

# SPIN-LATTICE EFFECTS IN Ni—Mn—Ga SHAPE-MEMORY ALLOYS

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The unusual magnetic properties and a recently discovered giant magnetically induced deformation of Ni—Mn—Ga shape-memory alloys have been considered. A magnetostrictive mechanism of the magnetic and magnetomechanical effects observed in Ni—Mn—Ga alloys has been substantiated, and a consistent theory of these effects has been developed starting from the fundamental conception of magnetoelasticity and commonly known principles of the theory of ferromagnetism. A quantitative agreement between the theoretical and experimental results has been achieved, and a complete adequacy of the developed theory has been proved in this way. A correspondence of the magnetostrictive mechanism to the crystallographic features of a giant magnetically induced deformation and the basic relationships of thermodynamics of solids has been discussed.

## 1. Introduction

The present paper is devoted to the application of the consistent theory of magnetoelasticity developed by Bar'yakhtar *et al.* [1–5] to ferromagnetic Ni—Mn—Ga alloys. It will be shown below that these alloys belong to the family of various magnetic solids whose properties are not only dependent but also originated and strictly controlled by magnetoelastic interaction (for more examples of this kind, see [6]).

Ferromagnetic Ni—Mn—Ga shape-memory alloys are intensively studied now due to the recent observation of the record-breaking magnetically induced deformation  $\varepsilon \sim 5\%$ , which exceeds the deformation of industrial magnetostrictive materials by two orders of magnitude [7–9]. A considerable deformation of a Ni—Mn—Ga single crystal under the action of an applied magnetic field is known as the magnetostrain effect (MSE) or magnetic shape memory (MSM). The first observation of MSE [10] initiated an avalanche-like increase of the number of papers dealing with the Ni—Mn—Ga alloys and the formulation of a few different theoretical models of giant magnetically induced deformation (see [11] and references therein).

The MSE essentially is a transformation of the microstructure of an alloy specimen in an increasing magnetic field. The microstructure arises on the

cooling of Ni—Mn—Ga alloys as a result of the first-order phase transition from the high-temperature cubic (austenitic) phase to the low-temperature tetragonal (martensitic) one. This phase transition is commonly known as the martensitic transformation (MT) of a crystal lattice. In accordance with the existing experimental data, the microstructure of specimens exhibiting MSE can be modeled by the alternating domains (variants) of a tetragonal crystal lattice which form the quasiperiodic sequence of crystallographic twins (Fig. 1). Let the axes of coordinate frame be aligned with (100) crystallographic directions and the magnetic field be applied in the  $y$  direction. The field application breaks the equivalency of twin components, initiates the growth of the volume fraction of the  $y$ -variant of martensite, and, hence, induces a deformation of the alloy specimen. An absolute value of the appropriate strain tensor component may be estimated as  $\varepsilon \sim (1 - c/a)/2$ . Moreover, the magnetic field application to the specimen, which was preliminary brought to the one-variant state, results in its twinning and subsequent detwinning. In this case, the observed deformation may be close to the theoretical limit  $1 - c/a$ .

In this paper, a magnetoelastic theory of MSE will be substantiated and its quantitative agreement with the existing experimental data will be demonstrated. It will be argued that the models of MSE, which disregard the basic principles of magnetoelasticity formulated in the fundamental monograph by Bar'yakhtar *et al.* [3], are incompatible with the fundamental principles of linear elasticity of solids and/or internally contradictory.

