INTEGRABLE COSMOLOGICAL MODELS IN THE EINSTEIN — CARTAN THEORY WITH TWO SOURCES OF TORSION

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In the framework of the two-torsion Einstein–Cartan theory (ECT), spatially flat cosmological models with a nonminimally coupled scalar field, perfect fluid, and stiff one are considered. Exact partial solutions for the above models are obtained for an arbitrary coupling constant. It is shown that both singular and nonsingular models are possible depending on the type of scalar field. For the obtained solutions, some restrictions on the coupling constant are found, and the case of two sources of torsion is discussed.

1. Introduction

The creation of a physical picture of the world is now connected with the construction of a unified theory of all fundamental physical interactions, in particular. An important stage towards it is the development of a gauge theory of gravity. In the framework of this program, the Poincaré gauge theory of gravity is gaining a great topicality, its simplest variant being the ECT, in particular. The ECT is an extension of General Relativity (GR) to a space-time with torsion and it reduces to GR when torsion vanishes. The ECT finds applications in cosmology, the particle theory and the theory of strong interactions (see, e.g., [1—3] and references therein).

Recently, we have proposed a variant of the two-torsion ECT, in which a perfect fluid and a nonminimally coupled scalar field are the sources of torsion [4]. As an application of the theory, static models [4, 5] and cosmological ones [6—8] for matter distributions in the two-torsion ECT have been considered.

In the Minkowski space for a negative nonminimal coupling constant, soliton-like solutions for the scalar field with polynomial potential were obtained [4]. It was shown that the case of axion field is specific. For each solution, a measure of the scalar field localization was determined.

In [5], the exact partial static spherically symmetric solutions in the one-torsion ECT and the two-torsion one were obtained, and some consequences of the version with two sources of torsion were discussed.

The exact partial solutions for all the types of Friedman's metrics for the universes filled with a nonminimally coupled scalar field with nonlinear potential, perfect fluid, and ultrarelativistic gas were derived in [6]. It was shown that, in the one-torsion case where the torsion is generated by a perfect fluid with the vacuum equation of state, only singular models are possible for spatially flat and open cosmological ones. Both singular and nonsingular models are possible when the second source of torsion generated by a nonminimally coupled scalar field is taken into account.

In [7, 8], the cosmological models with a nonminimally coupled scalar field with nonlinear potential and two perfect fluids, one of which is a stiff fluid and the other is the source of torsion were considered. The exact partial solutions for two-torsion models have been derived for spatially flat [7] and closed [8] models. The role of each sources of torsion in the evolution of models has been elucidated.

In this work, we analyze the other exact partial solutions for the spatially flat models with a nonminimally coupled scalar field with nonlinear potential, perfect fluid, and stiff one.

2. Basic Equations

The Lagrangian Ł of the model is chosen as the sum of Lagrangians of the gravitational field $Ł_g$, scalar field $Ł_s$, and perfect fluids: $Ł_{\mathrm{fl}(1)}$ and $Ł_{\mathrm{fl}(2)}$:

$$Ł_g = -R(\Gamma)/2æ, \tag{1}$$

$$Ł_s = \frac{1}{4\pi}\Big\{\frac{\alpha_s}{2}\Big[\Phi_{,k}\Phi^{,k} + \xi R(\Gamma)\Phi^2\Big] - V(\Phi)\Big\}, \tag{2}$$

$$Ł_{\mathrm{f\,l}(1)} = -\rho(c^2 + \Pi(\rho, e)) + k \overset{\Gamma}{\nabla}_i (\rho u^i) +$$
$$+ k_1(u_i u^i - 1) + k_2 u^i \partial_i X + k_3 u^i \partial_i e \, . \tag{3}$$

Here, $R(\Gamma)$ is the curvature scalar obtained from the full connection $\Gamma^k_{ij} = \{^{\,k}_{ij}\} + S_{ij\cdot}^{\ \ \,k} + S^k_{\cdot ij} + S^k_{\cdot ji}$; $\{^{\,k}_{ij}\}$ are

