bulk thermal conductivity χ_1, light absorption coefficient β_1, and the light refraction coefficient n_1. The corresponding parameters of the second layer are denoted as d_2, χ_2, β_2, and n_2.

Similarly to [16], we suppose that the energy of the light is converted to heat instantly at every point of the sample. At the same time, we neglect the light absorption at the interface plane $x = d_1$ for the sake of simplicity. We consider that each layer is uniform and isotropic, having unit cross-section. By hypothesis, the left surface $x = 0$ is adiabatically insulated (the condition of the highest possible photo-thermal effect under the given laser perturbation). The opposite surface $x = d = d_1+d_2$ is in isothermal contact with the ambient heat reservoir at the equilibrium temperature T_0.

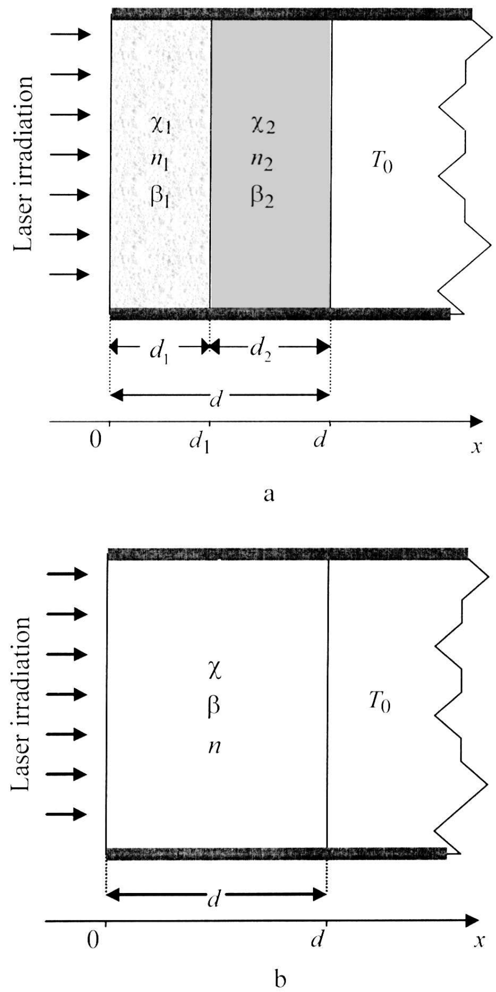

Fig. 2. *a* — real two-layer structure; *b* — effective one-layer structure

At small intensities of the light, the temperatures in the first and the second layers can be described by the following linear thermal diffusion equations:

$$-\chi_i \frac{\partial^2 T_i(x)}{\partial x^2} = Q_i(x), \quad i = 1, 2, \tag{11}$$

where $T_i(x)$ is the temperature in the i-th layer; $Q_i(x)$ is the intensity of the local heat sources caused by the light-heat conversion at the same layer; and $\frac{T_i - T_0}{T_0} \ll 1$. Due to the last inequality, χ_i depend only on the temperature T_0.

Equations (11) must be complemented by thermal boundary conditions. In accordance with [4], they can be written in the following form:

$$\left.\frac{dT_1(x)}{dx}\right|_{x=0} = 0, \tag{12a}$$

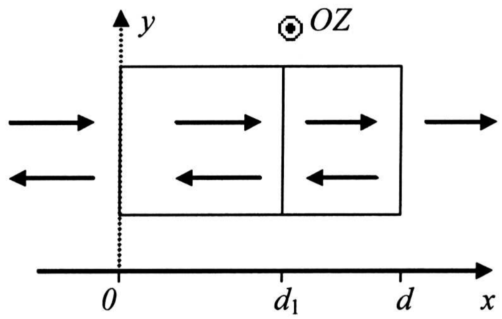

Fig.3. Sketch of the electromagnetic wave propagation in two-layer structure

$$-\chi_1 \left. \frac{dT_1(x)}{dx} \right|_{x=d_1} = \eta \left(T_1(x)|_{x=d_1-0} - T_2(x)|_{x=d_1+0} \right), \tag{12b}$$

$$\chi_1 \frac{dT_1(x)}{dx} = \chi_2 \frac{dT_2(x)}{dx}, \tag{12c}$$

$$T_2(x)|_{x=d} = T_0. \tag{12d}$$

Equation (12a) determines the adiabatic contact at the surface $x = 0$; Eq. (12b) determines the heat flux through the plane interface $x = d_1$ with surface heat conductivity η; Eq. (12c) is the condition for the thermal flux continuity at the interface (it is worth to note that the temperature at the interface plane does not have a continuous value in the general case and can have a gap here [6]); and, finally, Eq. (12d) is the condition for isothermal contact at the surface $x = d$.

