SPONTANEOUS BREAKING OF CHIRAL SYMMETRY IN FRUSTRATED SPIN CHAINS IN MAGNETIC FIELD

A.K. KOLEZHUK[1,2], T. VEKUA[3,4], R.S. KHYMYN[5,2]

[1]Institut für Theoretische Physik, Universität Hannover
(2, Appelstraße, Hannover 30167, Germany; e-mail: kolezhuk@itp.uni-hannover.de),

[2]Institute of Magnetism, Nat. Acad. Sci. and Ministry of Education of Ukraine
(36b, Academician Vernadsky Blvd., Kyiv 03142, Ukraine),

[3]Université Louis Pasteur, Laboratoire de Physique Théorique
(3, Rue de l'Université, Strasbourg 67084, France),

[4]Andronikashvili Institute of Physics
(380077 Tbilisi, Georgia),

[5]Taras Shevchenko Kyiv National University, Faculty of Radiophysics
(2, Bild. 5, Academician Glushkov Prosp., Kyiv 03127, Ukraine)

UDC 537.622.5, 538.955

It is predicted that the quantum chiral phase should exist in frustrated spin-S chains in the presence of an external magnetic field. The qualitative form of the phase diagram in the large-S limit and for $S = \frac{1}{2}$ and 1 is discussed.

Phases with broken vector chirality in quantum spin chains have been intensively studied in the recent years [1—6]. They are characterized by a nonzero expectation value of the vector product of two adjacent spins

$$\vec{\kappa}_n = \langle \vec{S}_n \times \vec{S}_{n+1} \rangle \,, \tag{1}$$

so that spins in a chirally ordered phase tend to rotate in a certain preferred plane predominantly clockwise or counterclockwise. This type of order breaks only parity, leaving the U(1) in-plane rotation symmetry intact, and thus is allowed in one dimension, in contrast to the long-range helical spin order [7,8].

In [1], the existence of a *gapless chiral* phase with incommensurate power-law in-plane spin correlations $\langle S_0^+ S_n^- \rangle \propto n^{-\eta} e^{iQn}$ for an antiferromagnetic $S = \frac{1}{2}$ chain with frustrating next-nearest neighbor (NNN) interaction J_2 and easy-plane anisotropy Δ was predicted. After this first prediction, several attempts to find this phase in numerical calculations [2, 9] for $S = \frac{1}{2}$ were unsuccessful, but, quite surprisingly, *two* different types of chiral phases, *gapped* and *gapless*, were found in a frustrated $S = 1$ chain [2,3]. For general S, the emergence of two chiral phases was explained and the qualitative form of the (Δ, J_2) phase diagram in the large-S limit was theoretically established with the help of the large-S mapping [5, 10]. Later, the existence of chiral phases was confirmed for $S = \frac{1}{2}$ [3,4], as well as for $S = \frac{3}{2}$ and $S = 2$ [3]. The chiral gapped phase was found to occupy a very narrow region of the phase diagram, and the general appearance of the $S = 2$ phase diagram agrees with the large-S prediction of Ref. [5]. For $\Delta = 0$ (purely XY case), in the strong frustration limit $J_2 \to \infty$, the bosonization-based theoretical analysis [6] yields the value of the critical exponent $\eta = \frac{1}{8S}$, in a good agreement with the numerical data [3].

The chiral gapped phase was not found in anisotropic half-integer S chains [3], which can be explained [10] by the role of the topological term much in the same way as in the case of the well-known difference between integer and half-integer S in the unfrustrated case [11]. On the other hand, the absence of the chiral gapped phase for half-integer S is hard to reconcile with the results of the bosonization analysis [6].

Experimentally, there are indications [12] that a gapless phase exists in the $S = 1$ zigzag chain material CaV_2O_4; however, it is not clear whether the observed phase is not 3D ordered. The chiral ordering transition has possibly been observed experimentally in the quasi-1D anisotropic organic magnet Gd(hfac)$_3$NITiPr [13].