## 2. Magnetoelastic Model of Magnetostrain Effect

### 2.1. Preliminary considerations

A large value of the magnetostriction constant  $\lambda \approx 1.3 \times 10^{-4}$  was reported, and the reversible magnetostrictive deformation  $\varepsilon^{(me)} \approx -0.01\%$  was observed for the cubic (austenitic) phase of Ni—Mn—Ga alloy in the saturating magnetic field [10]. On the other hand, the compressive

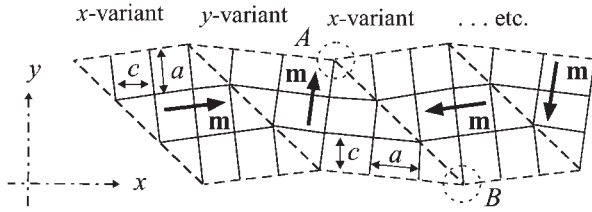


Fig. 1. Schematic representation of a twinned ferromagnetic single crystal

axial stress  $\sigma \approx 2$  MPa applied to the single-variant martensitic state along one of the  $\langle 100 \rangle$  crystallographic directions is sufficient for the accomplishment of the twinning-detwinning cycle [12]. The abnormally low values of the shear elastic modulus  $C' < 10$  GPa reported for the number of Ni—Mn—Ga alloys [13] result in the following estimation of the elastic strain needed for the accomplishment of the twinning-detwinning process:  $\varepsilon^{(f)} = \sigma/2C' > -10^{-4} = -0.01\%$ . The estimated value is equal to the magnetostrictive deformation of the cubic phase.

It may be assumed for the estimations that the magnetostriction of the tetragonal phase is close in value to the magnetostriction of the cubic one. In this case, a magnetostrictive deformation of the twinned crystal lattice exceeds the value needed for the start of the twinning-detwinning process when the increasing magnetic field exceeds some critical value. In other words, the magnetostrictive deformation triggers the twinning-detwinning process. So, the magnetoelastic interaction is the physical origin of the magnetostrain effect observed in Ni—Mn—Ga alloys [14 – 17]. This statement was verified by the qualitative and quantitative comparison of the theoretical results with experimental data [12, 17 – 19].

It can be argued, further, that the magnetoelastic model of MSE is compatible with the fundamental crystallographic conceptions of martensitic transformation [20]. According to these conceptions, a spatial orientation of twin boundaries can be definitely calculated from the values of lattice parameters, and the small changes in these values result in the appreciable reorientation of the boundaries.

The magnetoelastic model is compatible with the microscopic mechanism of MSE (see, e.g., [21]). This statement can be explained using Fig. 1: a magnetic field applied in the  $y$  direction rotates the magnetic moment of the  $x$ -variant and deforms this variant while a rotation of the magnetic moment and a magnetostrictive deformation of the  $y$ -variant is absent; in this case, the resultant lattice misfit between points  $A$  and  $B$  is

estimated as  $\Delta(AB) \approx \varepsilon^{(me)}(AB) \sim 10^{-4}(AB)$ . Thus, the lattice misfit exceeds the lattice parameter  $a$  when  $AB > 10^4 a \sim 5 \mu\text{m}$ . According to the microscopic model of MSE, the ends of twin boundaries are pinned at the cores of twinning dislocations, and the distance between the centers of pinning may be substantially larger than  $5 \mu\text{m}$ . The dimensions of cores are of the order of lattice parameter, and therefore, the estimated lattice misfit is sufficient for the rearrangement of twinning dislocations and the initiation of the detwinning process.

## 2.2. Model formulation

The cubic-tetragonal martensitic transformation is characterized by the diagonal components of the strain tensor  $\varepsilon_{ii}$ , and the nondiagonal components may be disregarded. According to the commonly recognized theory of magnetoelasticity [3], the free energy  $F$  of the cubic ferromagnetic phase is the sum of the elastic, magnetic, and magnetoelastic energies denoted as  $F_e$ ,  $F_m$ , and  $F_{me}$ , respectively. Thus,

$$F(\varepsilon_{ii}, \mathbf{M}, \mathbf{H}) = F_e(\varepsilon_{ii}) + F_m(\mathbf{M}, \mathbf{H}) + F_{me}(\varepsilon_{ii}, \mathbf{M}), \quad (1)$$

where  $\mathbf{M}$  is the magnetization vector and  $\mathbf{H}$  is a magnetic field applied to a specimen. The expression for the elastic energy is

$$F_e = 3(C_{11} + 2C_{12})u_1^2/2 + C'(u_2^2 + u_3^2)/6, \quad (2)$$

where  $C_{11}$ ,  $C_{12}$  and  $C' = (C_{11} - C_{12})/2$  are elastic moduli,

$$\begin{aligned} u_1 &= (\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz})/3, & u_2 &= \sqrt{3}(\varepsilon_{xx} - \varepsilon_{yy}), \\ u_3 &= 2\varepsilon_{zz} - \varepsilon_{yy} - \varepsilon_{xx}. \end{aligned} \quad (3)$$