To calculate $Q_i(x)$, it is necessary to take into account the complicated process of light propagation through the two-layer structure, accounting for the light absorption in each medium and the reflection at all surfaces.

To obtain the effective thermal conductivity of the two-layer structure in accordance with [4], we need to imagine some one-layer sample with the length $d = d_1 + d_2$ and the unit cross-section (Fig. 2,*b*). Let us suppose that the thermal conductivity of this sample is χ, the light absorption coefficient is β, and the light refraction coefficient is n. The boundary conditions at the surfaces $x = 0$ and $x = d$ of that one-layer sample must be the same as at the surfaces $x = 0$ and $x = d$ of the real two-layer structure, namely

$$\left. \frac{dT(x)}{dx} \right|_{x=0} = 0, \tag{13a}$$

$$T(x)|_{x=d} = T_0, \tag{13b}$$

where $T(x)$ is the temperature in the one-layer sample. This temperature has to satisfy the following thermal diffusion equation:

$$-\chi \frac{\partial^2 T(x)}{\partial x^2} = Q(x), \tag{14}$$

where $Q(x)$ is the intensity of the bulk heat sources in the one-layer sample.

5. Electromagnetic Waves and Heat Sources in a Real Two-layer Structure and the Effective One-layer Sample

The arrows in Fig. 3 denote the wave propagation directions. We suppose that the incident wave falls normally on the surface $x = 0$, is plane and monochromatic, has the polarization direction along the y-axis, and propagates along the x-axis (Fig. 3). The magnetic fields of these waves are directed along the z-axis. We limit ourselves to nonmagnetic media (magnetic permittivity $\mu \approx 1$), so the magnetic induction of the magnetic field is $\vec{B} \approx \mu_0 \vec{H}$, where μ_0 is the vacuum magnetic permittivity.

We restrict ourselves by the case of weak wave attenuation where the imaginary part of the dielectric permeability is much more less than its real part or, in other words, where the effective decay length of the light is much more than the light wavelength ($\beta_{1,2} \ll kn_{1,2}$), where k is the wave number in vacuum. Then the electric fields in front of the first layer E_{y0}, in the first layer E_{y1}, in the second layer E_{y2}, and behind the sample E_{y3} can be written without the oscillation factor $e^{i\omega t}$ as follows:

$$E_{y0}(x) = E_0 \left(e^{-ikx} + Re^{ikx} \right), \tag{15a}$$

$$E_{y1}(x) = E_1 \left\{ \exp\left(-\left(in_1 k + \frac{1}{2}\beta_1 \right) x \right) + \right.$$
$$\left. + R_1 \exp\left(\left(in_1 k + \frac{1}{2}\beta_1 \right)(x - d_1) \right) \right\}, \tag{15b}$$

$$E_{y2}(x) = E_2 \left\{ \exp\left(-\left(in_2 k + \frac{1}{2}\beta_2 \right)(x - d_1) \right) + \right.$$
$$\left. + R_2 \exp\left(\left(in_2 k + \frac{1}{2}\beta_2 \right)(x - d) \right) \right\}, \tag{15c}$$

$$E_{y3}(x) = E_3 \exp(-ik(x-d)). \tag{15d}$$

Here, E_0 is the amplitude of oscillations of the electric field of the incident electromagnetic wave in vacuum, and the index y means the projection on the y-axis. We find the unknown quantities E_1, E_2, E_3, R, R_1, and R_2 from the boundary conditions [16] (the equality of tangential components of the vectors of the electric and magnetic fields). Based on the Maxwell equations for linearly polarized waves [19]

$$-i\omega\vec{B} = \mathrm{rot}\vec{E} \tag{16}$$

and on the geometry of the problem (see Fig. 3), it is easy to obtain the boundary conditions as the following system of equations:

