INTRASUBBAND PLASMONS IN A FINITE ARRAY OF QUANTUM WIRES CONTAINING A DISPLACED QUANTUM WIRE

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The paper deals with the theoretical investigation of intrasubband plasmons in an array of quantum wires (QWs) consisting of a finite number of QWs with the same one-dimensional density of electrons. We consider that all QWs are arranged at an equal distance from each other except one QW, which is displaced from its position of periodicity. The existence of two local plasmon modes in the plasmon spectrum is found. It is shown that, under certain conditions, the existence of plasmon modes, whose spectrum does not practically depend on the position of the displaced QW in the array, is possible.

1. Introduction

Quasi-one-dimensional electron systems (1DES) or QWs are artificial structures, in which the motion of charge carriers is confined in two transverse directions but is essentially free (in the effective mass sense) in the longitudinal direction [1—3]. Usually, QWs are produced by adding an additional one-dimensional confinement of a two-dimensional electron system (2DES). This additional confinement is, in general, weaker than the strong confinement of original 2DES [4]. One of the motivations to study QWs is the fact that the mobility of charge carriers is higher than that in 2DES, on which they are built. The reason for this is that the impurity content and distribution around QWs can be selectively controlled, producing the enhanced mobility [5].

Collective charge-density excitations or plasmons in QWs are the objects of great physicist's interest. Earlier, plasmons in QWs were investigated both theoretically [5–9] and experimentally [10, 11, 13]. In those works, it was shown that plasmons in QWs have some new unusual dispersion properties. First, the plasmon spectrum strongly depends on the width of QWs. Secondly, 1D plasmons are free from the Landau damping [6,9] in the whole range of wavevectors.

From the point of view of practical applications, the so-called weakly disordered arrays of low-dimensional systems are the objects of interest. So, the plasmons in weakly disordered superlattices formed of a finite number of two-dimensional electron systems (2DES) have been theoretically investigated [14—20]. The weakly disordered superlattice is characterized by the fact that all 2DES are arranged periodically and possess the equal density of electrons except one 2DES, which can possess the density of electrons different from other 2DES ("defect" 2DES) [14—17] or be displaced from the position of periodicity (displaced 2DES) [18—20]. It was found that the plasmon spectrum of such a superlattice contains the local plasmon mode (LPM), whose properties differ from those of other plasmon modes. The existence of LPM is completely analogous to the existence of a local phonon mode, first obtained by Lifshitz in 1947 for the problem of the phonon modes in a regular crystal containing a single isotope impurity [21]. In a weakly disordered superlattice containing defect 2DES, the LPM lies in the higher- or lower-frequency region in comparison with the other plasmon modes depending on whether the defect 2DES have higher or lower electron density, correspondingly. In a weakly disordered superlattice containing displaced 2DES, the LPM lies in the lower-frequency region only in comparison with the other plasmon modes. Notice that, as was shown in [17,20], practically the whole flow of the electromagnetic energy of plasmons, which correspond to the LMP, is concentrated in the vicinity of a defect or displaced 2DES. At the same time, works [17, 20] indicated the opportunity to determine the parameters of defects in the superlattice using peculiarities of the plasmon spectrum.

Plasmons in a finite weakly disordered array of QWs containing a defect QW have been earlier investigated theoretically in works [22, 23]. It has been supposed that the defect QW can occupy an arbitrary position in the array. It was shown in works [22, 23] that the position of the defect QW in the array does not strongly affect the spectrum of the local plasmon mode but it exerts an significant influence on the spectrum of other plasmon modes. At the same time, when the defect QW is arranged inside the array, the plasmon spectrum contains modes, whose dispersion properties do not depend on the value of the electron density in the defect QW.

This paper deals with the theoretical investigation of plasmons in a finite weakly disordered array of QWs containing one QW displaced from the position of periodicity. It is found that the two LPMs exist in the plasmon spectrum: one of the LPMs lies in the higher-frequency region in comparison with the other plasmon modes and another LPM lies in the lower-frequency region. This situation is distinct from the case of an array of QWs with one defect QW, where only one LPM exists [22]. It is shown that, under certain conditions, the existence of plasmon modes, whose spectrum does not practically depend on the position of the displaced QW, is possible.