In this paper we show that a chiral phase emerges in *isotropic* frustrated spin chains as well, if they are subjected to a strong external magnetic field. We consider the model of a zigzag chain defined by the Hamiltonian

$$\mathcal{H} = J_1 \sum_j \vec{S}_j \cdot \vec{S}_{j+1} + J_2 \sum_j \vec{S}_j \cdot \vec{S}_{j+2} - h \sum_j S_j^z, \tag{2}$$

where $\vec{S}_j$ are spin-S operators at the j-th site, and $J_{1,2} > 0$. It is easy to analyze the classical counterpart of the above model, where spins are represented by vectors, $(S_n^\pm, S_n^z) \mapsto (S \sin\theta_n e^{\pm i\varphi_n}, S\cos\theta_n)$. The applied field selects a preferred plane, reducing the symmetry to U(1). Depending on the frustration strength

$$\alpha = J_2/J_1,$$

the in-plane ground state configuration is given by $\varphi_n = (\pi \pm \lambda)n$ with

$$\lambda = \begin{cases} 0, & \alpha < 1/4, \\ \arccos(1/4\alpha), & \alpha > 1/4. \end{cases} \tag{3}$$

The spins are canted towards the field, $\cos\theta_n = h/h_s$, where

$$h_s = 4S\{J_1 \cos^2(\lambda/2) + J_2 \sin^2 \lambda\} \tag{4}$$

is the saturation field. The classical ground state is a canted antiferromagnet for α below the Lifshits point $\frac{1}{4}$, while, for $\alpha > \frac{1}{4}$, one has two degenerate helical ground states, as reflected by the $\pm$ signs above, which correspond to the left and right chirality $\kappa = \pm S^2(1 - h^2/h_s^2)\sin\lambda$. Thus, for $\alpha > \frac{1}{4}$ in the presence of a field, the initial SU(2) symmetry is reduced to $\mathrm{U}(1) \times \mathrm{Z}_2$.

In the quantum case, the U(1) symmetry cannot be broken, but it is allowed to break the discrete Z_2 chiral symmetry. Such a scenario is indeed realized in anisotropic chains [1—3,6,5]. A natural question arises, namely, whether an external magnetic field can act similarly to the *xy* anisotropy [14, 15], favoring the chiral order in *isotropic* spin chains? Recent numerical studies [16, 17] propose the scenario of a *two-component Luttinger liquid* (LL) without any breaking of the Z_2 symmetry, casting doubts on the above idea. The aim of the present paper is to show that the correct high-field physics of isotropic frustrated chains is indeed determined by the spontaneous breaking of the chiral symmetry.

Large-S case. We start with the case of a large-S frustrated chain close to the saturation field h_s. In the vicinity of the saturation field, the emergence of chirality can be analyzed for an arbitrary spin value S. In the coherent state path integral representation, the effective Lagrangian is given by $\mathcal{L} = -\hbar S \sum_n (1 - \cos\theta_n)\partial_t \varphi_n - \langle \mathcal{H} \rangle$. One can introduce the variables

$$z_n = (-1)^n \sin(\theta_n/2) e^{i\varphi_n}, \tag{5}$$

then the dynamical part of the Lagrangian can be rewritten as $i\hbar S \sum_n (z_n^* \dot{z}_n - \dot{z}_n^* z_n)$, where the dot denotes differentiation with respect to time. It is also easy to show that the scalar spin product $\vec{S}_n \cdot \vec{S}_{n+d}$ in terms of the new variables takes the form

$$\vec{S}_n \cdot \vec{S}_{n+d} = (1 - 2|z_n|^2)(1 - 2|z_{n+d}|^2) +$$
$$+ 2(-1)^d (z_n^* z_{n+d} + \text{c.c.}) \sqrt{1 - |z_n|^2} \sqrt{1 - |z_{n+d}|^2}. \tag{6}$$

Near the saturation field, one can expect that the deviations from the fully polarized state are small, $|z_n| \ll 1$, so one can expand the square roots in (6), keeping terms up to the quartic ones. In order to pass to the continuum properly, we use the following ansatz for the z-variables:

$$z_n = \psi_{R,n} e^{i\lambda n} + \psi_{L,n} e^{-i\lambda n}, \tag{7}$$

where λ is the classical helix pitch given by (3). Treating $\psi_{R,L}$ as smooth fields and keeping only non-oscillating terms, one obtains for the Hamiltonian average