Christoffel symbols of the second kind; $S_{ij\cdot}^{\ \ k} = \Gamma^k_{[ij]}$ is the torsion tensor; $æ = 8\pi G/c^4$ is Einstein's constant; $\alpha_s = +1$ conforms to the material scalar field; $\alpha_s = -1$ corresponds to the "gravitational" scalar field [9, 10]; ξ is the coupling constant; $V(\Phi)$ is the scalar field potential; ρ is the perfect fluid mass density; $\Pi(\rho, e)$ is the internal energy; k, k_1, k_2, k_3 are the Lagrange multipliers; X is the Lagrangian coordinates of the matter particles; e is the entropy per volume [11]; u^i is the four-velocity; and $\overset{\Gamma}{\nabla}_i$ is the covariant derivative of the Riemann – Cartan space. The Lagrangian $\text{L}_{\text{f\,l}(2)}$ for the stiff fluid is not indicated since there is no torsion vector for it in the derivative of the term which regulates the conservation of the number of particles.

The metric g_{ik} has signature $(-,-,-,+)$, the Riemann and Ricci tensors are defined as $R^m_{ijk\cdot} = \Gamma^m_{jk,i} - \Gamma^m_{ik,j} + \Gamma^m_{ip}\Gamma^p_{jk} - \Gamma^m_{jp}\Gamma^p_{ik}$, $R_{jk} = R^i_{ijk\cdot}$. We should note that, in the framework of the ECT, a scalar field nonminimally coupled to the gravitational field gives rise to torsion, even thought the scalar field has zero spin. It follows from (3) that the torsion can interact with a perfect fluid only through its trace: $S_i = S_{ik\cdot}^{\ \ k}$ (the vector of torsion). An analogous result was derived in [10] for a scalar field. Hence, the curvature scalar can be presented in the form [10]

$$R(\Gamma) = R(\{\}) + 4\nabla_k S^k - \frac{8}{3} S_k S^k \ , \tag{4}$$

where $R(\{\})$ is the Riemannian part of the curvature built from Christoffel symbols; ∇_k is the covariant derivative of the Riemannian space.

One can note that Lagrangian (2) in the torsionless case at $\xi = 1/6$ and $V(\Phi) = 0$ conforms to the conformally invariant scalar field. As shown in [10], when $\alpha_s = -1$, $\xi = -1/6$, $V(\Phi) = -(1/2)\mu^2\Phi^2$, the scalar field corresponding to Lagrangian (2) is the axion field in GR. From the viewpoint of QCD, the interest to the axion field is based on the fact that it leads to the compensation of the strong CP violation effect; from the viewpoint of cosmology, it is a cold dark matter candidate (see, e.g., [10, 12] and references therein).

Varying the action with the Lagrangian $\text{L} = \text{L}_g + \text{L}_s + \text{L}_{\text{f\,l}(1)} + \text{L}_{\text{f\,l}(2)}$ in g_{ij}, S_k, Φ, ρ, k, k_i, X, e, u^i, we obtain the following set of equations for the gravitational fields and matter:

$$G_{ij}(\{\}) = æ(T^{\text{f\,l}(1)}_{ij} + T^{\text{f\,l}(2)}_{ij} + T^s_{ij}) + \Lambda_{ij} \ , \tag{5}$$

$$S^k = (3/2)\Psi(2\pi\alpha_s\Theta u^k + \xi\Phi\Phi^{,k}) \ , \tag{6}$$

$$\Box\Phi - \xi\Phi R(\Gamma) + \alpha_s V' = 0 \ , \tag{7}$$

$$\varepsilon_{\text{f\,l}(1)} + P_{\text{f\,l}(1)} + \rho u^i(\partial_i + 2S_i)k = 0 \ , \tag{8}$$

$$\overset{\Gamma}{\nabla}_i \, (\rho u^i) = 0 \ , \tag{9}$$

$$u_i u^i = 1 \ , \tag{10}$$

$$u^i \partial_i X = 0 \ , \tag{11}$$

$$u^i \partial_i e = 0 \ , \tag{12}$$

$$\nabla_i(k_2 u^i) = 0 \ , \tag{13}$$

$$\partial\varepsilon_{\text{f\,l}(1)}/\partial e + \nabla_i(k_3 u^i) = 0 \ , \tag{14}$$