2. Dispersion Relation

We consider the weakly disordered array of QWs consisting of a finite number M of QWs arranged at planes $z = z_l$ ($l = 0,...,M-1$ is the number of QWs). The 1D density of electrons, N, is supposed to be equal in all QWs. We consider that all QWs in the array are arranged periodically with period d except one interior QW (with the number $p = 1,...,M-2$) which is displaced relative to the position of periodicity by the distance Δ. So, the z-coordinate of the l-th QW can be expressed as $z_l = ld + \delta_{pl}\Delta$. Here, δ_{pl} is the Kronecker delta. QWs are considered to be placed into the uniform dielectric medium with the dielectric constant ε. We consider the movement of electrons is free in the x-direction and is considerably confined in the directions y and z. At the same time we suppose that the width of all QWs is equal to a in the y-direction and is equal to zero in the z-direction.

In other words, each QW can be represented as a square quantum well with infinite barriers at $y = -a/2$ and $y = a/2$ and a zero thickness in the z-direction. Meanwhile, we take into account only the lowest subband in each QW. In this case, the single-particle wave function of an electron can be written as

$$\psi_{k_x,l}(\mathbf{r}) = |k_x, l\rangle = \frac{e^{ik_x x}}{\sqrt{2\pi}}\varphi(y)\left[\delta(z - z_l)\right]^{1/2}, \qquad (1)$$

where $\varphi(y) = \sqrt{\dfrac{2}{a}}\cos\dfrac{\pi y}{a}$, and k_x is the one-dimensional wave vector describing the motion in the x-direction. In this case, the single-particle energy $E_{k_x,l} = E_0 + \frac{\hbar^2 k_x^2}{2m^*}$. Here, E_0 is the energy of the subband bottom (for simplicity, we can put $E_0 = 0$), and m^* is the effective mass of an electron.

To obtain the spectrum of collective excitations, we start with a standard linear-response theory in the random phase approximation. We consider $\delta n(\mathbf{r})$ which is a deviation of the electron density from its equilibrium value. After using the standard linear-response theory and the random phase approximation, $\delta n(\mathbf{r})$ can be related to the perturbation by

$$\delta n(\mathbf{r}) = \sum_{\alpha,\alpha'} \frac{f_{\alpha'} - f_\alpha}{E_{\alpha'} - E_\alpha + \hbar\omega} V_{\alpha\alpha'}\psi^*_{\alpha'}(\mathbf{r})\psi_\alpha(\mathbf{r}), \qquad (2)$$

where $\alpha = (k_x, l)$ is a composite index which is defined by (1) f_α is the Fermi distribution function, $V_{\alpha,\alpha'} = \langle\alpha|V|\alpha'\rangle$ are the matrix elements of the perturbing potential $V = V^{\rm ex} + V^{\rm H}$, and $V^{\rm ex}$ and $V^{\rm H}$ are the external and Hartree potentials, respectively.

Note that the matrix elements of the Hartree potential can be expressed through the perturbation [6] as

$$V^{\rm H}_{\beta\beta'} = \frac{e^2}{\varepsilon}\int d\mathbf{r}\psi^*_\beta(\mathbf{r})\psi_{\beta'}(\mathbf{r})\int \frac{d\mathbf{r_1}}{|\mathbf{r} - \mathbf{r}_1|}\delta n(\mathbf{r}_1). \qquad (3)$$

Here, $\beta = (k_{1x}, n)$. Substituting (2) in (3), we get

$$V^{\rm H}_{\beta\beta'} = \sum_{\alpha\alpha'} W_{\beta\beta'\alpha\alpha'}\frac{f_{\alpha'} - f_\alpha}{E_{\alpha'} - E_\alpha + \hbar\omega}\left[V^{\rm ex}_{\alpha\alpha'} + V^{\rm H}_{\alpha\alpha'}\right], \qquad (4)$$

where

$$W_{\beta\beta'\alpha\alpha'} = \frac{e^2}{\varepsilon}\int d\mathbf{r}\psi^*_\beta(\mathbf{r})\psi_{\beta'}(\mathbf{r})\times$$