$$E_{y0}(0) = E_{y1}(0), \tag{17a}$$

$$E_{y1}(d_1) = E_{y2}(d_1), \tag{17b}$$

$$E_{y2}(d) = E_{y3}(d), \tag{17c}$$

$$\left.\frac{\partial E_{0y}(x)}{\partial x}\right|_{x=0} = \left.\frac{\partial E_{1y}(x)}{\partial x}\right|_{x=0}, \tag{17d}$$

$$\left.\frac{\partial E_{1y}(x)}{\partial x}\right|_{x=d_1} = \left.\frac{\partial E_{2y}(x)}{\partial x}\right|_{x=d_1}, \tag{17e}$$

$$\left.\frac{\partial E_{2y}(x)}{\partial x}\right|_{x=d} = \left.\frac{\partial E_{3y}(x)}{\partial x}\right|_{x=d}. \tag{17f}$$

Using the condition of a weak attenuation ($\beta_{1,2} \ll kn_{1,2}$) in each layer, we obtain

$$E_1 \underset{1}{=} \alpha_1 E_0, \tag{18a}$$

$$E_2 = \alpha_2 E_1, \tag{18b}$$

$$R = \frac{1-n_1 f_1}{1+n_1 f_1}, \tag{18c}$$

$$R_1 = \frac{n_1 - n_2 f_2}{n_1 + n_2 f_2} \exp\left(-\left(in_1 k + \frac{\beta_1}{2}\right) d_1\right), \tag{18d}$$

$$R_2 = \frac{n_2 - 1}{n_2 + 1} \exp\left(-\left(in_2 k + \frac{\beta_2}{2}\right) d_2\right), \tag{18e}$$

where

$$\alpha_1 = 2\left(1 + n_1 + (1-n_1)\frac{n_1 - n_2 f_2}{n_1 + n_2 f_2} e^{-(2in_1k+\beta_1)d_1}\right)^{-1}, \tag{19a}$$

$$\alpha_2 = \frac{2\exp\left(-\left(in_1 k + \frac{\beta_1}{2}\right) d_1\right)}{1 + \frac{n_2}{n_1} + \left(1 - \frac{n_2}{n_1}\right)\frac{n_2-1}{n_2+1} e^{-(2in_2k+\beta_2)d_2}}, \tag{19b}$$

$$f_1 = \frac{n_2 f_2 + n_1 \mathrm{th}\left(\left(in_1 k d_1 + \frac{\beta_1}{2}\right) d_1\right)}{n_1 + n_2 f_2 \mathrm{th}\left(\left(in_1 k d_1 + \frac{\beta_1}{2}\right) d_1\right)}, \tag{19c}$$

$$f_2 = \frac{1 + n_2 \mathrm{th}\left(\left(in_2 k + \frac{\beta_2}{2}\right) d_2\right)}{n_2 + \mathrm{th}\left(\left(in_2 k + \frac{\beta_2}{2}\right) d_2\right)}. \tag{19d}$$

Here, $|R|^2$, $|R_1|^2$, and $|R_2|^2$ are the coefficients of light reflection from the surfaces $x = 0$, $x = d_1$, and $x = d$, respectively; and $|\alpha_1|^2$ and $|\alpha_2|^2$ are the coefficients of light propagation through the surfaces $x = 0$ and $x = d_1$, respectively.

As to the one-layer effective sample, the electric field strength in front of the sample, E_{y0}, inside the sample, E_y, and behind the sample, E'_y, are

$$E_{y0}(x) = E_0\left(e^{-ikx} + R_F e^{ikx}\right), \tag{20a}$$

$$E_y(x) = E\left(e^{-\left(ink+\frac{\beta}{2}\right)x} + \tilde{R} e^{\left(ink+\frac{\beta}{2}\right)(x-d)}\right), \tag{20b}$$

$$E'_y(x) = E' e^{-ik(x-d)}. \tag{20c}$$

We find the unknown quantities R_F, $\tilde{R}$, E, and E' by analogy with the determination of those for a two-layer structure:

$$E = \alpha E_0, \tag{21a}$$

$$R_F = \frac{1-nf}{1+nf}, \tag{21b}$$

$$\tilde{R} = \frac{n-1}{n+1}\exp\left(-\left(ink + \frac{\beta}{2}\right)d\right), \tag{21c}$$

where

$$\alpha = 2(1+n)\left((1+n)^2 - (1-n)^2 e^{-(2ink+\beta)d}\right)^{-1}, \tag{22a}$$

$$f = \left\{1 + n\,\mathrm{th}\left(\left(ink + \frac{\beta}{2}\right)d\right)\right\}/$$

$$/\left\{n + \mathrm{th}\left(\left(ink + \frac{\beta}{2}\right)d\right)\right\}. \quad (22b)$$