$$-\rho\partial_i k - 2k\rho S_i + 2k_1 u_i + k_2\partial_i X + k_3\partial_i e = 0 \ , \tag{15}$$

where

$$T^{\text{f\,l}(1)}_{ij} = (\varepsilon_{\text{f\,l}(1)} + P_{\text{f\,l}(1)})u_i u_j - P_{\text{f\,l}(1)} g_{ij} \ , \tag{16}$$

$$T^{\text{f\,l}(2)}_{ij} = (\varepsilon_{\text{f\,l}(2)} + P_{\text{f\,l}(2)})u_i u_j - P_{\text{f\,l}(2)} g_{ij} \ , \tag{17}$$

$$T^s_{ij} = \frac{\alpha_s}{4\pi}\Big\{\Phi_{,i}\Phi_{,j} - \frac{1}{2}\Big[\Phi_{,m}\Phi^{,m} + \xi R(\{\})\Phi^2 - 2\alpha_s V(\Phi)\Big]g_{ij} + \xi\Big[-2S_i\nabla_j - 2S_j\nabla_i + 2g_{ij}S^n\nabla_n - \nabla_i\nabla_j + g_{ij}\Box + R_{ij}(\{\}) - \Lambda_{ij}\Big]\Phi^2\Big\} \ , \tag{18}$$

$$\Lambda_{ij} = \frac{8}{3}S_i S_j - \frac{4}{3}S_k S^k g_{ij} \ . \tag{19}$$

$$\varepsilon_{\text{f\,l}(1)} = \rho(c^2 + \Pi(\rho, e)) \ , \qquad P_{\text{f\,l}(1)} = \rho^2\partial\Pi/\partial\rho \ . \tag{20}$$

Here, $\varepsilon_{\text{f\,l}}$ is the perfect fluid energy density; $P_{\text{f\,l}}$ is its pressure; $\Box$ is the D'Alembertian operator of the Riemannian space; $\Psi = æ(4\pi\alpha_s - \xi æ\Phi^2)^{-1}$; $\Theta = k\rho$; and $V' = \partial V/\partial\Phi$.

By contracting Eq. (15) with u^i and using Eqs. (8), (10) – (12), we find

$$2k_1 = -(\varepsilon_{\text{f\,l}(1)} + P_{\text{f\,l}(1)}) \ . \tag{21}$$

Finally, Eqs. (8) and (9) give

$$\nabla_i(\Theta u^i) = -(\varepsilon_{\text{f\,l}(1)} + P_{\text{f\,l}(1)}) \ . \tag{22}$$

Excluding torsion with the help of Eq. (6), a closed subsystem of Eqs. (5), (7) and (22) is derived. In the framework of GR, this sybsystem describes the gravitational interactions of two perfect fluids and a nonminimally coupled scalar field with the potential $V(\Phi)$.

For spatially flat homogeneous isotropic models with the metric

$$ds^2 = a^2(\eta)[-dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) + d\eta^2], \qquad (23)$$

Eqs. (5) and (7) take the form:

$$2\frac{a''}{a} - \frac{a'^2}{a^2} = \Psi\Big[2\xi\Phi\Phi'' + 2\xi\frac{a'}{a}\Phi\Phi' +$$

$$+\Big(-\frac{1}{2} + 2\xi + 3\xi^2\Phi^2\Psi\Big)\Phi'^2 + \alpha_s a^2 V(\Phi) -$$

$$-12(\pi\Theta a)^2\Psi\Big] - 4\pi\alpha_s a^2\Psi(P_{\mathrm{f\,l(1)}} + P_{\mathrm{f\,l(2)}}) \ , \qquad (24)$$

$$\frac{a'^2}{a^2} = \Psi\Big[\Big(\frac{1}{6} - \xi^2\Phi^2\Psi\Big)\Phi'^2 + 2\xi\frac{a'}{a}\Phi\Phi' +$$