The magnetic energy is expressed as [17]

$$F_m = Jy^2/2 + M^2(\mathbf{m} \cdot \mathbf{D} \cdot \mathbf{m})/2 - \mathbf{mH}M, \quad (4)$$

where the first, second, and third terms are the exchange, magnetostatic, and Zeeman energies, respectively,  $J$  is the spin exchange parameter, the dimensionless variables  $y = M(T)/M(0)$  and  $\mathbf{m} = \mathbf{M}(T)/M(T)$  characterize the absolute value and direction of the magnetization vector, respectively. The expression for the magnetoelastic energy has the form [22,23]

$$\begin{aligned} F_{me} &= -\delta_0 y^2 u_1 - \delta_1 [\sqrt{3}(m_x^2 - m_y^2)u_2 + \\ &+ (2m_z^2 - m_y^2 - m_x^2)u_3], \end{aligned} \quad (5)$$

where  $\delta_0$  and  $\delta_1$  are the magnetoelastic energy parameters characterizing the isotropic and anisotropic parts of the spin-lattice interaction. The energy terms of the fourth order in the magnetic vector and of the second order in strain tensor components are comparatively small and therefore are omitted in Eqs. (4) and (5).

The cubic-tetragonal transformation of a Ni<sub>2</sub>MnGa single crystal results in a spontaneous deformation of the cubic lattice, which is characterized by strains  $u_{2,3}^M \sim c/a - 1 \approx -0.05$ . Let a deformation of the stressed tetragonal lattice be counted off the spontaneous value, i.e.  $u_\alpha \rightarrow u_\alpha^M + u_\alpha$ ,  $\alpha = 1, 2, 3$ . In view of the rather small values of spontaneous strains, Eq. (2) for the elastic energy is approximately valid for a tetragonal lattice, i.e.  $F_e^{(t)} \approx \text{const} + F_e$  as long as  $u_\alpha^M$  values are constant. The magnetic energy is independent from strains, and therefore  $F_m^{(t)} = F_m$ . Finally, using the condition  $\mathbf{m}^2 = 1$ , the magnetoelastic energy of the  $j$ -variant of the tetragonal phase can be expressed as [22]

$$F_{me}^{(t)} = -\delta_0(\Delta V/V)y^2/3 - K_u m_j^2 + F_{me}, \quad (6)$$

where  $\Delta V/V$  is the rational change of the volume of a specimen, the magnetic anisotropy constant  $K_u = 6\delta_1(c - a)/a$  is positive for Ni—Mn—Ga alloys with  $c < a$ , because the fourfold symmetry axis is parallel to the direction of easy magnetization. Hence, the magnetoelastic parameter  $\delta_1$  is negative. Rigorously speaking, the tetragonal phase is characterized by two magnetoelastic parameters, but the difference in these parameters is of the order of  $\delta_1(1 - c/a) \ll \delta_1$ .

According to the fundamental principle of thermodynamics of solids, the equilibrium values of elastic strains correspond to a minimum of the Gibbs potential

$$G = F - (\sigma_1 u_1 + \sigma_2 u_2 + \sigma_3 u_3)/6, \quad (7)$$

where

$$\begin{aligned} \sigma_1 &= (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})/3, & \sigma_2 &= \sqrt{3}(\sigma_{xx} - \sigma_{yy}), \\ \sigma_3 &= 2\sigma_{zz} - \sigma_{yy} - \sigma_{xx}. \end{aligned} \quad (8)$$

Hence, the equilibrium values of strain tensor components satisfy the conditions

$$\partial G / \partial u_\alpha = 0. \quad (9)$$

Conditions (9) result in the Hooke's law establishing a direct proportionality between strains and stresses. It means that every stressed state of solid is elastically deformed and every elastically deformed state is stressed.