$$\times\int\frac{d\mathbf{r_1}}{|\mathbf{r} - \mathbf{r}_1|}\psi^*_{\alpha'}(\mathbf{r}_1)\psi_\alpha(\mathbf{r}_1) =$$

$$= \frac{\delta(k_x + q - k'_x)}{2\pi}\delta_{n,n'}\delta_{l,l'}U_{n,l}, \qquad (5)$$

$$U_{n,l} = \frac{8e^2}{\varepsilon a^2}\int\limits_{-a/2}^{a/2}{\rm d}y_1\cos^2\left(\frac{\pi y_1}{a}\right)\int\limits_{-a/2}^{a/2}{\rm d}y\cos^2\left(\frac{\pi y}{a}\right)\times$$

$$\times K_0\left(q\left[(y - y_1)^2 + (z_n - z_l)^2\right]^{1/2}\right), \qquad (6)$$

$q = k'_{1x} - k_{1x}$, $K_0(x)$ is the zero-order modified Bessel function of the second kind. After some algebra, expression (4) can be rewritten as

$$V^{\rm H}_n = 2\sum_l \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} dk_x \frac{f_{k_x+q,l} - f_{k_x,l}}{E_{k_x+q,l} - E_{k_x,l} + \hbar\omega}\times$$

$$\times\left[V_l^{\mathrm{ex}}+V_l^{\mathrm{H}}\right]U_{n,l}, \tag{7}$$

where $V_n^{\mathrm{H}}\equiv\langle k_{1x},n|V^{\mathrm{H}}|k_{1x}+q,n\rangle$, $V_l^{\mathrm{H}}\equiv\langle k_x,l|V^{\mathrm{H}}|k_x+q,l\rangle$, $V_l^{\mathrm{ex}}\equiv\langle k_x,l|V^{ex}|k_x+q,l\rangle$. The factor 2 before the summation symbol in (7) comes from the spin degeneracy.

Collective excitations of the array of QWs exist when Eq. (7) has a nonzero solution V^{H} in the case where the external perturbation $V^{\mathrm{ex}}=0$. Hence, the intrasubband plasmon dispersion relation takes the form

$$\det\left|\delta_{n,l}-\Pi^l U_{n,l}\right|=0, \tag{8}$$

where $\Pi^l=\dfrac{1}{\pi}\displaystyle\int_{-\infty}^{\infty}dk_x\frac{f_{k_x+q,l}-f_{k_x,l}}{E_{k_x+q,l}-E_{k_x,l}+\hbar\omega}$ is the noninteracting 1D polarizability ("bare bubble") function. At the zero temperature, the function Π^l can be written as

$$\Pi^l=\frac{m^*}{q\pi\hbar^2}\ln\frac{\omega^2-\left(\hbar qk_{\mathrm{F}}/m^*-\hbar q^2/2m^*\right)^2}{\omega^2-\left(\hbar qk_{\mathrm{F}}/m^*+\hbar q^2/2m^*\right)^2}. \tag{9}$$

Here, $k_{\mathrm{F}}=\dfrac{\pi N}{2}$ is the Fermi wavenumber in a QW. In the long-wavelength limit (where $q\to 0$), the function Π^l can be written as $\Pi^l=\dfrac{N}{m^*}\dfrac{q^2}{\omega^2}$.

It should be noted that, for $M=2$, the dispersion relation (8) coincides with that for plasmons in a double-layer system of QWs [6].

3. Numerical Results

Fig. 1 shows the intrasubband plasmon spectrum (solid lines) in the array of QWs with parameters $M=5$, $d=15a^*$, $a=20a^*$ ($a^*=\varepsilon\hbar^2/m^*e^2$ is the effective Bohr radius), $p=2$ and for the value of the QW displacement $\Delta=0.5d$. The y-axis gives the dimensionless frequency ω/ω_0 ($\omega_0^2=2Ne^2/\varepsilon m^*a^2$ is the plasma frequency), and the x-axis gives the dimensionless wavevector qa^*. For comparison, we present in Fig. 1 (by dashed curves *1* and *2*) also the dispersion curves for the intrasubband plasmons in a QWs array consisting of two QWs with the same parameters as was described above except the distance between QWs which is supposed to be equal to $d-\Delta$, i.e. the distance between the displaced QW and adjacent QW closest to it.

