COLLECTIVE GREEN FUNCTION AND LONGITUDINAL SUSCEPTIBILITY OF ANISOTROPIC HEISENBERG FERROMAGNET AT LOW TEMPERATURES

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The longitudinal dynamic susceptibility defining the linear response of an "easy axis" anisotropic Heisenberg ferromagnet on the presence of a weak external magnetic field (supposed time dependent and spatially non-uniform) is studied in the region of low temperatures (relative to the Curie temperature). Linearized quantum equations of motion for the Fourier transforms of the longitudinal components of the spin operators are used to construct the dynamic random phase approximation (DRPA) for the magnon collective Green function. The one-parameter class of the bosonic representations of the spin operators (including as particular cases the Dyson—Maleev and Holstein—Primakoff representations) is used. The dispersion equation, describing the poles of the longitudinal dynamic susceptibility, is studied and it is shown that there are no long-wavelength excitations of the "zero-magnon" type. Moreover, it is shown that the kinematic interaction doesn't contribute to this dispersion equation, which accords completely with the well-known result of Baryakhtar et al. for the isotropic case.

1. Introduction

The problem of a correct account for the so-called *kinematic interaction*, which is due to the peculiar — neither bosonic nor fermionic — commutation properties of the spin operators, is one of the most interesting (but yet not fully solved) problems of the theory of magnetism (some most elaborated approaches of this kind may be found, e.g., in monographs [1—3]). Note that almost all the Bose or Fermi representations for the spin operators are intended to "convert" the kinematic interaction into the dynamical one, because the last may be treated by more conventional and well established means.

One of the most simple as well as effective means to deal with the kinematic interaction is undoubtedly due to Baryakhtar, Krivoruchko and Yablonsky [4] (see also [3, Ch. 4]). These authors more than twenty years ago suggested the combined Bose-Fermi representation for the spin operators. The bosonic part in [4] is given by the Dyson [5]—Maleev [6] term, whereas the fermionic part — by the so-called *spurion* term; then the projection or metric operator is simply expressed in terms of only spurionic Fermi-operators. In particular, the authors of work [4] (see also [3, Ch. 6]) succeeded to show that the spurionic degrees of freedom don't contribute into the transverse and longitudinal components of the tensor of dynamic susceptibility.

The aim of the present paper is to generalize the results obtained in [3, 4] for the longitudinal dynamic susceptibility in two respects. First, we consider the anisotropic Heisenberg model of the easy-axis type and, secondly, we employ a more general form of the bosonic part representation of spin operators. Namely, we consider the one-parameter family of these representations which are in general not unitary transformations (and so the appropriate Hamiltonian is non-Hermitian). This family was suggested by Dembinski [7] and includes as special cases such well-known representations as the Dyson—Maleev and Holstein—Primakoff ones. We will not consider explicitly the spurionic terms (and their combinations with bosonic ones) because, as was proved in [3, 4], the spurionic spectrum is separated from the bosonic one by a gap of the order of the exchange energy and so (in full accord with the assertion of Dyson [5]) gives only exponentially small contributions at low temperatures.

Physically, our paper pursues the following line of thought. As is well known (see, e.g., [11, 12]), in condensed Bose or Fermi systems, some collective motions like zero sound or plasma oscillations can arise under certain conditions. At low temperatures, the spin system of a Heisenberg ferromagnet is very like to a nonideal gas of magnons which are similar (but not identical) to Bose-particles. So one might expect some "zero-magnon" excitations to arise in the spin density under these conditions (see, e.g., [13—15]). Naturally, one should anticipate the final physical results to be fully independent upon a specific form of account for the kinematic interaction (i.e., upon a particular form of the spin-bosonic representation), but nevertheless it should be proved in every specific problem.

One of the main results of the present paper is that such an independence really takes place at least for a rather wide class of spin-bosonic representations. In order to investigate these questions in connection with the "zero-magnon" problem, it is natural to introduce the magnon collective Green function and to construct some appropriate approximation for it. We take this to be DRPA and consider simultaneously quantum equations of motion for the Fourier transforms of the longitudinal components of the original spin operators and of their bosonic counterparts.

We show that a zero-magnon does not appear in both cases even if the Heisenberg exchange anisotropy is taken into account; the absence of a zero-magnon in the isotropic case was proved in [14—16] and in the anisotropic case (but only for the Dyson representation) in [17]. All these statements were obtained in the lowest approximation with respect to temperature (i.e., linear in the magnon occupation number) and in the long wavelength limit.

It should be noted that, in spite of the presence of kinematic (i.e., non-bosonic) corrections, the effective magnon interaction potential stays to be short-range and repulsive at small momenta and so uncapable to produce the "zero-magnon" excitation. However, as the momentum increases, the sign of the relevant potential may be changed, in principle, thus making the existing of a "zero-magnon" more probable in the neighborhood of the Brillouin zone edge above the one-particle spin-wave continuum. The problems of two-particle or spin-complex bound states as well as of the "second magnon" are also briefly discussed.