In particular, for  $\alpha = 2$ , Eq. (9) results in the relationship

$$u_2 = \left( \sigma_2^{(me)} + \sigma_2 \right) / 2C', \quad (10)$$

where  $C' = (C_{11} - C_{12})/2$  is the shear modulus and

$$\sigma_2^{(me)} = 6\sqrt{3}\delta_1(m_x^2 - m_y^2) \quad (11)$$

must be interpreted as a magnetomechanical stress which is linearly related to a magnetostrictive deformation  $u_2^{(me)} = \sigma_2^{(me)} / 2C'$ .

It should be clearly emphasized now that the magnetic anisotropy energy  $K_u m_j^2$  and Zeeman energy **mHM** cannot be the sources of a magnetomechanical stress, because both energies are not the explicit functions of strains and their partial derivatives with respect to  $u_\alpha$  are equal to zero. This means that these energies do not contribute to Eq. (9) and Hooke's law (10). Therefore, all theoretical models, which disregard magnetostrictive strains and derive a magnetomechanical stress from the anisotropy energy or Zeeman energy, are incompatible with the minimum principle for the Gibbs potential and the Hooke's law.

### 2.3. Model results

#### 2.3.1. Effects of the isotropic spin-lattice interaction

The isotropic part of the spin-lattice interaction expresses itself in an appreciable shift of the Curie temperature under the action of hydrostatic pressure; the experimental value of the shift results in the estimation  $(dT_C/dP) = 8$  K/GPa. The shift is caused by the magnetoelastic renormalization of the spin exchange parameter  $J(T) \rightarrow J^*(T) - 2\delta_0(\Delta V/V)/3$  (see Eqs. (4) and (6)). The volume change accompanies not only the compression, but also the martensitic transformation (MT) of the alloy. Due to this, the exchange parameter jumps when the temperature of a cooled/heated specimen reaches the martensitic transformation temperature  $T_M$ . This jump results in the jump of a magnetization in the saturating magnetic field. The magnetoelastic model involves the orthodox temperature dependence of the exchange parameter  $J(T) = \zeta(T - T_{CA})/T_{CA}$  and standard equations for the magnetization in the austenitic and martensitic phases

$$\begin{aligned} y(T) &= \tanh [(T_{CA}/T)y(T)], \\ y(T) &= \tanh [(T_{CM}/T)y(T)]. \end{aligned} \quad (12)$$

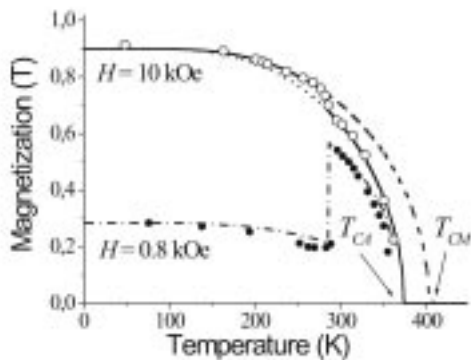


Fig. 2. Experimental (circles) and theoretical (solid line) temperature dependences of the saturation magnetization obtained for Ni–Mn–Ga alloy with  $c/a = 0.94$ ,  $T_C = 375$  K, and  $T_M = 285$  K (the dotted and dashed lines show the magnetization curves computed for the labile phases)

For the alloys with  $c < a$ , the ferromagnetic ordering temperature exceeds the MT temperature, and therefore,  $T_{CA}$  is the Curie temperature which can be determined in experimental way. The temperature

$$T_{CM} = T_{CA}[1 + 2(\delta_0/\zeta)(\Delta V/V)3] \quad (13)$$

is a characteristic parameter which prescribes the temperature dependence of a magnetization below the MT temperature.