Here, $|R_F|^2$ and $\left|\tilde{R}\right|^2$ are the coefficients of light reflection in the one-layer structure from the surfaces $x = 0$ and $x = d$, respectively, and $|\alpha|^2$ is the coefficient of light propagation through the surface $x = 0$ of the one-layer sample.

Given all the parameters of an electromagnetic wave propagating through the two-layer structure and the effective medium, we can obtain the intensities of the bulk heat sources in these media. They are equal to $-\mathrm{div}\vec{q}$, where $\vec{q} = \frac{1}{\mu_0}\left[\vec{E}\vec{B}\right]$ is the Poynting vector. Thus, the intensity of the heat sources in the first layer averaged by the coordinate x within the wavelength λ with regard for the weak attenuation ($\beta_{1,2} \ll kn_{1,2}$) is equal to

$$Q_1(x) = \beta_1 n_1 |\alpha_1|^2 I_0 \left(e^{-\beta_1 x} + |R_1|^2 e^{-\beta_1(d_1 - x)}\right). \quad (23a)$$

The intensity of such sources in the second layer looks as

$$Q_2(x) = \beta_2 n_2 |\alpha_1 \cdot \alpha_2|^2 I_0 \left(e^{-\beta_2(x - d_1)} + |R_2|^2 e^{-\beta_2(d - x)}\right). \quad (23b)$$

Here, $I_0 = \frac{\varepsilon_0 E_0^2}{2}c$ is the intensity of the incident emission, ε_0 is the vacuum dielectric permittivity, and c is the velocity of light in vacuum.

The first terms in Eqs. (23) are the heat sources arising from the incident waves, and the second ones are the heat sources arising due to the reflected waves. The interfering waves do not create heat sources, since they disappear due to the self-averaging within the light length.

For the effective sample, we have

$$Q(x) = \beta n |\alpha|^2 I_0 \left(e^{-\beta x} + |R|^2 e^{-\beta(d - x)}\right). \quad (24)$$

The physical sense of the terms in Eq. (24) is the same as in Eqs. (23).

6. Temperature Distributions in the Two-layer Structure and the Effective One-layer Sample

To simplify the calculations which are aimed only at the illustration of the principal idea and do not break the general qualitative picture, we consider the case where the intensities of reflected waves are much smaller than those of the incident waves. This consideration assumes that the reflection coefficients (see Eqs. (18d) and (18e)) must satisfy the conditions

$$|R_{1,2}| \ll 1, \quad (25)$$

or the wave has attenuated before coming up to the interface, or, in other words, the wave decay length β_1^{-1} must be much less than the thickness of the first layer d_1:

$$\beta_1 d_1 \gg 1. \quad (26)$$

Inequality (25) is valid, for example, in the case of a small difference in the optical densities of the bordering media, i.e. if the conditions

$$|n_1 - n_2| \ll n_1, n_2 \quad (27a)$$

hold in the first layer, and the inequality

$$|n_2 - 1| \ll n_2 \quad (27b)$$

holds in the second one.

As a result, the equations for $Q_{1,2}(x)$ are as follows [see Eq. (23)]:

$$Q_1(x) = \beta_1 n_1 I_0 |\alpha_1|^2 e^{-\beta_1 x}. \quad (28a)$$

$$Q_2(x) = \beta_2 n_2 I_0 |\alpha_1 \alpha_2|^2 e^{-\beta_2(x - d_1)}. \quad (28b)$$