The plan of this paper is the following. After the introduction (Section 1), the Hamiltonian and the equations of motion for spin and quasi-bosonic operators are set down in Section 2. In Section 3, the DRPA for the collective Green function is formulated. Section 4 is devoted to the study of the dispersion equation defining the poles of the collective Green function (and, thus, the possible existence of a zero-magnon), and Section 5 contains the discussion of the results.

2. Hamiltonian and Equations of Motion

The following Hamiltonian describes the spin system of an easy-axis Heisenberg ferromagnet:

$$\mathcal{H} = -\mu H \sum_f S_f^z - \\ -\frac{1}{2}\sum_{fg} I_{fg}\left\{S_f^z S_g^z + \xi\left(S_f^x S_g^x + S_f^y S_g^y\right)\right\}. \qquad (1)$$

All the notations in Eq. (1) are standard: the exchange integral $I_{fg} \geq 0$ will be assumed, for simplicity, non-zero only for nearest-neighbor sites f and g; H is the external magnetic field, and $0 \leq \xi \leq 1$ is the anisotropy parameter (the value $\xi = 0$ corresponds to the Ising model, whereas $\xi = 1$ – to the isotropic Heisenberg model). Assuming a perfectly periodic lattice, we go to the Fourier representation upon the lattice sites and transform Hamiltonian (1) from spin operators to Pauli operators for the case of spin 1/2:

$$S_k^+ = S_k^x + iS_k^y = b_k, \\ S_k^- = S_k^x - iS_k^y = b_k^+, \qquad (2) \\ S_k^z = \frac{\sqrt{N}}{2}\delta_{k0} - \rho_k.$$

Here, the operator ρ_k is an analog of the collective density operator, which is so important in the theory of Bose and Fermi systems:

$$\rho_k = \frac{1}{\sqrt{N}}\sum_q b_q^+ b_{q+k}, \quad \rho_k^+ = \rho_{-k} \qquad (3)$$

The operators b_k, b_k^+ and ρ_k obey the commutation relations

$$\left[b_k, b_{k'}^+\right] = \delta_{kk'} - \frac{2\varepsilon}{\sqrt{N}}\rho_{k-k'}, \quad \left[b_k, \rho_{k'}\right] = \frac{b_{k-k'}}{\sqrt{N}},$$

where the formal (not small!) parameter $\varepsilon = 1$ has been introduced to explicitly isolate the kinematic effects due to the distinction of b_k and b_k^+ from purely bosonic operators.

In terms of the Pauli operators, Eq. (1) becomes

$$\mathcal{H} = E^0 + \sum_k E_k^B b_k^+ b_k - \frac{1}{2}\sum_k I(k)\rho_k\rho_{-k}, \qquad (4)$$

where E_k^B is the single-particle, or Bloch spin-wave, spectrum and E^0 is the ground state energy:

$$E^0 = -\frac{N}{2}\mu H - \frac{N}{8}I(0); \quad E_k^B = \mu H + \frac{1}{2}I(0)\left(1 - \xi\gamma_k\right),$$

where

$$I(k) = I(0)\gamma_k, \quad \gamma_k = \frac{1}{z}\sum_b e^{ikb}, \quad I(0) = Iz.$$

The equation of motion for b_k reads

$$i\dot{b}_k = E_k^B b_k + J_k, \quad J_k = -\frac{1}{\sqrt{N}}\sum_q V_{kq}\rho_q b_{k-q}, \qquad (5)$$

(and its Hermitian conjugate for b_k^+), where

$$V_{kq} = I(0)\left(\gamma_q - \varepsilon\xi\gamma_{k-q}\right) \tag{6}$$

is the magnon interaction potential.

For ρ_k, one obtains analogously

$$i\dot{\rho}_k = \frac{1}{\sqrt{N}}\sum_q E_{kq} b_q^+ b_{q+k}, \tag{7}$$

where the quantity

$$E_{kq} = E^B_{q+k} - E^B_q = \frac{I(0)}{2}\xi\left(\gamma_q - \gamma_{q+k}\right) \tag{8}$$

determines a band of the two-particle spin-wave continuum. Note that ρ_0 is an integral of motion for any value of ξ, whereas the same is true for b_0 or b_0^+ only for $\xi = 1$.

However, it turns to be more appropriate to use the equation of motion not for the operator ρ_k (defined by Eq. (3)) itself, but only for its "non-summed" part, i.e.

$$i\frac{d}{dt}\left(b_p^+ b_{p+k}\right) = E_{kp} b_p^+ b_{p+k} + J_{kp}, \tag{9}$$

where

$$\frac{1}{\sqrt{N}}\sum_p J_{kp} = 0, \quad J_{kp} = b_p^+ J_{p+k} - J_p^+ b_{p+k}.$$

Thus, we escape the necessity to exclude some spurious "$1/E^2$" poles which otherwise arise in the collective Green function.