The experimental value  $dT_{CA}/dP$  (see above) and the relationship  $\zeta = nk_B T_{CA}$  ( $n$  is the number of magnetic atoms in the unit volume) result in the estimations  $\zeta \approx 0.1$  GPa and  $\delta_0 \approx -0.4$  GPa. Moreover, the value  $\Delta V/V = -3 \times 10^{-2}$  can be accepted [24]. In this case,  $T_{CM} \approx T_{CA} + 30$  K.

The theoretical temperature dependence of the magnetization computed from Eqs. (12),(13) is presented in Fig. 2 (upper branch) together with experimental values measured in the saturating magnetic field. The computations were carried out with account of a statistical spread of “local” Curie temperatures in the spatially inhomogeneous martensitic state (for more details, see [24]). Figure 2 demonstrates an excellent agreement between the model results and experimental data.

### 2.3.2. Effects of the anisotropic spin-lattice interaction

An abrupt drop of the magnetization measured in the external magnetic field of about 1 kOe accompanies the martensitic transformation in Ni<sub>2</sub>MnGa alloy [25] (see

the lower branch in Fig. 2). The magnetoelastic model relates this effect to the first term in energy (6) and explains as follows: i) the martensitic transformation results in a tetragonal deformation of the cubic crystal lattice and the appearance of an internal anisotropy field  $H_A = 6\delta M(c - a)/a \sim 10$  kOe [22,24] (the dimensionless magnetoelastic constant  $\delta = \delta_1/M^2 \approx -23$  is introduced here for the sake of convenience); ii) for the  $x$ - and  $z$ -variants of martensite, the anisotropy field is transversally directed to the external field  $H_y$ , and the magnetic moments of these variants are almost perpendicular to the magnetic vector of the  $y$ -variant if  $H \ll H_A$ ; iii) the magnetic anisotropy of the cubic phase is small, and, therefore, the direction of the magnetic vector of a whole specimen is close to the direction of an external field, and the magnetization is approximately 3 times larger than that of the tetragonal phase.

If the experimental specimen is not magnetically saturated, the model expression for the magnetization is

$$M(H, T) = M_0 y(T, s)[(1/3)(H/H_D) + (2/3)(H/H_S)], \quad (14)$$

where  $H_D \approx 1$  kOe is the demagnetization field and  $H_S \approx H_A = 10$  kOe is the field of magnetic saturation (for more details, see [17,22,24]). The theoretical dependence  $M(T)$  computed from Eq. (14) for  $H = 0.82$  kOe agrees with the experimental one (see Fig. 2).

The magnetoelastic model relates a giant field-induced deformation of Ni–Mn–Ga alloy to the field-induced stress defined as the difference  $\sigma_2^{(me)}(H) - \sigma_2^{(me)}(0)$ . As it was proved in [12], the application of a field  $\mathbf{H} \parallel y$  to the martensitic structure depicted in Fig. 1 is physically equivalent to an axial compression in the  $y$  direction when the compressive stress is equal to

$$\sigma_{yy}^{(eq)}(H) = [\sigma_2^{(me)}(H) - \sigma_2^{(me)}(0)]/2\sqrt{3}. \quad (15)$$

The equivalent stress may be computed from Eqs. (4), (6), (11), and (15). Equations (4) and (6) result in the linear dependence  $m_x(H) = H/H_S$ , where  $H_S = (2K_u/M) + (D_{yy} - D_{xx})M$ , and the theoretical field dependence of the stress is quadratic. The quadratic dependence which was computed for  $M(H_S) = 0.59$  T and  $\mu_0 H_S = 5$  T is presented in Fig. 3. In the field range  $\mu_0 H_S < 3$  T, this dependence agrees with the experimental equivalent stress determined from the stress–strain loops taken in an external magnetic field (details of the experimental procedure are described in [12]).