$$\langle \mathcal{H} \rangle = \int \frac{dx}{a} \Big\{ 2S(h - h_s)(|\psi_R|^2 + |\psi_L|^2) +$$
$$+ 2S h_s (|\psi_R|^2 + |\psi_L|^2)^2 +$$
$$+ 4S\big[h_s - 4SJ_1(1 + J_1^2/J_2^2)\sin^2\lambda\big] |\psi_R|^2 |\psi_L|^2 +$$
$$+ 8J_2 S^2 a^2 \sin^2\lambda \big(|\nabla\psi_R|^2 + |\nabla\psi_L|^2\big) \Big\}, \tag{8}$$

and the dynamic part of the Lagrangian acquires the form $\int \frac{dx}{a} \big\{ i\hbar S(\psi_R^* \dot{\psi}_R - \dot{\psi}_R^* \psi_R) + i\hbar S(\psi_L^* \dot{\psi}_L - \dot{\psi}_L^* \psi_L) \big\}$. Here a is the lattice constant (from now on set to 1), and $\nabla \equiv \partial/\partial x$.

It is convenient to rescale the bosonic fields

$$(2S)^{1/2} \psi_{R,L} \to \psi_{1/2},$$

then one finally arrives at a Lagrangian of the form

$$\mathcal{L} = \int dx \sum_{\sigma=1,2} \Big\{ i\psi_\sigma^* \partial_t \psi_\sigma - \frac{1}{2m} |\partial_x \psi_\sigma|^2 + \mu |\psi_\sigma|^2 \Big\} -$$
$$- \frac{1}{2} \int dx \{ u(|\psi_1|^2 + |\psi_2|^2)^2 + w|\psi_1|^2 |\psi_2|^2 \}, \tag{9}$$

which was recently discussed in the context of 1D binary Bose-condensate mixtures [18]. The Lagrangian parameters are given in our case by

$$\mu = h_s - h, \quad m^{-1} = 8J_2 S \sin^2\lambda,$$

$$u = h_s/S, \quad w = 2\{u - 4J_1(1 + J_1^2/J_2^2)\sin^2\lambda\}, \tag{10}$$

where we have set the Planck constant to 1 for the sake of clarity.

In the harmonic fluid approach [19], the field operators and densities can be expressed through scalar bosonic fields ϑ, φ as

$$|\psi_\sigma|^2 = \{\rho_\sigma + \partial_x \phi_\sigma/\pi\} \sum_m e^{2im(\pi\rho_\sigma x + \phi_\sigma)},$$

$$\psi_\sigma = \{\rho_\sigma + \partial_x \phi_\sigma/\pi\}^{1/2} e^{i\vartheta_\sigma} \sum_m e^{2im(\pi\rho_\sigma x+\varphi_\sigma)}, \qquad (11)$$

and, for $\mu > 0$, Lagrangian (9) corresponds to two Gaussian (Luttinger liquid) models with Hamiltonians of the form

$$\mathcal{H} = \frac{v}{2}\int dx \left\{ \frac{1}{K}(\partial_x\phi)^2 + K(\partial_x\vartheta)^2 \right\}, \qquad (12)$$

coupled by the density-density interaction $w|\psi_1|^2|\psi_2|^2$. In contrast to [18], the total particle numbers of the components $n_{1,2} = \int dx |\psi_{1,2}|^2$ (which are separately conserved) are not fixed in our case, but are chosen by the system so as to minimize the energy at $\mu > 0$ (i.e., $h < h_s$). It is easy to show that, for $w > 0$, the system is unstable against any perturbation making $\rho_1 \neq \rho_2$: indeed, e.g., the interaction term for $\rho_1 > \rho_2$ leads to the renormalization $\rho_\sigma \mapsto \rho_\sigma + \langle\partial_x\varphi_\sigma\rangle/\pi$ with $\langle\partial_x\varphi_1\rangle > \langle\partial_x\varphi_2\rangle$. In terms of the two-component Bose-condensates, this corresponds to the "demixing"instability [18]. As a result, the chiral Z_2 symmetry breaks spontaneously and one of the bands σ gets fully depleted. The effective theory is a *single* Luttinger liquid with the parameter $K > 1$ depending on the dimensionless coupling constant

$$\gamma = \frac{mu}{\rho_0} \simeq \frac{\pi}{2S\sin\lambda}\Big(\frac{h_s}{4J_2 S(1-h/h_s)}\Big)^{1/2}, \qquad (13)$$

where $\rho_0 = (2\mu m)^{1/2}/\pi$ is the equilibrium density for small μ (i.e., in the vicinity of the saturation field). For $h \to h_s$, when $\gamma \gg 1$, the LL parameter tends to 1 and is given by $K \simeq 1 + 4/\gamma$, and for $\gamma \ll 1$ (which, despite the condition $\rho_0 \ll 1$, is formally possible for large S) one has $K \simeq \pi/\sqrt{\gamma}$ [20].