Along with Eq. (2), where Pauli operators are introduced, one may consider also a general one-parameter family of transformations[1] [7] from spin operators $S_f^\pm$, S_f^z (with the spin quantum number S) to bosonic operators a_f, a_f^+:

$$S_f^+ = \sqrt{2S}\varphi_f^{1-\lambda}, \quad S_f^- = \sqrt{2S}a_f^*\varphi_f^\lambda, \quad \left[a_f, a_{f'}^*\right] = \delta_{ff'},$$

$$S_f^z = S - n_f,\ \varphi_f = 1 - \varepsilon n_f/2S,\ n_f = a_f^* a_f. \tag{10}$$

This nonlinear transformation with $0 \le \lambda \le 1/2$ is an irrational one in the general case $\lambda \ne 0$. Moreover, at $\lambda \ne 1/2$, it is not unitary[2], so that the bosonic operators a_f, a_f^+ are conjugated to one another in some non-Hermitian sense (for details, see [18], e.g.).

In particular cases $\lambda = 0$ and $\lambda = 1/2$, the transformation in Eq. (10) gives the Dyson [5]—Maleev [6] or the Holstein—Primakoff [8] transformations respectively; thus, the parameter λ may be called "non-Dyson". Both the limiting cases mentioned above are most often used, because, in the case $\lambda = 0$, transformation (10) is rational (though highly non-symmetric and non-unitary) for any value of S, whereas in the case $\lambda = 1/2$ it is symmetric and unitary, but remains to be irrational[3].

Applying the transformation of Eq. (10) to the spin Hamiltonian in Eq. (1), we obtain a one-parameter family of quasibosonic Hamiltonians which, for $\lambda \ne 0$, take the form of convergent infinite series arising when φ_f^λ and $\varphi_f^{1-\lambda}$ are formally expanded in terms of the operator $\varepsilon n_f/2S$.

Considering only the case of sufficiently low temperatures, we follow Oguchi [9] and truncate the series at the second term, thus obtaining the following approximate Hamiltonian:

$$\mathcal{H}^{(\lambda)} = E^0 + \sum_k E_k^B a_k^* a_k -$$

$$-\frac{1}{4N}\sum_{kqp} V^{(\lambda)}_{kqp} a^*_{p+q} a^*_{k-q} a_k a_p. \tag{11}$$

Here, E^0 and E_k^B are defined above, and the symmetrized magnon-magnon interaction potential $V^{(\lambda)}_{kqp}$ takes the form

$$V^{(\lambda)}_{kqp} = I(0)\{\gamma_q + \gamma_{-q+k-p} -$$

$$-\varepsilon\xi[\lambda(\gamma_k + \gamma_p) + (1-\lambda)(\gamma_{p+q} + \gamma_{k-q})]\} \tag{12}$$

which possesses some symmetry properties:

$$V^{(\lambda)}_{kqp} = V^{(\lambda)}_{k,-q+k-p,p} = V^{(\lambda)}_{p,-q,k}.$$

Note that if the kinematic interaction is neglected by setting $\varepsilon = 0$ in Eq. (10), then $V^{(\lambda)}_{kqp}$ goes over to $I(0)(\gamma_q + \gamma_{-q+k-p})$, and $\mathcal{H}^{(\lambda)}$ of Eq. (11) turns into the usual Hermitian Hamiltonian of a nonideal Bose gas of Eq. (4).

[1] Of course, it is possible to use some other Bose or Fermi representations of spin operators (see, e.g., [17—19]), but one may suggest them to give the same result as in the present paper. Note, that the interest in these problems has arosen recently in connection with the "bosonization" of Lie algebra generators for quantum groups (see, e.g., [34—36])

[2] The limits $1/2 \le \lambda \le 1$ for λ is not essential because the values $0 \le \lambda \le 1/2$ correspond obviously to the change $S_f^\pm \to S_f^\mp$; on the other hand, the values $\lambda < 0$ and $\lambda > 1$ are excluded, because the nonlinear operators φ_f^λ and $\varphi_f^{1-\lambda}$ become unbounded.

[3] Note that both the limiting cases mentioned above lead to identical results for the spin-wave spectrum and the magnetization for the isotropic Heisenberg model up to the fourth order in the expansion of φ_f^λ [10].

The following natural representation of $V^{(\lambda)}_{kqp}$ is rather useful:

$$V^{(\lambda)}_{kqp} = V^{(0)}_{kqp} + \lambda W_{kqp}; \quad V^{(0)}_{kqp} = V_{kq} + V_{k,-q+k-p},$$

$$W_{kqp} = I(0)\varepsilon\xi\left(\gamma_{p+q} + \gamma_{k-q} - \gamma_p - \gamma_k\right), \tag{13}$$

Here, from the general expression for $V^{(\lambda)}_{kqp}$, we have isolated the Dyson potential[4] $V^{(0)}_{kqp}$ which is a symmetrized form of the spin model potential V_{kq} from Eq. (6). The additional potential W_{kqp} bears completely the dependence upon the kinematic parameter ε as well as upon the anisotropy parameter ξ. Note that W_{kqp} is proportional to the "non-Dyson" parameter λ.