However, the experimental dependences  $M_x(H)$  show that the function  $m_x(H)$  is not linear as a rule

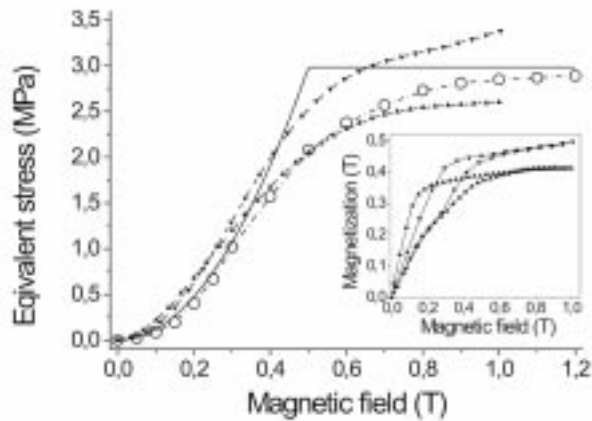


Fig. 3. Equivalent stress found from the experimental stress-strain loops [12] (open circles), Eq. (15) and linear field dependence of magnetization (solid line), Eq. (15) and experimental magnetization curves [26] (triangles). The magnetization curves measured in the increasing field (the right branches of the loops presented in the inset) were used for computation

(see the inset in Fig. 3). To account this fact, the equivalent stresses were computed also from the experimental magnetization curves (measured in [26]) on the assumption that  $m_x(H) = M_x(H)/M_x(H_S)$ . The magnetization branches obtained in the increasing field were used in computations. The resultant field dependences of the equivalent stress reasonably agree with experimental values (see Fig. 3) despite the fact that the measurements of stress and magnetization were carried out for different alloys, and hence, the comparison has the illustrative (nonrigorous) character.

When the model function of the equivalent stress fitted to experimental values is known, a sufficiency of the magnetostrictive deformation for the transformation of twinned martensite and triggering of MSE can be proved. To this end, the experimental strain–stress and strain–field dependences would be used. The strain–stress dependence shows that the transformation starts at the stress value  $\sigma^{(s)} \approx 0.9$  MPa and finishes at  $\sigma^{(f)} \approx 1.75$  MPa (see Fig. 4). Moreover, the final segment of a stress–strain loop enables the evaluation of the elastic stiffness of a martensitic specimen  $S \approx 1.9$  GPa (see the inset in Fig. 4) and the extraction of the elastic part of the total deformation. The elastic deformation which was found in such a way is shown in Fig. 5 by a straight line. The elastic strains  $\varepsilon^{(s)} \approx 4.5 \times 10^{-4}$  and  $\varepsilon^{(f)} \approx 9.2 \times 10^{-4}$ , corresponding to the start and finish of the detwinning process are shown by horizontal dashed

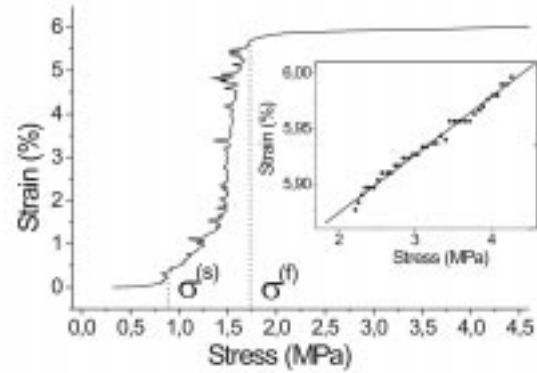


Fig. 4. Experimental stress-strain curve taken for Ni–Mn–Ga alloy [12] and a linear extrapolation of its final segment