As seen from Fig. 1, the intrasubband plasmon spectrum in the finite array of QWs contains M modes. Thus, the number of modes in the spectrum is equal to the number of QWs in the array [14]. The plasmon frequency ω increases with the wavenumber q. At the same time the propagation of plasmons in the array of

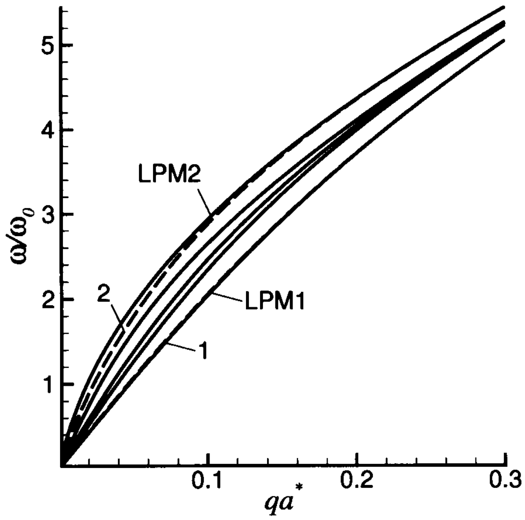

Fig. 1. Spectrum of intrasubband plasmons (solid curves) in the array of QWs with parameters $M=5$, $d=15a^*$, $a=20a^*$, $p=2$ and for the value of the QW displacement $\Delta=0.5d$

QWs with one displaced QW is characterized by the presence of two LPM, one of which (LPM1) lies in the lower-frequency region in comparison with the other plasmon modes and the other one (LMP2) lies in the higher-frequency region. It should be emphasized that, at large values of q when the Coulomb interaction between electrons in adjacent QWs is negligible, the LPM dispersion curves are close to those for the plasmons in the array consisting of two QWs, the distance between which is equal to $d-\Delta$.

Let us consider now the dependence of the intrasubband plasmon spectrum on the value of the QW displacement. This dependence is depicted in Fig. 2 for the fixed value of the wavenumber $qa^*=0.05$ and for different numbers of the displaced QW in the array: $p=1$ (Fig. 2,*a*), $p=2$ (Fig. 2,*b*) and $p=3$ (Fig. 2,*c*). The y-axis gives the dimensionless frequency ω/ω_0 and the x-axis gives the dimensionless displacement Δ/d. As seen from Fig. 2, the LPM1 frequency monotonically decreases when the displaced QW is moved away from the position of periodicity (when the value of $|\Delta|$ increases). In addition, the LPM1 spectrum is weakly dependent on the number of the displaced QW in the array: at a fixed value of Δ, changing the number of the displaced QW in the array does not affect significantly the LPM1 frequency (this fact is evident from the comparison of Figs. 2,*a*,*b*,*c*). However, the other plasmon mode spectra depend strongly on the number of the displaced QW in the array. We note that the