$$+\frac{1}{3}\alpha_s a^2 V(\Phi) + 4(\pi\Theta a)^2\Psi\Big] +$$

$$+\frac{4}{3}\pi\alpha_s a^2\Psi(\varepsilon_{\mathrm{f\,l(1)}} + \varepsilon_{\mathrm{f\,l(2)}}), \qquad (25)$$

$$(1 - 6\xi^2\Phi^2\Psi)(\Phi'' + 2\frac{a'}{a}\Phi') + \alpha_s a^2 V' -$$

$$-12\pi\alpha_s\xi a^2\Phi\Psi\nabla_k(\Theta u^k) + +6\xi\frac{a''}{a}\Phi +$$

$$+24\pi\xi\Phi\Psi^2(-æ^{-1}\alpha_s\xi\Phi'^2 + \pi a^2\Theta^2) = 0 \ , \qquad (26)$$

where the prime denotes differentiation with respect to η.

We confine ourselves to considering the perfect fluid which induces torsion with the vacuum equation of state: $P_{\mathrm{f\,l(1)}} = -\varepsilon_{\mathrm{f\,l(1)}}$. Then, from (20) and (22), we obtain

$$\varepsilon_{\mathrm{f\,l(1)}} = C_1 \ , \qquad \Theta = C_\Theta a^{-3} \ , \qquad (27)$$

where C_1, C_Θ are integration constants ($C_1 > 0$).

For the stiff fluid, we have

$$\varepsilon_{\mathrm{f\,l(2)}} = P_{\mathrm{f\,l(2)}} = C_2 a^{-6} \ , \qquad C_2 = \mathrm{const} \ . \qquad (28)$$

The scalar field potential $V(\Phi)$ is chosen in the form

$$V(\Phi) = \beta\frac{\mu^2}{2}\Phi^2 + \frac{\lambda}{4!}\Phi^4 \ . \qquad (29)$$

Here, μ and λ are constants; $\beta = +1$ conforms to a massive scalar field, and $\beta = -1$ corresponds to a Higgs-type nonlinearity.

The exact solutions have been obtained for positive Einstein's effective constant,

$$æ_{\mathrm{eff}} = æ\{1 - (\alpha_s\xi/4\pi)æ\Phi^2\}^{-1} > 0, \qquad (30)$$

provided that

$$\lambda = (6/\pi)(æ\xi)^2 C_1 \ , \qquad \mu^2 = 4æ|\xi|C_1 \ . \qquad (31)$$

3. Solutions with Material Scalar Field

3.1. Solution for $\xi > 0$, $\beta = -1$

The exact partial solution is obtained by quadrature:

$$a(\eta) = \beta_1 F^{-1/2}\cosh u \ , \Phi(\eta) = B\tanh u,$$

$$\int\frac{F^{1/2}dF}{\sqrt{sF^3 + s_1}} = 2n\int\frac{du}{W_1(u)\cosh u} \ ,$$

$$t = \frac{\beta_1}{c}\int\frac{du}{W_1(u)F^{3/2}}, \qquad (32)$$

where $\beta_1 = (æ|\xi|)^{1/4}/(4\pi)^{1/2}$, $B = (4\pi/æ|\xi|)^{1/2}$; $s = (D_1 + 32\pi^2 C_2 + 24æ\pi^2 C_\Theta^2)/6|\xi|$; D_1 is an integration constant; $s_1 = (æC_1/3)\beta_1^2$; $n = \pm 1$; $W_1(u) = (D_1 + 32\pi^2 C_2\tanh^2 u)^{1/2}$; and $W_1(u) > 0$.

Note that, in [7], the exact partial solutions with a material scalar field and a "gravitational" one have been obtained for $D_1 = 32\pi^2 C_2$. Here, we consider the other meanings of the constant D_1.

1.$\mathbf{D_1 > 0}$. The investigation of solution (32) has demonstrated that it describes nonsingular symmetric cosmological models of two types:

$$1)\ a|_{t\to\pm\infty} \sim t^{4/3} \ , \qquad \Phi|_{t\to\pm\infty} \simeq \pm B \ , \qquad (33)$$

$$2)\ a|_{t\to\pm\infty} \sim \exp(\pm H_1 t) \ , \qquad \Phi|_{t\to\pm\infty} \simeq \pm B \ , \qquad (34)$$

where t is the cosmological time ($a(\eta)d\eta = cdt$), $H_1 = c(æC_1/3)^{1/2} = \sigma$.