The solutions of Eqs. (11) with the boundary conditions (12) and the intensities of heat sources (28) are

$$T_1(x) = T_2(d_1) + \frac{I_0 n_1}{\eta}|\alpha_1|^2\left(1 - e^{-\beta_1 d_1}\right) +$$

$$+\frac{|\alpha_1|^2 I_0 n_1}{\chi_1 \beta_1}\left(\beta_1(d_1 - x) + \left(e^{-\beta_1 d_1} - e^{-\beta_1 x}\right)\right) \quad (29a)$$

on the interval $0 \le x < d_1$, where $T_2(d_1)$ is defined by formula (29*b*) at the point $x = d_1$ and

$$T_2(x) = T_0 + \frac{|\alpha_1 \cdot \alpha_2|^2 I_0 n_2}{\chi_2 \beta_2}\times$$

$$\times\left\{\beta_2(d - x)\left(1 + \frac{n_1}{n_2}\frac{1}{|\alpha_2|^2}\left(1 - e^{-\beta_1 d_1}\right)\right) +\right.$$

$$+ e^{-\beta_2 d_2} - e^{-\beta_2 (x-d_1)} \Big\} \tag{29b}$$

on the interval $d_1 < x \leq d$.

It is obvious from Eqs. (29) that the two temperatures depend on the optical parameters of both layers. The temperature $T_1(x)$ is determined by both thermal conductivities $\chi_{1,2}$, while the temperature $T_1(x)$ is determined only by the thermal conductivity of its own layer χ_2. The thermal conductivity η of the interface influences only the temperature distribution of the first layer.

For the effective one-layer sample, we also consider the case of weak reflection. That is, the refraction coefficient n satisfies the inequality

$$|n-1| \ll 1, \tag{30}$$

or the wave has attenuated before coming up to the end of the effective sample, which means

$$\beta d \gg 1. \tag{31}$$

Under such conditions, formula (24) takes the simpler form

$$Q = \beta n I_0 |\alpha|^2 e^{-\beta x}. \tag{32}$$

It is easy to obtain from Eqs. (13),(14), and (32) that

$$T(x) = T_0 + \frac{I_0 n |\alpha|^2}{\chi\beta} \left(\beta (d-x) + e^{-\beta d} - e^{-\beta x}\right). \tag{33}$$

7. Effective Thermal Conductivity

Based on the results of works [4, 6] and Introduction, the calculational procedure of the effective parameters can be realized in different ways depending on the specificity of the photothermal experiment. One of them requires the equality of the temperatures at the front surfaces of the two-layer structure and the one-layer sample. The second possibility is related to the equality of the temperatures at the rear surfaces. In both cases, the obtained thermal conductivity of the one-layer sample is the effective thermal conductivity of the real two-layer structure by definition. For this reason, we call the one-layer sample as the "effective sample". We denote the corresponding effective thermal conductivities obtained in different ways as χ_F and χ_R.

The effective thermal conductivity χ_F can be obtained from the following equation:

$$T_1(0) = T(0). \tag{34}$$

Since the isothermal condition holds at the rear surface, we must equate the temperatures at points very close to the surface $x = d$ in order to calculate the effective thermal conductivity χ_R. So the effective thermal conductivity χ_R can be obtained from the equation

$$T_2(d-\delta) = T(d-\delta), \tag{35}$$

where δ is a small value, $\frac{\delta}{d} \ll 1$.

Substituting Eqs. (29a) and (33) in Eq. (34), we obtain

$$\frac{1}{\chi_F} = \frac{|\alpha_1|^2}{|\alpha|^2} \left(\frac{n_1\beta}{n} \left(\frac{1}{\eta} + \frac{d_2}{\chi_2}\right) \frac{1-e^{-\beta_1 d_1}}{\beta d - 1 + e^{-\beta d}} + \right.$$

$$+ \frac{1}{\chi_1} \frac{\beta n_1}{\beta_1 n} \frac{\beta_1 d_1 - 1 + e^{-\beta_1 d_1}}{\beta d - 1 + e^{-\beta d}} +$$

$$\left. + \frac{|\alpha_2|^2}{\chi_2} \frac{\beta n_2}{\beta_2 n} \frac{\beta_2 d_2 - 1 + e^{-\beta_2 d_2}}{\beta d - 1 + e^{-\beta d}} \right). \tag{36}$$

It is evident from this equation that the effective thermal conductivity depends on the thermal and optical parameters of both layers.