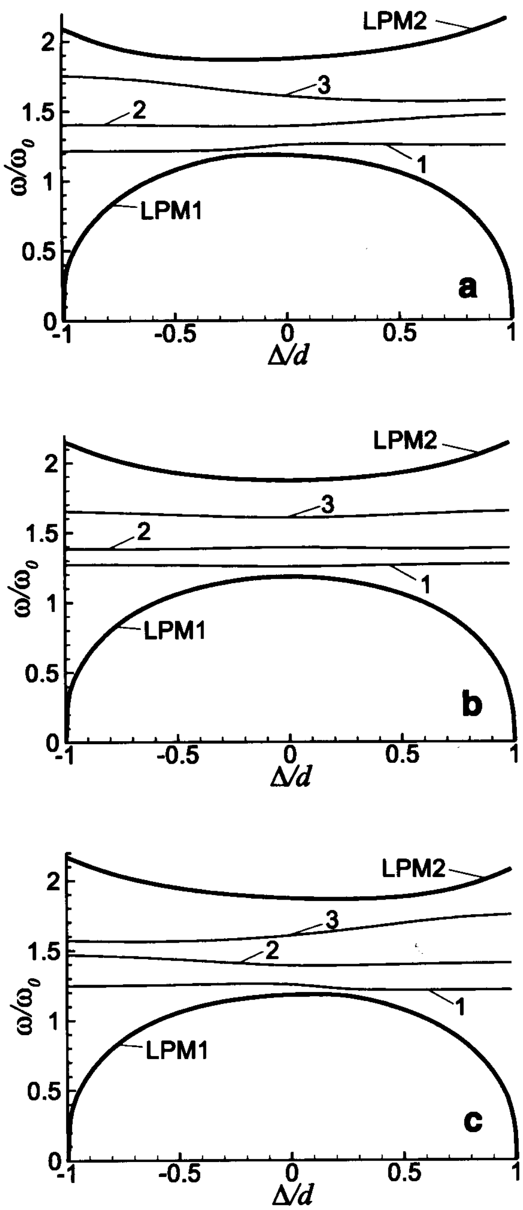

Fig. 2. Dependence of the intrasubband plasmon spectrum upon the value of the QW displacement for $M = 5$, $d = 15a^*$, $a = 20a^*$, $qa^* = 0.05$ and for different numbers of the displaced QW in the array: $p = 1$ (a), $p = 2$ (b) and $p = 3$ (c)

LPM2 frequency increases when the central ($p = 2$) QW in the array is displaced from its position of periodicity (with an increase of the value of $|\Delta|$) (Fig. 2,b). At the same time, the dependence of the LPM2 frequency on the displacement Δ in the case where $p = 1$ has a minimum at $\Delta/d \approx -0.2$ (Fig. 2,a). Likewise, when $p = 3$, the dependence of the LPM2 frequency on the value of Δ has the minimum at $\Delta/d \approx 0.2$ (Fig. 2,c). It should be noted that the intrasubband plasmon spectrum in the array of QWs with one displaced QW contains modes, the frequency of which does not practically vary with a variation of the Δ. In the case where $p = 1$ (Fig. 2,a) or $p = 3$ (Fig. 2,c), modes 1 and 2 have this property (at $p = 1$, mode 2 has this property only when $\Delta < 0$, but mode 2 has this property at $p = 3$ only when $\Delta > 0$). At the same time, when the central QW in the array ($p = 2$) is displaced from its position of periodicity (Fig. 2,b), the frequencies of all plasmon modes except LPM1, LPM2 almost do not change. Summing up, it should be emphasized that at any number of displaced QW in the array the frequency of plasmon mode 1 (Fig. 2,a—c) is weakly sensitive to the value of the QW displacement Δ.

The reason for the weak sensitivity of the frequency of some plasmon modes to the QW displacement is evident from Fig. 3 which presents the spatial distribution of the Hartree potential for different plasmon modes and for different numbers of the displaced QW in the array: $p = 1$, plasmon mode 1 (Fig. 3,a); $p = 2$, plasmon mode 1 (Fig. 3,b); $p = 3$, plasmon mode 1 (Fig. 3,c) and plasmon mode 2 (Fig. 3,d). All depicted Hartree potential distributions are calculated in the case where $\Delta/d = 0.5$. The vertical axis in Figs. 3,a—d gives the dimensionless Hartee potential $V^{\mathrm{H}}(q, y, z)/V^{\mathrm{H}}(q, 0, 0)$, and the horizontal axes give the dimensionless coordinates z/d and y/a. Positions of QWs in the array are depicted in Figs. 3,a—d by vertical rectangles. As can be seen from Fig. 3, the spatial distributions of Hartree potentials for the chosen plasmon modes are characterized by the fact that the absolute values of the Hartree potential in the vicinity of the displaced QW are much less than ones in the other part of the array. So, the electromagnetic field of plasmons, corresponding to that modes, is concentrated for the most part outside the defect region, and this fact gives rise to a weak dependence of this mode frequency on the QW displacement Δ.