The chirality order parameter is directly related to the density difference, $\kappa \simeq \langle|\psi_1|^2 - |\psi_2|^2\rangle \sin\lambda$. Neglecting the depleted field and using the known expression for a density correlator [20], one obtains the leading asymptotics of the chirality correlation function:

$$\langle\kappa(x)\kappa(0)\rangle \simeq \frac{S^2}{\pi^2}\Big\{\frac{h_s-h}{J_2 S} - \frac{2K\sin^2\lambda}{x^2}\Big\}. \qquad (14)$$

The longitudinal spin correlator $\langle S^z(x)S^z(0)\rangle$ is also related to the density and behaves similarly to (14). The leading part of the transversal spin correlator can be expressed through $\langle\psi^\dagger(x)\psi(0)\rangle$ and is given by

$$\langle S^+(0)S^-(x)\rangle \simeq 2S\rho_0\Big(\frac{K}{\pi\rho_0 x}\Big)^{\frac{1}{2K}} e^{i\lambda x}. \qquad (15)$$

$S = \frac{1}{2}$ *chain.* The "extreme quantum" case of $S = \frac{1}{2}$ can be treated using a field-theoretical description based on the bosonization approach. Let us consider the limit of strong frustration $\alpha \gg 1$ and strong magnetic fields $h \sim J_2$. The system may be viewed as two chains weakly coupled by the zigzag interaction J_1. A single spin-$\frac{1}{2}$ chain in a uniform magnetic field is known to be critical, its low-energy physics being effectively described [21] by the LL model of the form (12), where ϕ is a compactified scalar bosonic field and ϑ is its dual, $\partial_t\phi = v\partial_x\vartheta$, with the commutation relations $[\phi(x),\vartheta(y)] = i\Theta(y-x)$, where $\Theta(x)$ is the Heaviside function and the regularization $[\phi(x),\vartheta(x)] = i/2$ is assumed. Integrability of the $S = \frac{1}{2}$ chain model makes possible to relate explicitly the coupling constants of the theory, the spin wave velocity v, and the LL parameter K, to the microscopic parameters J_2, h. The exact functional dependences $v(h)$ and $K(h)$ are known (see [22] and references therein) from the numerical solution of the Bethe ansatz integral equations [23]. Particularly, K increases with the magnetic field from $K(h=0) = \frac{1}{2}$ to $K = 1$ for h approaching the saturation value $2J_2$.

In the infrared limit, the following representation of the lattice spin operators holds [21]:

$$\begin{aligned} S_n^z &= \frac{1}{\sqrt{\pi}}\partial_x\phi + \frac{a}{\pi}\sin\{2k_{\rm F}x + \sqrt{4\pi}\phi\} + m, \\ S_n^- &= (-1)^n e^{-i\theta\sqrt{\pi}}\{c + b\sin(2k_{\rm F}x + \sqrt{4\pi}\phi)\}, \end{aligned} \qquad (16)$$

Here, $m(h)$ is the ground state magnetization per spin which determines the Fermi wave vector $k_F = (\frac{1}{2} - m)\pi$ and is known exactly from the Bethe ansatz results [23]. Nonuniversal constants a, b, and c for general h have been extracted numerically from the density matrix renormalization group (DMRG) calculations [24].