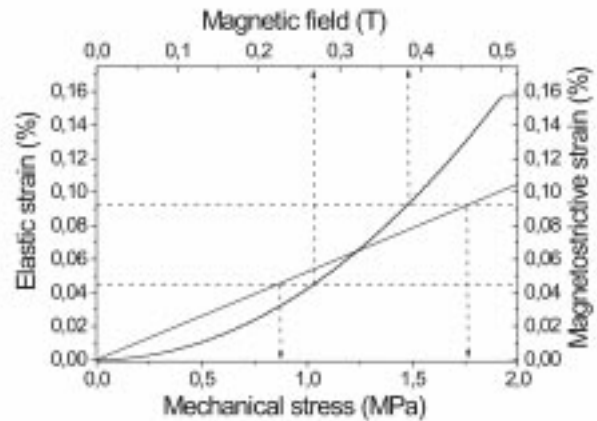


Fig. 5. Elastic part of the strain obtained from the final segment of the stress-strain loop (straight solid line), stresses, and elastic strains corresponding to the start and finish of detwinning process (down arrows and horizontal dashed lines, respectively), magnetostrictive strain (curve) and magnetic field values (up arrows) corresponding to the start and finish of a magnetically induced deformation

lines. (These strains exceed the values mentioned in Section 2.1 due to a very low value of the stiffness coefficient inherent in the studied alloy specimen). Further, the magnetoelastic strain

$$u^{(eq)}(H) \equiv \sigma^{(eq)} / S \quad (16)$$

was computed and depicted by the curve in Fig. 5. The cross points of this curve with the horizontal dashed lines result in the magnetic field values  $H^{(s)} = 0.27$  T and

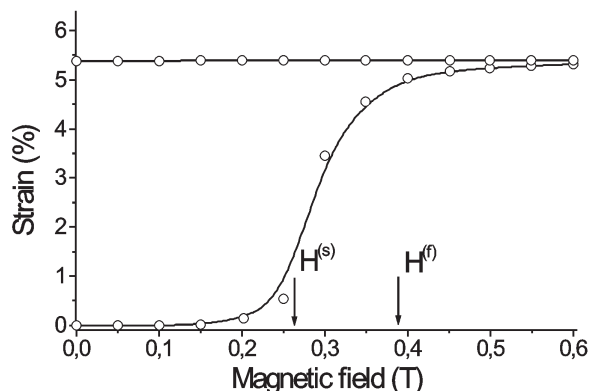


Fig. 6. Giant magnetically induced deformation versus magnetic field (the field values determined from the elastic and magnetostrictive strains are shown by the arrows)

$H^{(f)} = 0.38$  T, which must to coincide with the fields of the start and finish of a magnetically induced deformation of the specimen.

The experimental strain—field curve measured for this specimen by P. Müllner is shown in Fig. 6. The estimated values  $H^{(s)}$  and  $H^{(f)}$  are shown by the arrows. It is seen that these values approximately correspond to the start and finish of a magnetically induced deformation of the specimen.

### 3. Discussion

**3.1.** The magnetoelastic model states that the giant magnetically induced deformation of Ni—Mn—Ga martensitic alloys is originated by a large magnetostriction inherent in them. The model enables the evaluation of both a magnetostrictive elastic strain and a magnetomechanical stress which is linearly related to the strain in accordance with the Hooke's law. The results presented in Figs. 4—6 prove that the magnetostrictive deformation is sufficiently large for the accomplishment of the twinning/detwinning cycle in the single-variant state of the experimental specimen due to a low value of the elastic stiffness of the alloy. It should be emphasized that these results may be reformulated in terms of a magnetomechanical stress using the Hooke's law expressed by Eq. (16). In contrast to this, the theoretical models which disregard the magnetoelastic interaction and deduce the magnetomechanical stress from the difference of the magnetic anisotropy energies

as

$$\sigma^{(mi)} = (K_u m_x^2 - K_u m_y^2)/(1 - c/a) \quad (17)$$

are internally contradictory because

- i) in the absence of the magnetoelastic interaction, a magnetoelastic strain is equal to zero and, according to the Hooke's law, the stiffness coefficient of an alloy must be equal to infinity, to provide the finite stress (17);
- ii) stress (17) tends to infinity when  $1 - c/a$  goes to zero, and this means, that Eq. (17) is obviously wrong in this limiting case.

However, the magnetoelastic model “saves” relationship (17) when it establishes a direct proportionality between  $K_u$  and  $1 - c/a$ : in this case,  $1 - c/a$  vanishes from the formula. That's why this relationship often gives rise to the reasonable results. In particular, the form of a magnetically induced stress function, which was determined in [27] using Eq. (17) and experimental magnetization curves, is close enough to the functions presented in Fig. 3.