It is easy to see that

$$\left(\mathcal{H}^{(\lambda)}\right)^* - \mathcal{H}^{(\lambda)} = -\frac{1-2\lambda}{4N}\sum_{kqp} W_{kqp} a^*_{p+q} a^*_{k-q} a_k a_p,$$

i.e., the Hamiltonian $\mathcal{H}^{(\lambda)}$ is not self-adjoint when $\lambda \neq 1/2$. This causes certain differences in the equations of motion for the operators a_k and a^*_k

$$i\dot{a}_k = E^B_k a_k + J^{(\lambda)}_k, \quad -i\dot{a}^*_k = E^B_k a^*_k + J^{*(\lambda)}_k, \tag{14}$$

where

$$J^{(\lambda)}_k = -\frac{1}{\sqrt{N}}\sum_q V_{kq}\nu_q a_{k-q} -$$

$$-\frac{1-\lambda}{2N}\sum_{qp} W_{kqp} a^*_p a_{p+q} a_{k-q},$$

$$J^{*(\lambda)}_k = -\frac{1}{\sqrt{N}}\sum_q V_{kq}\nu_{-q} a^*_{k-q} -$$

$$-\frac{\lambda}{2N}\sum_{qp} W_{kqp} a^*_{k-q} a^*_{p+q} a_p. \tag{15}$$

It is easy further to obtain the equations of motion for the operator $a^*_p a_{p+k}$.

Noting that $\nu_k = \nu^*_{-k} = (1/\sqrt{N})\sum_q a^*_q a_{q+k}$, we see that, analogously to Eq. (3), $\left(J^{(\lambda)}_k\right)^* \neq J^{*(\lambda)}_k$ because of the λ-dependent corrections which distinguish Eq. (15) from the interaction operators J_k in the spin model of Eq. (5). Later on, Eq. (14) can be used also in order to obtain an equation of motion for the operator $a^* a_{q+k}$ (which is similar to Eq. (9)).

3. Collective Green Function in DRPA

The dynamical properties of a spin system are described by the complex susceptibility tensor $\chi^{\alpha\beta}(k,\omega)$:

$$-\chi^{\alpha\beta}(k,\omega) = \langle\langle S^\alpha_k, S^\beta_{-k}\rangle\rangle_{\omega+i\varepsilon} =$$

$$= \int_0^\infty dt e^{-\varepsilon t} e^{i\omega t}\langle[S^\alpha_k(t), S^\beta_{-k}]\rangle, \tag{16}$$

where (Im$E > 0$) is the time Fourier transform of the retarded temperature-dependent Green function. We will study the properties of the longitudinal component ($\alpha = \beta = z$) of the tensor in Eq. (13) in the complex plane $E = \omega + i\varepsilon$

$$-\chi^{zz}(k,E) = \langle\langle S^z_k, S^z_{-k}\rangle\rangle_E = \begin{cases} \langle\langle\rho_k, \rho_{-k}\rangle\rangle_E & \text{(spin)}, \\ \langle\langle\nu_k, \nu_{-k}\rangle\rangle_E & \text{(Bose)}. \end{cases}$$

To describe the collective Green's function (13) of a spin system, we use DRPA, which was successfully used in the theory of Fermi and Bose systems (see, e.g., [11,12]).

Let us consider the equations of motion for collective Green functions $A_{kp}(E)$ in the spin and bosonic models:

$$(E - E_{kp})\langle\langle b^+_p b_{p+q}, \rho_{-k}\rangle\rangle_E =$$

$$= \frac{1}{\sqrt{N}}(\bar{n}_p - \bar{n}_{p+k}) + \langle\langle J_{kp}, \rho_{-k}\rangle\rangle_E, \tag{17}$$

$$(E - E_{kp})\langle\langle a^*_p a_{p+q}, \nu_{-k}\rangle\rangle_E =$$

$$= \frac{1}{\sqrt{N}}(n_p - n_{p+k}) + \langle\langle J^{(\lambda)}_{kp}, \nu_{-k}\rangle\rangle_E, \tag{18}$$

where $\bar{n}_p = \langle b^+_p b_p\rangle = 2\sigma n_p$, $n_p = \langle a^*_p a_p\rangle$. In the generalized Hartree—Fock approximation, $n_p = \left(e^{E_p/\theta} - 1\right)^{-1}$; at low temperatures, we assume that $E_p \simeq E^B_p$, $2\sigma \simeq 1$, so that $\bar{n}_p \simeq n_p \simeq \left(e^{E^B_p/\theta} - 1\right)^{-1}$.

In DRPA, one linearizes J_{kp} through the pairing which isolates the collective operator ρ_q; in our case this approximation is valid because the necessary condition $\omega \ll 1/\tau$ is satisfied at all frequencies ω if one neglects a damping in the single-particle spectrum (i.e., the magnon lifetime $\tau \to \infty$). Then

$$J_{kp} \simeq -\frac{\rho_k}{\sqrt{N}}\{V_{p+k,k} n_p - V_{p,-k} n_{p+k}\}. \tag{19}$$

[4] Strictly speaking, the potential $V^{(0)}_{kqp} \equiv \Gamma^q_{kp}$ in Dyson's paper [5] was obtained only for the case $\xi = 1$; Dyson's potential is a "soft" one, i.e. $\Gamma^q_{kp} \to 0$ if $k \to 0$ and $p \to 0$ for any q.