The spatial distribution of the Hartree potential of the LMPs is depicted in Fig. 4. These spatial distributions are calculated for LPM1 (Fig. 4,a) and for LPM2 (Fig. 4,b) in the case where $p = 2$, $\Delta/d = 0.5$. As seen from Fig. 4, the particularity of the Hartree potential spatial distribution of the LMPs is the fact that the LPM electromagnetic field is localized mainly inside the narrow interval between the displaced QW and the adjacent QW closest to it. It should be emphasized that the signs of the Hartree potentials of LPM1 in the vicinity of the displaced QW and in the vicinity of the adjacent QW closest to it are opposite. So, the LPM1 Hartree potential spatial distribution is similar to the

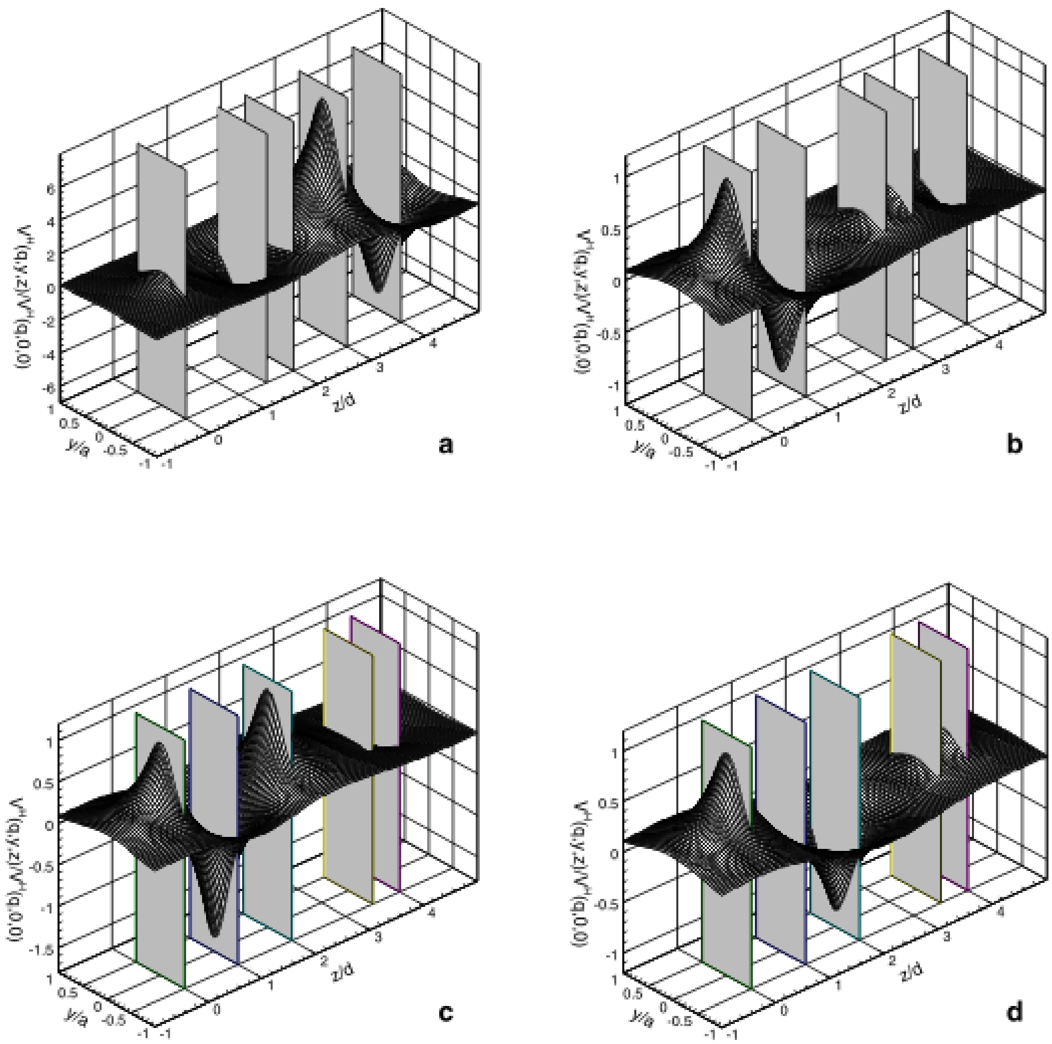

Fig. 3. Spatial distribution of the Hartree potential for different plasmon modes and for different numbers of the displaced QW in the array with parameters $M = 5$, $d = 15a^*$, $a = 20a^*$, $\Delta = 0.5d$: (a) $p = 1$, plasmon mode 1; (b) $p = 2$, plasmon mode 1; (c) $p = 3$, plasmon mode 1; (d) $p = 3$, plasmon mode 2

antisymmetric plasmon mode in the array consisting of two QWs (see Fig. 1). At the same time, the signs of the Hartree potentials of LPM2 in the vicinity of the displaced QW and in the vicinity of the adjacent QW closest to it are the same (Fig. 4,b), and that mode is similar to the symmetric mode in the array consisting of two QWs.