It is easy to verify that models (33)—(34) are regular due to the violation of the strong energy condition with a scalar-torsion field. It is not difficult to show that, for minima of the scale factor $a(t)$ and for the asymptotics $a \sim t^{4/3}$, the contribution of the scalar-torsion field dominates, while for $a \sim \exp(H_1 t)$ the contribution of the perfect fluid which is the source of torsion dominates.

2.$\mathbf{D_1 < 0}$. In this case, physically permissible models do not exist.

3.$\mathbf{D_1 = 0}$. It is not difficult to show that, in this case, it is possible to express $F = F(u)$ from (32) explicitly

$$F^{3/2} = (1/2)(s_1/s)^{1/2}(V(u) - V^{-1}(u)), \qquad (35)$$

where $V(u) = D_2(|\tanh(u/2)|)^{n\gamma}$, $V^2 \geq 1$; D_2 is an integration constant; $\gamma = 3(1 + 3æC_\Theta^2/4C_2)^{1/2}(6\xi)^{-1/2}$.

Note that there exist variable solutions for the scale factor a according to the value of the parameter n.

With $\mathbf{n = +1}$, the solution exists for $D_2 > 1$ only for $u \geq \ln[(1 + D_2^{-1/\gamma})/(1 - D_2^{-1/\gamma})]$ and $u \leq -\ln[(1+D_2^{-1/\gamma})/(1-D_2^{-1/\gamma})]$. This solution describes a nonsingular model with the asymptotics

$$a|_{t\to+\infty} \sim \exp(H_2 t), \quad \Phi|_{t\to+\infty} \simeq B \ ,$$

$$a|_{t\to-\infty} \sim \exp(-H_3 t), \quad \Phi|_{t\to-\infty} \simeq Bm_1, \tag{36}$$

where $H_2 = (3\sigma/2\gamma)(D_2 - D_2^{-1})$; $H_3 = \sigma(1 - D^{-2/\gamma})/(1 + D_2^{-2/\gamma})$; $m_1 = 2(D_2^{1/\gamma} + D_2^{-1/\gamma})^{-1}$.

It is not difficult to see that Hubble's constant H_2 can take great values at $\xi \gg 1$. As the models are considered in the framework of the classical theory of gravitation, they will be physically permissible provided that $\varepsilon < \varepsilon_{Pl}$, where $\varepsilon_{Pl} = c^7/\hbar G^2$. As a result, we get the restriction on ξ: $\xi < 2(1 + 3æC_\Theta^2/4C_2)\varepsilon_{Pl}/3C_1(D_2 - D_2^{-1})^2$.

It should be noted here that Hubble's constant H_3 does not reach Planck's values for any ξ.

It is necessary to point out that, for a de Sitter-like asymptotics, the contribution of two sources of torsion dominates and reverse asymptotics are possible.

With $\mathbf{n = -1}$ solutions exist for $D_2 \geq 1$ at $u \in (-\infty, \infty)$ and for $D_2 < 1$ at $-\ln[(1+D_2^{1/\gamma})/(1-D_2^{1/\gamma})] \leq u \leq \ln[(1 + D_2^{1/\gamma})/(1 - D_2^{1/\gamma})]$.

The analysis has shown that, for $n = -1$, the singular models of three types are possible. For all the types of models at early times, the scale factor and the scalar field behave identically as

$$a|_{t\to0} \sim t^{1/3}, \ \Phi|_{t\to0} \sim t^{1/\gamma}, \tag{37}$$

but, at the late times, the expansion laws are:

1) $D_2 < 1$

$$a|_{t\to+\infty} \sim \exp(H_4 t), \ \Phi|_{t\to+\infty} \simeq Bm_1, \tag{38}$$