The operator $J_{kp}^{(\lambda)}$ is somewhat more complicated; following the same procedure, we find

$$J_{kp}^{(\lambda)} \simeq -\frac{\nu_k}{\sqrt{N}}\{V_{p+k,k}n_p - V_{p,-k}n_{p+k}\}-$$

$$-[(1-\lambda)n_p + \lambda n_{p+k}]\frac{1}{2N}\sum_q W_{p+k,k,q}a_q^* a_{q+k} \qquad (20)$$

(note that $W_{p+k,k,q} = 2\varepsilon(E_{kp} - E_{kq})$).

Due to the approximations made in Eqs. (19) and (20), Eqs. (17) and (18) take the form of inhomogeneous Fredholm integral equations of the second type,

$$A_{kp}(E) = F_{kp}(E) + \frac{1}{N^2}\sum_{qr} Q_{kpqr}(E)A_{rq}(E), \qquad (21)$$

with a degenerate kernel. In the spin and bosonic models, the kernel takes the forms

$$Q_{kpqr}(E) = -N\delta_{rk}G_{kp}(E) \qquad (22)$$

and

$$Q_{kpqr}^{(\lambda)}(E) = -N\delta_{rk}\Big\{\Big[G_{kp}(E) + \Delta G_{kp}^{(\lambda)}(E)\Big] -$$

$$-g_{kp}^{(\lambda)}(E)E_{kq}.\Big\}, \qquad (23)$$

respectively. Here, we have used the following notations:

$$F_{kp}(E) = \frac{1}{\sqrt{N}}\frac{n_p - n_{p+k}}{E - E_{kp}};$$

$$G_{kp}(E) = I(0)\left\{\sqrt{N}\gamma_k F_{kp}(E) + H_{kp}(E)\right\},$$

$$\Delta G_{kp}^{(\lambda)}(E) = g_{kp}^{(\lambda)}(E)E_{kp};$$

$$g_{kp}^{(\lambda)}(E) = \varepsilon\frac{(1-\lambda)n_p + \lambda n_{p+k}}{E - E_{kp}};$$

$$H_{kp}(E) = -\frac{\varepsilon\xi\left(\gamma_p n_p - \gamma_{p+k}n_{p+k}\right)}{E - E_{kp}}.$$

For the spin model of Eq. (22) we obtain a closed equation for the longitudinal susceptibility $\chi(k,E) = (1/\sqrt{N}\sum_p A_{kp}(E))$; its solution is

$$\chi(k,E) = \frac{F_k(E)}{1 + G_k(E)},$$

$$F_k(E) = \frac{1}{\sqrt{N}}\sum_p F_{kp}(E) \qquad (24)$$

[$G_k(E)$ and $H_k(E)$ are defined in a similar way].

In the case of the bosonic model of Eq. (23) it is necessary to introduce still another unknown function $\psi(k,E) = (1/\sqrt{N})\sum_p E_{kp}A_{kp}(E)$, and we arrive at a system of two equations

$$\hat{\Phi}^{(\lambda)}(k,E)\begin{pmatrix}\chi(k,E)\\ \psi(k,E)\end{pmatrix} = \begin{pmatrix}F_k(E)\\ N^{-1/2}\sum_p E_{kp}F_{kp}(E)\end{pmatrix}, \qquad (25)$$

where

$$\hat{\Phi}^{(\lambda)}(k,E) =$$

$$= \begin{pmatrix} 1 + G_k(E) + \Delta G_k^{(\lambda)}(E) & -g_k^{(\lambda)}(E) \\ \frac{1}{N}\sum_p E_{kp}\left\{G_{kp}(E) + \Delta G_{kp}^{(\lambda)}(E)\right\} & 1 - G_k^{(\lambda)}(E)\end{pmatrix}.$$

Then

$$\chi(k,E) = \frac{\Delta_1(k,E)}{\Delta(k,E)}, \quad \Delta(k,E) = \mathrm{Det}\hat{\Phi}^{(\lambda)}(k,E), \qquad (26)$$

$\Delta_1(k,E)$ is the determinant of the matrix obtained when the first column of $\Phi^{(\lambda)}(k,E)$ is replaced with the column of free terms from Eq. (25).

We will now derive our principal result which states the following: to the lowest approximation in temperature (taking into account no more than the terms linear in the magnon occupation numbers), Eqs. (24) and (26) for $\chi(k,E)$ are the same[5] for any value of λ. In other words, the spin model and the one-parametric family of bosonic models are *completely equivalent* in the context of calculations of the longitudinal dynamical susceptibility at low temperatures, which is physically quite appealing.

To prove this result, it is sufficient to note that

$$\Delta_1(k,E) = F_k(E)\left\{1 - \Delta G_k^{(\lambda)}(E)\right\} +$$

$$+g_k^{(\lambda)}(E)\frac{1}{N}\sum_p E_{kp}F_{kp}(E) = F_k(E)(1+\varepsilon n);$$

$$\Delta(k,E) = \left\{1 + G_k(E) + \Delta G_k^{(\lambda)}(E)\right\}\left\{1 - \Delta G_k^{(\lambda)}(E)\right\} +$$

[5] It was shown in [16] that the expressions for $\chi(k,E)$ when $\lambda = 0$ and $\lambda = 1/2$ are identical in the isotropic case $\xi = 1$. This result was obtained by the help of only two iterations of the integral equation, whereas our result is *exact* in this sense; moreover, it is valid for any values of λ and ξ.