4. Conclusion

We have calculated the plasmon spectrum of the finite array of QWs which contains one QW, displaced from its position of periodicity. It is found that two LPMs whose properties differ from those of other modes exist in the plasmon spectrum: one of the LPMs lies in the higher-frequency region in comparison with the other plasmon modes and another LPM lies in the lower-frequency region. We point out that the spectrum of the low-frequency LPM is slightly sensitive to the number of the displaced QW in the array. It is shown that, under certain conditions, the existence of plasmon modes, whose spectrum does not practically depend on the position of the displaced QW, is possible. The spatial distribution of the Hartree potential for those modes has

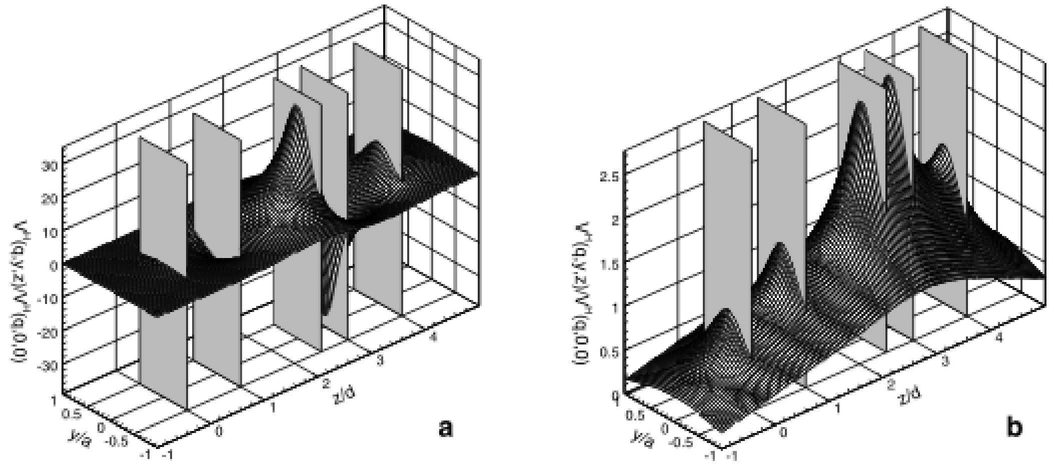

Fig. 4. Spatial distribution of the Hartree potential for LPMs in the QWs array with parameters $M = 5$, $d = 15a^*$, $a = 20a^*$, $\Delta = 0.5d$, $p = 2$: (a) LPM1; (b) LPM2

a peculiarity, the absolute value of the Hartree potential in the vicinity of the defect in the array being negligible. Therefore, the value of the QW displacement does not exert a significant influence on the dispersion properties of plasmon modes.

To conclude, it should be emphasized that the above-mentioned features of plasmon spectra can be used for the diagnostics of defects in QW structures. Hence, the fact of the presence of two LPMs can give an information about the type of a defect (a displacement of one of the QWs in the array from its position of periodicity), and the frequency of LPMs can be used for the determination of the QW displacement.

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ВНУТРІШНЬОПІДЗОННІ ПЛАЗМОНИ У СКІНЧЕННОМУ МАСИВІ КВАНТОВИХ ДРОТІВ ЗІ ЗМІЩЕНИМ КВАНТОВИМ ДРОТОМ

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Р е з ю м е

Теоретично досліджено внутрішньопідзонні плазмони у масиві квантових дротів, який містить скінченну кількість квантових дротів з однаковою одновимірною концентрацією електронів. Вважалося, що усі квантові дроти розташовані на однаковій відстані один від одного за винятком одного, зміщеного зі свого положення періодичності. Виявлено існування у спектрі плазмонів двох локальних плазмонних мод. Показано, що за певних умов можливе існування плазмонних мод, спектр яких практично не залежить від положення зміщеного квантового дроту у масиві.