2) $D_2 = 1$

$$a|_{t\to+\infty} \sim t^{4/3}, \ \Phi|_{t\to+\infty} \simeq B, \tag{39}$$

3) $D_2 > 1$

$$a|_{t\to+\infty} \sim \exp(H_2 t), \ \Phi|_{t\to+\infty} \simeq B, \tag{40}$$

where $H_4 = \sigma(1 - D_2^{2/\gamma})/(1 + D_2^{2/\gamma})$ as H_3 does not reach Planck's values for any ξ, and the expression for H_2 coincides with that from (36). Since solutions (38)—(40) have been obtained in two cards, $t \in (-\infty, 0)$ and $t \in 0, +\infty)$, reverse asymptotics are possible. It is not difficult to show that, for $t \to 0$, there dominates the contribution of the stiff fluid.

Note that, for $n = -1$, $D_2 = 1$, and $\gamma = 1$, the solution with an asymptotics of type (39) can be expressed in elementary functions:

$$a(t) = (32\pi^2\beta_1^4 C_2)^{1/6}(1 + 9\sigma^2 t^2)^{1/2}(c|t|)^{1/3},$$

$$\Phi(t) = 3\sigma Bt(1 + 9\sigma^2 t^2)^{-1/2}. \tag{41}$$

3.2. Solution for $\xi < 0$, $\beta = +1$

The exact partial solution has the form of quadratures:

$$a(\eta) = \beta_1 F^{-1/2} W_2^{-1/2}(u), \ \Phi = B \tanh u,$$

$$\int \frac{F^{1/2} dF}{\sqrt{sF^3 + s_1}} = 2n \int \frac{du}{W_2^{1/2}(u) W_3^{1/2}(u) \cosh^2 u},$$

$$t = \frac{\beta_1}{c} \int \frac{du}{W_2(u) W_3^{1/2}(u) F^{3/2} \cosh^2 u}, \tag{42}$$

where $W_2(u) = 1 + \tanh^2 u$, $W_3(u) = D_1 - 32\pi^2 \times C_2 \tanh^2(u)$, $W_3(u) > 0$.

It follows from (42) that, for $D_1 > 32\pi^2 C_2$, this solution describes nonsingular symmetric cosmological models of two types:

$$1) \ a|_{t\to\pm\infty} \sim \exp(\pm H_1 t), \ \Phi|_{t\to\pm\infty} \sim \mp\exp(\mp 3\sigma t), \tag{43}$$

$$2) \ a|_{t\to\pm\infty} \sim \exp(\pm\sqrt{2} H_1 t), \quad \Phi|_{t\to\pm\infty} \simeq \pm B. \tag{44}$$

Note that the qualitative picture of the evolution of models for $\xi > 0$, $\beta = -1$ $(D_1 > 0)$ and $\xi < 0$, $\beta = +1$ $(D_1 > 32\pi^2 C_2)$ is the same as for $D_1 = 32\pi^2 C_2$ with these signs of ξ and β [7].

4. Solutions with "Gravitational" Scalar Field

4.1. Solution for $\xi > 0$, $\beta = +1$

The exact partial solution is obtained by quadrature:

$$a(\eta) = \beta_1 F^{-1/2} \cosh^{-1} u, \qquad \Phi(\eta) = B \sinh u,$$

$$\int \frac{F^{1/2} dF}{\sqrt{\tilde{s} F^3 + s_1}} = 2n \int \frac{du}{W_4^{1/2}(u)},$$

$$t = \frac{\beta_1}{c} \int \frac{du}{W_4^{1/2}(u) F^{3/2} \cosh u}, \tag{45}$$

where $\tilde{s} = (-D_1 + 32\pi^2 C_2 + 24\pi^2 C_\Theta^2 æ)(6|\xi|)^{-1}$, $W_4(u) = D_1 + 32\pi^2 C_2 \sinh^2 u$.

Let us consider the possible models that admits solution (45).

1. $\mathbf{D_1 = 0}$. In this case, it is possible to present $F = F(u)$ as

$$F^{1/2} = (1/2)(s_1/\tilde{s})^{1/2}(V(u) - V^{-1}(u)). \tag{46}$$

We note that, in this case, the restrictions on D_2 and u are such that have been written out for the models with a material scalar field ($\xi > 0, D_1 = 0, n = \pm 1$).