We treat the J_1 interchain coupling term perturbatively, representing two decoupled chains in terms of LL models of the form (12). It is convenient to pass to the symmetric and antisymmetric combinations of the fields describing the individual chains, $\phi_\pm = (\phi_1 \pm \phi_2)/\sqrt{2K}$ and $\vartheta_\pm = (\vartheta_1 \pm \vartheta_2)\sqrt{K/2}$. The effective Hamiltonian describing low-energy properties of model (2) takes the following form:

$$\begin{aligned} &\mathcal{H}_{\rm eff} = \mathcal{H}_0^+ + \mathcal{H}_0^- + \mathcal{H}_{\rm int}, \\ &\mathcal{H}_0^\pm = \frac{v}{2}[(\partial_x\vartheta_\pm)^2 + (\partial_x\phi_\pm)^2], \\ &\mathcal{H}_{\rm int} = g_1\cos k_{\rm F}\cos\big(k_F + \sqrt{8\pi K_-}\phi_-\big) \\ &-g_2\partial_x\vartheta_+\sin\big(\sqrt{2\pi/K_-}\vartheta_-\big). \end{aligned} \qquad (17)$$

Only the relevant terms are shown here, including the "twist operator" with nonzero conformal spin [1]. The

Fermi velocity $v \propto J_2$, while the couplings $g_{0,1,2} \propto J_1 \ll v$. The renormalized LL parameter is given by

$$K_- = K(h)\Big\{1 + J_1 K(h)/\big(\pi v(h)\big)\Big\}. \tag{18}$$

Note that, in the first order in J_1/J_2, the correction to K_- for the zigzag type of interchain coupling is twice larger compared to that for the ladder type of coupling.

The inter-sector part of (17) contains a term which can be identified as the infrared limit of the product of in-chain and interchain chiralities: one can show that

$$\partial_x \vartheta_+ \sin\sqrt{\frac{2\pi}{K_-}}\vartheta_- \propto (\kappa^z_{2i-1,2i+1} + \kappa^z_{2i,2i+2})\,\kappa^z_{2i,2i+1}, \tag{19}$$

where $\kappa^z_{i,j} \equiv (\vec{S}_i \times \vec{S}_j)^z$. All the other terms omitted in (17) are made either irrelevant or incommensurate by the external magnetic field. Hamiltonian (17) gives the minimal effective field theory describing the low-energy dynamics of a strongly frustrated ($\alpha \gg 1$) spin-$\frac{1}{2}$ zigzag chain for a nonzero magnetization m. For small m, the LL parameter $K_- \simeq \frac{1}{2}$, and the inter-sector g_2 term has a higher scaling dimension than the strongly relevant g_1 term in the antisymmetric sector. In this case, the system is in a phase with a relevant coupling in the antisymmetric sector, as discussed for the first time in [25] (later dubbed EO phase [17]). In contrast to that, at $h = 0$, all terms generated by the zigzag coupling are only marginal.

When h increases, the chirality product operator (19) can become more relevant than the g_1 term controlling the field ϕ_-; the latter term becomes less relevant with increase in h as well as with increase in the zigzag *antiferromagnetic* coupling J_1. To study this situation, one can apply a mean field decoupling procedure to the inter-sector term in the spirit of [1]. At the mean field level, the interaction $\mathcal{H}_{\rm int}$ takes the form

$$\mathcal{H}_{\rm MF} = g_1 \cos k_{\rm F} \cos(k_{\rm F} + \sqrt{8\pi K_-}\phi_-) \tag{20}$$
$$-g_2\partial_x\vartheta_+\langle\sin\sqrt{\frac{2\pi}{K_-}}\vartheta_-\rangle - g_2\langle\partial_x\vartheta_+\rangle\sin\sqrt{\frac{2\pi}{K_-}}\vartheta_-.$$

Remarkably, the mean field Hamiltonian reveals a competition between the basic and dual field terms of the form $\sin(\gamma\phi_-)$ and $\sin(\delta\vartheta_-)$ with $\gamma\delta = 4\pi$, exactly the value where the Ising quantum phase transition takes place [26,27]. To find the critical magnetic field $h_{\rm cr}$ which corresponds to this transition, we equate the RG masses produced by the operators $\sin(\gamma\phi_-)$ and $\sin(\delta\vartheta_-)$:

$$\Big(\frac{g_1}{v}\Big)^{\frac{1}{2-d_1}} \sim \Big(\frac{g_2}{v}\langle\partial_x\vartheta_+\rangle\Big)^{\frac{1}{2-d_2}}, \tag{21}$$

where $d_1 = 2K_-$ and $d_2 = 1/(2K_-)$ are the scaling dimensions of the corresponding operators. The averages in (20) can be determined from the mean-field self-consistency conditions, which yields

$$\langle\partial_x\vartheta_+\rangle \sim \frac{g_2}{v}\langle\sin\sqrt{\frac{2\pi}{K_-}}\vartheta_-\rangle,$$
$$\langle\sin\sqrt{\frac{2\pi}{K_-}}\vartheta_-\rangle \sim \Big(\frac{g_2}{v}\langle\partial_x\vartheta_+\rangle\Big)^{\frac{d_2}{2-d_2}}, \tag{22}$$

resulting in

$$\langle\partial_x\vartheta_+\rangle \sim \Big(\frac{g_2}{v}\Big)^{\frac{1}{1-d_2}}. \tag{23}$$