We note that the calculation of the effective thermal conductivity by Eq. (35), which corresponds to measurements by the "open-cell" method, gives another result, namely

$$\frac{1}{\chi_R} = \frac{1}{\chi_2} \frac{|\alpha_1|^2}{|\alpha|^2} \left(\frac{n_2}{n} |\alpha_2|^2 \frac{1-e^{-\beta_2 d_2}}{1-e^{-\beta d}} + \frac{n_1}{n} \frac{1-e^{-\beta_1 d_1}}{1-e^{-\beta d}} \right). \tag{37}$$

In this case, the effective thermal conductivity depends only on the thermal conductivity of the second layer, and χ_R depends on the optical parameters of the structure as well, but according to another law.

From expressions (36) and (37), we see that the effective values of thermal conductivities depend on the optical parameters of the effective sample. To achieve the standardization of effective thermal conductivity, it is necessary to choose some values of the optical parameters of the effective sample. Moreover, two basic tasks to introduce the effective parameter which were posed in Introduction, are executed. For simplicity, we suggest that the effective sample absorbs all radiation on the frontal surface. It can be attained by accepting

$n = 1$ and $\beta \to \infty$. In this case, formulas (36) and (37) become

$$\frac{1}{\chi_F} = |\alpha_1|^2 \left(\left(\frac{1}{\eta} + \frac{d_2}{\chi_2} \right) \frac{n_1 \left(1 - e^{-\beta_1 d_1}\right)}{d} + \right.$$

$$\left. + \frac{n_1}{\chi_1} \frac{\beta_1 d_1 - 1 + e^{-\beta_1 d_1}}{\beta_1 d} + \frac{n_2 |\alpha_2|^2}{\chi_2} \frac{\beta_2 d_2 - 1 + e^{-\beta_2 d_2}}{\beta_2 d} \right). \quad (38)$$

$$\frac{1}{\chi_R} = \frac{1}{\chi_2} |\alpha_1|^2 \left(n_2 |\alpha_2|^2 \left(1 - e^{-\beta_2 d_2}\right) + n_1 \left(1 - e^{-\beta_1 d_1}\right) \right). \quad (39)$$

8. Recommendations as for the Comparison of Theory and Experiment

In the comparison of the theoretical predictions with experimental data, the following items seem to be important.

1. One must have a collection of standard samples with various heat conductivities, whose dimensions are the same as the dimensions of a two-sample structure. The frontal surface of these standard samples needs to be painted in the black color to absorb the laser radiation.

2. One must measure the temperature on the frontal surface of the two-sample structure in the photothermal experiment. Let it be T_F which is the same value given by formula (29a) at the point $x = 0$.

3. Among the standard samples with various heat conductivities, one should pick up one, whose temperature on the frontal surface is equal to T_F, when the laser intensity is the same as that in the case of the two-sample structure. Let the heat conductivity of this standard sample be equal to $\chi_F^{\exp}$.

4. The agreement of the theory with the experiment will be attained, when the value $\chi_F^{\exp}$ becomes equal to that given by formula (38).

Similar steps can be also done for the back surface.

9. Conclusions

We have discussed the general theoretical method of determination of the effective parameters of heterogeneous physical solids and have presented a new approach to the calculation of the effective thermal conductivity of two-layer structures. It can be applied to the interpretation of the results of photothermal experiments in the case of bulk light absorption. We have concluded that, in the general case, the effective thermal conductivity depends on the optical parameters of both the two-layer structure and the effective sample. For the standardization of the effective thermal conductivity, it is offered to measure it on optically absolutely black effective samples. Thus, two basic problems of the introducion of effective thermal conductivity, which are discussed above, have been solved.

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ЗАГАЛЬНА МЕТОДИКА РОЗРАХУНКУ ЕФЕКТИВНИХ ПАРАМЕТРІВ НЕОДНОРІДНИХ ТВЕРДИХ ТІЛ

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Р е з ю м е

Обговорено загальну теоретичну методику відшукання ефективних параметрів неоднорідних фізичних тіл і їхні властивості. Продемонстровано її на простому прикладі розрахунку ефективного електричного опору, а також у складнішому випадку визначення ефективної теплопровідності двошарових структур у фототермічних експериментах. На відміну від попередніх робіт, присвячених проблематиці відшукання ефективної теплопровідності, тут розглянуто випадок об'ємного поглинання світла. Показано, що ефективна теплопровідність у загальному випадку залежить від оптичних параметрів шарів і від оптичних параметрів ефективного зразка. Запропоновано спосіб уніфікації ефективної теплопровідності.