**3.2.** The magnetoelastic model proves that the magnetically induced stress is a quadratic function of the magnetic field. This conclusion is very important in view of the numerous publications reporting the experimental observation or/and theoretical substantiation of the linear stress — field or strain — field relationship. For example, Eq. (2) in (29) yields, in the case of a rather low magnetic field, the relationship

$$\varepsilon(H) = M_S H/(C\varepsilon_0), \quad (18)$$

where  $M_S$  is the saturation magnetization,  $C$  is the elastic modulus, and  $\varepsilon_0$  is the MT strain.

Strain (18) obviously tends to infinity when  $\varepsilon_0$  tends to zero. Moreover, the linear dependence of a giant magnetostrain cannot be correct because in the reality the experimentally measured value of deformation is a diagonal component of the symmetric tensor. If this component is linearly related to the magnetic field, it must have a form  $\varepsilon_{yy} = B_y H_y$  where  $B_y$  is a component of some vector, which is independent of the field. Let  $B_y$  be positive, for the sake of definiteness. In this case, the magnetic field applied in parallel to the  $y$ -axis ( $H_y > 0$ ) results in the expansion of the specimen in the  $y$ -direction because  $\varepsilon_{yy} > 0$ , while a field applied in the opposite direction ( $H_y < 0$ ) results in the contraction of the specimen in the same direction, because  $\varepsilon_{yy} < 0$  in this case. However, this cannot be true for a giant magnetically induced deformation because, for the both field directions, this deformation is caused by an increase of the volume fraction of the  $y$ -variant of martensite and contraction of the specimen in the  $y$ -direction (the

inequality  $K_u > 0$  inherent in Ni—Mn—Ga alloys with  $c/a < 1$  is accepted).

It can be concluded therefore that the development of a consistent theory of ferromagnetic shape-memory alloys is possible only with the proper regard of the fundamental principles of magnetoelasticity elaborated by Bar'yakhtar *et al.* [1–5].

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#### СПИН-ГРАТКОВІ ЕФЕКТИ У СПЛАВАХ З ПАМ'ЯТТЮ ФОРМИ Ni—Mn—Ga

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#### Резюме

Розглянуто незвичайні властивості та нещодавно відкриту велетенську магнітоіндуковану деформацію сплавів Ni—Mn—Ga. Виходячи з фундаментального поняття магнітопружності та загальновідомих принципів теорії магнетизму, обґрунтовано магнітострикційний механізм магнітних та магнітомеханічних ефектів, спостережених у сплавах Ni—Mn—Ga, та побудовано послідовну теорію цих ефектів. Досягнуто кількісного узгодження теоретичних результатів з експериментальними, і у такий спосіб доведено цілковиту адекватність побудованої теорії. Обговорено відповідність магнітострикційного механізму кристалографічним особливостям велетенської магнітоіндукованої деформації та основним співвідношенням термодинаміки твердого тіла.

#### СПИН-РЕШЕТОЧНЫЕ ЭФФЕКТЫ В СПЛАВАХ С ПАМ'ЯТТЮ ФОРМЫ Ni—Mn—Ga

В.А. Львов

#### Резюме

Рассмотрены необычные свойства и недавно открытая гигантская магнитоиндуцированная деформация сплавов Ni—Mn—Ga. Исходя из фундаментального понятия магнитоупругости и общеизвестных принципов теории магнетизма, обоснован магнитоострикционный механизм магнитных и магнитомеханических эффектов, наблюдаемых в сплавах Ni—Mn—Ga, а также построена последовательная теория этих эффектов. Достигнуто количественное согласие теоретических результатов с экспериментальными, и таким путем доказана полная адекватность построенной теории. Обсуждено соответствие магнитоострикционного механизма кристаллографическим особенностям гигантской магнитоиндуцированной деформации и основным соотношениям термодинамики твердого тела.