Substituting (23) back into (21) and taking into account that g_1 and g_2 are of the same order (of J_1), one can just assume that, at the transition, the exponents on the right- and left-hand sides are equal. Doing so, one obtains the following equation for the renormalized LL parameter K_- at the transition:

$$4K_-^2 - 2K_- - 1 = 0,$$

whose solution happens to be related to the celebrated "golden mean" q:

$$2K_-(h_{\rm cr}) = q \equiv (\sqrt{5}+1)/2.$$

This leads to the following equation for h_{cr}:

$$K_-(h_{\rm cr}) = \frac{q}{2}\Big\{1 - \frac{J_1 K_-(h_{\rm cr})}{\pi v(h_{\rm cr})}\Big\}. \tag{24}$$

The fact that $K(h)$ is a monotonically increasing function [22, 28] implies that the critical field decreases with increasing the *antiferromagnetic* zigzag coupling J_1:

$$(\partial h_{\rm cr}/\partial J_1) < 0 \quad \text{for} \quad J_1 > 0. \tag{25}$$

Numerically solving Eq. (24), one obtains that the maximal value of $h_{\rm cr}$ achieved at $J_1 \to 0$ is approximately $h_{\rm cr} \simeq 1.7J_2$, and the spin wave velocity in this limit is still of the order of the bandwidth, $v(h_{\rm cr}) \simeq 0.6J_2$, which justifies the applicability of the bosonization formalism close to $h_{\rm cr}$. Within this approach, there is no indication that the chiral phase would be destabilized by a further increase of the magnetic field, so one may conclude that it extends from h_{cr} up to the saturation field h_s.

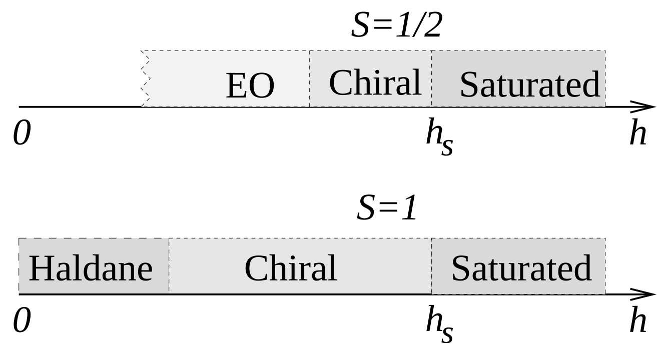
$S = 1$ *chain.* A similar bosonization analysis can be performed in the $S = 1$ case as well [15]. The LL parameter of a $S = 1$ chain turns out to be *increasing* from the free fermion value $K = 1$ at $h = \Delta$ (where Δ is the Haldane gap) with a further increase of the field, so that one has generally $K > 1$ for $h > \Delta$ [29,30]. Coupling two Luttinger liquids with $K > 1$ by a zigzag interchain coupling yields the same effective field theory (17), but the only relevant term is the product of chiralities (19) since $K > 1$. Thus, in contrast to the $S = \frac{1}{2}$ case, a strongly frustrated spin-1 chain *immediately* enters a chiral phase, described by a one component Luttinger liquid with spontaneously broken chiral symmetry, as far as the external field becomes higher than the gap value.

Discussion. In summary, we have shown that a sufficiently strong magnetic field applied to a spin-S isotropic J_1-J_2 zigzag chain induces a phase with spontaneously broken Z_2 symmetry, which is characterized by the long range vector chirality order and emerges immediately below the saturation field if the frustration strength J_2/J_1 exceeds the classical Lifshits point value $\frac{1}{4}$. This chiral phase is *gapless*, and its low-energy physics is effectively described by a *one-component* Luttinger liquid. The arising sequence of quantum phase transitions, as it follows from our study, is schematically displayed in the figure for $S = \frac{1}{2}$ and $S = 1$.