$$+g_k^{(\lambda)}(E)\frac{1}{N}\sum_p E_{kp}\left\{G_{kp}(E)+\Delta G_{kp}^{(\lambda)}(E)\right\}=$$

$$=\{1+G_k(E)\}(1+\varepsilon n)+R_k^{(\lambda)}(E),$$

$$n=\frac{1}{N}\sum_p n_p,$$

where

$$R_k^{(\lambda)}(E)=\overset{0}{g}{}_k^{(\lambda)}(E)\,\overset{2}{g}{}_k^{(\lambda)}(E)-\left(\overset{1}{g}{}_k^{(\lambda)}(E)\right)^2,$$

$$\overset{s}{g}{}_k^{(\lambda)}(E)\equiv\frac{1}{N}\sum_p g_{kp}^{(\lambda)}(E)E_{kp}^s,$$

$$n=\frac{1}{N}\sum_p n_p,$$

and take account for the fact that $R_k^{(\lambda)}(E)=O(n^2)$ when $\varepsilon n\ll 1$. Performing the transformation of the quantities Δ_1 and Δ, it was important to use the exact relationship $(1/\sqrt{N})\sum_p E_{kp}F_{kp}(E)=EF_k(E)$ as well as its analog with the change $F\to G$.

4. Analysis of the Dispersion Equation

Since the equivalence of the spin and the one-parametric family of bosonic models has been demonstrated, it is necessary and sufficient to consider the dispersion equation

$$1+G_k(E)=0 \tag{27}$$

when studying the possible existence of a "zero magnon". It is convenient to rewrite this equation as

$$\frac{1}{4}+\left[(1-\varepsilon\xi)-(1-\gamma_k)\right]I_1(k,\tilde{E})+\varepsilon\xi I_2(k,\tilde{E})=0,$$

where we have used the dimensionless energy variable $\tilde{E}=2E/I(0)$ and

$$I_1(k,\tilde{E})=\frac{1}{N}\sum_p\frac{\tilde{E}_{kp}n_p}{\tilde{E}^2-\tilde{E}_{kp}^2},$$

$$I_2(k,\tilde{E})=\frac{1}{N}\sum_p\frac{(1-\gamma_p)\tilde{E}_{kp}n_p}{\tilde{E}^2-\tilde{E}_{kp}^2}.$$

Here, with $\tilde{E}=\tilde{\omega}+i\eta$ $(\eta\to+0)$,

$$\begin{aligned}\mathrm{Re}I_1(k,\tilde{E})&=\tfrac{1}{N}P\sum_p\tfrac{\tilde{E}_{kp}n_p}{\tilde{\omega}^2-\tilde{E}_{kp}^2},\\ \mathrm{Im}I_1(k,\tilde{E})&=-\tfrac{\pi}{N}\sum_p\tilde{E}_{kp}n_p\delta\left(\tilde{\omega}^2-\tilde{E}_{kp}^2\right)\end{aligned} \tag{28}$$

[similar expression is obtained for $I_2(k,\tilde{E})$].

Clearly, when $k\to 0$, $E\neq 0$, both $I_1(0,\tilde{E})=I_2(0,\tilde{E})=0$, because $E_{op}=0$. Under these conditions, and noting that $\gamma_k\to 0$ when $k\to 0$, Eq. (27) has no solution[6]. Because of the factor n_p under the summation sign in I_1 and I_2, the largest contribution at low temperatures comes from the region of small p. Therefore, I_2 is the small quantity of a higher order (as compared to I_1) at low temperature because of the factor $1-\gamma_p$ (which goes to zero not less than p^2 when $p\to 0$), and so I_2 will be further omitted.

Because we are interested in small k, we use the following approximation for E_{kp} at low temperatures and small p (we assume the lattice is a simple cubic one with period a):

$$\tilde{E}_{kp}\approx(\xi a^2/6)\varepsilon_{kp},\quad \varepsilon_{kp}=k^2+2kp\mu,\quad \mu=\cos\widehat{(k,p)}.$$

Here, $\varepsilon_{kp}^{\min}=k^2-2kp$ and $\varepsilon_{kp}^{\max}=k^2+2kp$ are the lower and upper limits of the spin-wave continuum. The quantity $I_1(k,\tilde{E})$ is defined as

$$I_1(k,\tilde{E})=\frac{6}{\xi a^2}\frac{v}{(2\pi)^2}\int_0^{p_m}dpp^2I_1(k,p,\tilde{E}),$$

where $p_m=\pi/a$ is the momentum at the Brillouin zone edge (due to the presence of n_p, the value of p_m may be taken, as usual, as infinite). Here,

$$I_1(k,p,\tilde{E})=\int_{-1}^{1}d\mu\frac{k^2+2kp\mu}{\tilde{E}^2-(k^2+2kp\mu^2)^2}=$$

$$=-\frac{1}{4kp}\ln\left|\frac{\tilde{E}^2-(\varepsilon_{kp}^{\max})^2}{\tilde{E}^2-(\varepsilon_{kp}^{\min})^2}\right|.$$