Our results refute the two-component LL scenario proposed in [16, 17], and may necessitate, in fact, reconsidering the phase diagrams of other frustrated spin models, particularly of a biquadratic-bilinear spin-1 chain in a magnetic field [31]. One may suppose that the $S1$ phase in Fig. 2 of [31] should have a broken Z_2 symmetry in the region beyond the Lifshits point. To clarify this issue, we suggest to measure directly the chirality correlator $\langle \kappa_0^z \kappa_n^z \rangle$ in the limit $n \to \infty$ above the cusp singularity. For a spin-$\frac{1}{2}$ chain, such a correlator was calculated only for very short distances [32] and indicated the emergence of at least short-range chirality correlations for h directly below the saturation field h_s.

The chiral phase should be able to survive finite temperature effects since it involves a breaking of the discrete Z_2 symmetry. Less trivially, it has also a chance to survive the three-dimensional interaction without tranforming into a usual helical long-range order: as noted in [7], the chirality correlation length at finite temperatures is much larger than the spin correlation length. So, with decrease in temperature, the chiral order should set in before the helical spin order does.

Several materials are known which realize zigzag spin-$\frac{1}{2}$ chains (see Table 1 in [33]). A promising candidate substance for detecting the field-induced chirality would be $(N_2H_5)CuCl_3$, since its small



A schematic view of the phase diagram for a sufficiently large value of frustration $\alpha = J_2/J_1$, for the two lowest spin values $S = \frac{1}{2}$ and $S = 1$. EO denotes the phase with a gap in the antisymmetric sector [17,25]

exchange constants $J_1 \simeq 4$ K and $J_2 \simeq 16$ K make feasible the task of attaining magnetic fields comparable to J_2. Experimentally, the projection of vector chirality $\vec{\kappa}$ on the applied field direction could be detected by comparing the inelastic scattering intensities for oppositely polarized neutrons, as it was done for the triangular lattice antiferromagnet $CsMnBr_3$ [34]; a similar route can be employed with polarized light. We hope that our results will stimulate the further experimental work in this direction.

We thank D. Cabra, A. Honecker, G. Japaridze, and U. Schollwöck for stimulating discussions. AK is supported by the Heisenberg Fellowship of Deutsche Forschungsgemeinschaft.

1. *Nersesyan A. A., Gogolin A. O., Eßler F. H. L.* //Phys. Rev. Lett. — 1998. — **81**. — P. 910.

2. *Kaburagi M., Kawamura H., Hikihara T.* // J. Phys. Soc. Jpn. — 1999. — **68**. — P. 3185.

3. *Hikihara T., Kaburagi M., Kawamura H., Tonegawa T.* //Ibid. — 2000. — **69**. — P. 259; *T. Hikihara, M. Kaburagi, H. Kawamura* //Phys. Rev. B — 2001. — **63**. — Art.-Nr. 174430.

4. *Nishiyama Y.* //Europ. Phys. J. B. — 2000. — **17**. — P. 295.

5. *Kolezhuk A. K.* //Phys. Rev. B. — 2000. — **62**. — P. R6057.

6. *Lecheminant P., Jolicoeur T., Azaria P.* //Ibid. — 2001. — **63**. — Art.-Nr. 174426.

7. *Villain J.* //Ann. Isr. Phys. Soc. — 1978. — **2**. — P. 565.

8. *Chubukov A. V.* //Phys. Rev. B — 1991. — **44**. — P. R4693.

9. *Aligia A. A., Batista C. D., Eßler F. H. L.* //Phys. Rev. B — 2000. — **62**. — P. 6259.

10. *Kolezhuk A. K.* //Prog. Theor. Phys. Suppl. — 2002. — **145**. — P. 29.

11. *Affleck I.* //J. Phys.: Cond. Matter — 1989. — **1**. — P. 3047.

12. *Fukushima H., Kikuchi H., Chiba M.et al.* //Prog. Theor. Phys. Suppl. — 2002. — **145**. — P. 72.
13. *Affronte M., Caneschi A., Cucci C. et al.* //Phys. Rev. B — 1999. — **59**. — P. 6282.
14. *Khymyn R. S., Kolezhuk A. K.* //Visn. Kyiv Nat. Univ. Radiofiz. Elektr. — 2004. — **7**. — P. 41.
15. *Kolezhuk A. K., Vekua T.* [ArXiv preprint cond-mat/0502502].
16. *Okunishi K., Hieida Y., Akutsu Y.* //Phys. Rev. B. — 1999. — **60**. — P. R6953.
17. *Okunishi K., Tonegawa T.* //J. Phys. Soc. Jpn. — 2003. — **72**. — P. 479.
18. *Cazalilla M. A., Ho A. F.* //Phys. Rev. Lett. — 2003. — **91**. — Art.-Nr. 150403.
19. *Haldane F. D. M.* //Ibid. — 1981. — **47**. — P. 1840.
20. *Cazalilla M. A.* //J. Phys. B: At. Mol. Opt. Phys. — 2004. — **37**. — P. S1.
21. *Luther A., Peschel I.* //Phys. Rev. B. — 1975. — **12**. — P. 3908.
22. *Affleck I., Oshikawa M.* //Ibid. — 1999. — **60**. — P. 1038.
23. *Bogoliubov N. M., Izergin A. G., Korepin V. E.*// Nucl. Phys. B. — 1986. — **275**. — P. 687.
24. *Eßler F. H. L., Furusaki A., Hikihara T.* //Phys. Rev. B. — 2003. — **68**. — Art.-Nr. 064410; *Hikihara T., Furusaki A.* //Ibid. — 2004. — **69**. — Art.-Nr. 064427.
25. *Cabra D. C., Honecker A., Pujol P.* //Eur. Phys. J. B. — 2000. — **13**. — P. 55.
26. *Totsuka K.* //Eur. Phys. J. B. — 1998. — **5**. — P. 705.
27. *Arlego M., Cabra D. C., Drut J. E., Grynberg M. D.* //Phys. Rev. B — 2003. — **67**. — Art.-Nr. 144426.
28. *Cabra D. C., Honecker A., Pujol P.* //Ibid. — 1998. — **58**. — P. 6241.
29. *Konik R. M., Fendley P.* //Phys. Rev. B — 2002. — **66**. — Art.-Nr. 144416.
30. *Venuti L. C., Ercolessi E., Morandi G. et al.* //Int. J. Mod. Phys. B — 2002. — **16**. — P. 1363.
31. *G. Fáth and P. B. Littlewood* //Phys. Rev. B. — 1998. — **58**. — P. R14709.
32. *Yoshikawa S., Okunishi K., Senda M., Miyashita S.* //J. Phys. Soc. Jpn. — 2004. — **73**. — P. 1798.
33. *Hase M., Kuroe H., Ozawa K. et al.* //Phys. Rev. B — 2004. — **70**. — Art.-Nr. 104426.
34. *Maleyev S. V., Plakhty V. V., Smirnov O. P. et al.* //J. Phys.: Condens. Matter — 1998. — **10**. — P. 951; *S. V. Maleyev* //Phys. Rev. Lett. — 1995. — **75**. — P. 4682.

СПОНТАННЕ ПОРУШЕННЯ КІРАЛЬНОЇ СИМЕТРІЇ У ФРУСТРОВАНИХ СПІНОВИХ ЛАНЦЮЖКАХ В МАГНІТНОМУ ПОЛІ

О.К. Колежук, Т. Векуа, Р.С. Химин

Р е з ю м е

Передбачено виникнення квантової кіральної фази у фрустрованих ланцюжках спіну S в зовнішньому магнітному полі. Запропоновано якісний вигляд фазової діаграми в граничному випадку великого S і для $S = \frac{1}{2}$, 1.

СПОНТАННОЕ НАРУШЕНИЕ КИРАЛЬНОЙ СИММЕТРИИ В ФРУСТРИРОВАННЫХ СПИНОВЫХ ЦЕПОЧКАХ В МАГНИТНОМ ПОЛЕ

А.К. Колежук, Т. Векуа, Р.С. Химин

Р е з ю м е

Предсказано возникновение квантовой киральной фазы в фрустрированных цепочках спина S во внешнем магнитном поле. Предложен качественный вид фазовой диаграммы в пределе большого S и для $S = \frac{1}{2}$, 1.