So that, for $\tilde{E}=\tilde{\omega}+in$,

$$\mathrm{Re}I_1(k,p,\tilde{\omega})=-\frac{1}{4kp}\ln\left|\frac{\tilde{\omega}^2-(\varepsilon_{kp}^{\max})^2}{\tilde{\omega}^2-(\varepsilon_{kp}^{\min})^2}\right|, \tag{29}$$

$\mathrm{Im}I_1(k,p,\tilde{\omega})=$

[6] This is in contrast, e.g., with the case of a Coulomb gas, where a singularity in the interaction potential $\nu(k)\sim 1/k^2$ at $k>0$ leads to well-known plasma oscillations [11, 12].

$$= -\frac{\pi}{4kp}\left\{\Theta\left[\tilde{\omega}^2 - \left(\varepsilon_{kp}^{\min}\right)\right] - \Theta\left[\tilde{\omega}^2 - \left(\varepsilon_{kp}^{\max}\right)\right]\right\}. \qquad (30)$$

From Eq. (30) (or even from the more general Eq. (28)) it is seen that the Landau damping occurs (i.e., $\mathrm{Im}I_1(k_1, p, \tilde{\omega}) \neq 0$) only if the frequency $\tilde{\omega}$ lies within the band of two-particle spin-wave excitations $\varepsilon_{kp}^{\min} < \tilde{\omega} < \varepsilon_{kp}^{\max}$. We will assume that this is not the case, so that the Landau damping is absent, and consider two limiting cases for the quantity $I_1(k,p,\tilde{\omega}) = \mathrm{Re}I_1(k,p,\tilde{\omega})$: the high-frequency case with $k^2/\omega \ll 1$ where

$$I_1(k,p,\tilde{\omega}) \approx$$

$$\approx 2\left(\frac{k}{\tilde{\omega}}\right)^2 + \left(\frac{k}{\tilde{\omega}}\right)^4 (k^2 + 8p^2) + \ldots \left(\frac{k^2}{\omega} \ll 1\right), \qquad (31)$$

and the quasistatic case with $k^2/\omega \gg 1$:

$$I_1(k,p,\tilde{\omega}) \simeq -\frac{1}{2kp}\ln\left|\frac{k+2p}{k-2p}\right| -$$

$$-2\left(\frac{\tilde{\omega}}{k}\right)^2 \frac{1}{(k+2p)(k-2p)} + \ldots \left(\frac{k^2}{\omega} \gg 1\right). \qquad (32)$$

Clearly, one would expect a zero-magnon solution only in the first case. However, when Eq. (31) is substituted into the dispersion equation, Eq. (27), we find that already in the lowest order in the small parameter k^2/ω (with $\xi \neq 1$), there are only imaginary poles $\tilde{\varepsilon}_k = \pm ikn^{1/2}(1-\varepsilon\xi)^{1/2}$. It is easy to show that this result does not change when higher order terms are included; the same is true when the expansion in Eq. (32) for $\tilde{\omega} \neq 0$ is used. This indicates the instability and the violation of analytical properties of the retarded Green function $\chi(k,E)$ in Eq. (16) which cannot have poles in the upper half of the E-plane.

At $\xi = 1$ there is a stable solution to Eq. (27), $\tilde{\varepsilon}_k = k^2\sqrt{n}$, but it exists only for $k > k_0 \sim \sqrt{\theta}$, where θ is the dimensionless temperature. When $\theta \to 0$ and for all values of ξ, the pole $\tilde{\varepsilon}_k$ disappears because, unlike fermions or bosons, the number of spin waves is not conserved and, at $\theta \to 0$, their density $n \to 0$ (in particular, $n \sim \theta^{3/2}$ for $\xi = 1$, whereas if $\xi \neq 1$, $n \to 0$ exponentially with θ).

Thus, in the general case of an anisotropic ferromagnet, DRPA is not satisfactory for either the spin model or for the family of quasibosonic models. But the static RPA at $\tilde{E} = 0$ is of interest for describing the static longitudinal susceptibility when $h = 2H/I(0) \neq 0$, to the lowest order in θ (this approximation was considered, e.g. in (20)). Since $F_k(\tilde{E}) \propto I_1(k,\tilde{E})$, it is not difficult to show that, when $\tilde{E} = 0$ and $\xi \neq 1$, $h \neq 0$. Moreover, at $k = 0$, the quantity $F_k(0) \propto \tilde{E}_0^{-1/2}$, where $\tilde{E}_0 = h + (1-\xi)$. In the isotropic case at $\xi = 1$ and $h = 0$, one can demonstrate that $F_k(0) \propto 1/k$, and if $h \neq 0$, the quantity $F_k(0)$ diverges as $\min\{1/k, 1/\sqrt{h}\}$, which corresponds to the previous results [23, 24] and agrees with the general properties of three-dimensional degenerate systems (see, e.g., [25]).

5. Discussion

Let us make some concluding remarks connected with the presence of a large number of various collective excitations in a Heisenberg ferromagnet. The "zero magnon" problem considered here is related to spin density oscillations or the dynamics of the operator $b_p^+ b_{p+k}$ which, according to Eq. (7), describes the creation of a magnon with momentum p and the annihilation of a magnon with momentum $p + k$. Consequently, if a zero magnon did exist, it would appear as a singularity in the Green function $\langle\langle b_p^+ b_{p+k}, \rho_{-k}\rangle\rangle$ (or, equivalently, in $\langle\langle \rho_k, \rho_{-k}\rangle\rangle$).

Clearly, the above problem is not directly related to the existence of bound two-magnon states (or "spin complexes") (see [26–28]). The last states are described by the operator $b_{p+q}^+ b_{k-q}^+$ corresponding to the simultaneous creation (or annihilation) of a pair of magnons with total momentum $p + k$. Spin complexes appear as singularities in the second-order Green function $\langle\langle b_p^+ b_{p+q} b_{k-q}, b_k^+\rangle\rangle$, which, from Eq. (6), enters the equation of motion for the single-particle Green function $\langle\langle b_k, b_k^+\rangle\rangle$ within the approximation of the scattering T-matrix [27, 28] (see also [29]). In these papers, it was shown that the conditions for forming a bound state at small k become even more favorable as the anisotropy increases $\xi \to 0$. But even when $\xi = 1$, such states exist if k is large enough (note that this contradicts the Dyson's assumption [5]).

However, these facts have obviously no bearing on our problem; moreover, the limiting case $\xi = 0$ (the Ising model) cannot in general be treated with DRPA because $E_{kp} \equiv 0$ in Eq. (6), and so all further calculations become meaningless. This fact is due to a very high degree of degeneration of the Ising model, for which all ρ_k are constants of the motion for all k; thus, for this model, one must employ quite different methods of approximation (see, e.g., [25, 26]).

Finally, completely independent of the zero-magnon problem is that of the "second magnon" which arises, for example, in the presence of temperature fluctuations (see [25] and [30,31]). The "second magnon" would appear as a singularity in the thermal conductivity which, according to [32,33], is expressed in terms of higher-order Green functions constructed with the energy (not magnetization!) density operators (e.g., according to Eq. (4), in the form $\langle\langle \rho_k \rho_{-k}, A_k \rangle\rangle$).

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КОЛЕКТИВНА ФУНКЦІЯ ГРІНА ТА ПОЗДОВЖНЯ ДИНАМІЧНА СПРИЙНЯТЛИВІСТЬ АНІЗОТРОПНОГО ГЕЙЗЕНБЕРГІВСЬКОГО ФЕРОМАГНЕТИКА ПРИ НИЗЬКИХ ТЕМПЕРАТУРАХ

Ю.Г. Рудой

Р е з ю м е

Розглянуто поздовжню магнітну сприйнятливість, що визначає лінійний відгук анізотропного феромагнетика Гейзенберга типу "легка вісь" на наявність слабкого змінного і неоднорідного зовнішнього магнітного поля; розглянуто область температур, малих порівняно з температурою Кюрі. Для побудови динамічного наближення випадкових фаз для колективної функції Гріна магнонів використано лінеаризовані квантові рівняння руху для фур'є-образів поздовжніх і поперечних компонент спінових операторів. Аналогічну побудову виконано також із застосуванням однопараметричної сім'ї перетворень від спінових до бозе-операторів, яка перетворення Дайсона—Макєєва та Гольштейна—Примакова містить як частинні випадки. Отримано і проаналізовано дисперсійне рівняння, що визначає полюси поздовжньої динамічної сприйнятливості. Показано, що у довгохвильовій границі відсутнє колективне збудження типу "нульового магнона", а також відсутній внесок кінематичної взаємодії, що узгоджується з відомим результатом, отриманим раніше Бар'яхтаром із співробітниками для ізотропного випадку.

КОЛЛЕКТИВНАЯ ФУНКЦИЯ ГРИНА И ПРОДОЛЬНАЯ ДИНАМИЧЕСКАЯ ВОСПРИИМЧИВОСТЬ АНИЗОТРОПНОГО ГЕЙЗЕНБЕРГОВСКОГО ФЕРРОМАГНЕТИКА ПРИ НИЗКИХ ТЕМПЕРАТУРАХ

Ю.Г.Рудой

Р е з ю м е

Рассмотрена продольная магнитная восприимчивость, определяющая линейный отклик анизотропного ферромагнетика Гейзенберга типа “легкая ось” на наличие слабого переменного и неоднородного внешнего магнитного поля; рассмотрена область температур, малых по сравнению с температурой Кюри. Линеаризованные квантовые уравнения движения для фурье-образов продольных и поперечных компонент спиновых операторов использованы для построения динамического приближения случайных фаз (ДПСФ) для коллективной функции Грина магнонов. Аналогичное построение выполнено также с применением однопараметрического семейства преобразований от спиновых к бозе-операторам, включающего как частные случаи преобразования Дайсона—Малеева и Гольштейна—Примакова. Получено и проанализировано дисперсионное уравнение, определяющее полюса продольной динамической восприимчивости. Показано, что в длинноволновом пределе отсутствует коллективное возбуждение типа “нулевого магнона”, а также отсутствует вклад кинематического взаимодействия, что согласуется с известным результатом, найденным ранее для изотропного случая Барьяхтаром с сотрудниками.