With $\mathbf{n = +1}$, the solution admits only a singular model with the asymptotics

$$a|_{t\to 0} \sim t^{1/2}\ , \quad \Phi|_{t\to 0} \sim -t^{-1/2}\ ,$$

$$a|_{t\to +\infty} \sim \exp(H_5 t)\ , \quad \Phi|_{t\to +\infty} \simeq -Bm_2\ , \tag{47}$$

where $H_5 = \sigma(1 + D_2^{-2/\gamma})/(1 - D_2^{-2/\gamma})$; $D_2 > 1$; $m_2 = 2(D_2^{1/\gamma} - D_2^{-1/\gamma})^{-1}$. We note that reverse asymptotics are possible. Since Hubble's constant H_5 takes great values at $\xi \ll 1$, we get the restriction on ξ: $\xi > 3C_1(1 + 3æC_\Theta^2/4C_2)(2\varepsilon_{Pl}\ln^2 D_2)^{-1}$.

As in the previous variant of the material scalar field ($\alpha_s = +1$), for $\mathbf{n = -1}$, singular models of three types are possible. For all the types of models at early times, $a(t)$ and $\Phi(t)$ behave identically and are similar to (37). But, at the late times, the expansion laws are
1)$D_2 < 1$

$$a|_{t\to +\infty} \sim \exp(|H_5|t), \quad \Phi|_{t\to +\infty} \sim -B|m_2|, \tag{48}$$

2) $D_2 = 1$

$$a|_{t\to t_o} \sim (t_o - t)^{2/3}, \quad \Phi|_{t\to t_o} \sim (t_o - t)^{-1}. \tag{49}$$

It is worth to note that, in this case, the scalar-torsion field creates the effect similar to a curvature [13], since the evolution of model (49) is characteristic of one of the closed types.
3) $D_2 > 1$

$$a|_{t\to t_o} \sim (t_o - t)^{1/2}, \quad \Phi|_{t\to t_o} \sim (t_o - t)^{-1/2}. \tag{50}$$

It is easy to see that the evolution of model (50) corresponds of one of the closed types.

Note that, for $n = -1$, $D_2 = 1$, and $\gamma = 1$, the solution with an asymptotic of type (49) can be expressed in elementary functions as

$$a(t) = \beta_1(\tilde{s}/s_1)^{1/6}(1 + \tan^2(3\sigma t))^{-1/2}(|\tan(3\sigma t)|)^{1/3},$$

$$\Phi(t) = B\tan(3\sigma t). \tag{51}$$

2.$\mathbf{D_1 \neq 32\pi^2 C_2}$. In this case, we have the singular models (in two cards) with the asymptotics

$$a|_{t\to \pm 0} \simeq A, \qquad \Phi_{t\to \pm 0} \sim t,$$

$$a|_{t\to \pm t_o} \sim (t_o \mp t)^{1/2}, \quad \Phi|_{t\to \pm t_o} \sim \pm(t_o \mp t)^{-1/2}, \tag{52}$$

where $A = \text{const}$.

4.2. Solution for $\xi < 0$, $\beta = -1$

The exact partial solution has the following form:

$$a(\eta) = \beta_1 F^{-1/2}\cosh u, \qquad \Phi(\eta) = B\tanh u,$$

$$\int \frac{F^{1/2}dF}{\sqrt{\tilde{s}F^3 + s_1}} = 2n\int \frac{du}{W_3^{1/2}(u)\cosh u}\ ,$$

$$t = \frac{\beta_1}{c}\int \frac{du}{W_3^{1/2}(u)F^{3/2}}. \tag{53}$$

The analysis of solution (53) has shown that, for $D_1 \neq 32\pi^2 C_2$, the qualitative behaviour of $a(t)$ and $\Phi(t)$ is the same and coincides with that in the models for $\alpha_s = +1$, $\xi > 0$, $\beta = -1$, $D_1 > 0$.

5. Conclusion

In this article, in the framework of the two-torsion ECT, the exact partial solutions for spatially flat cosmological models with a stiff fluid, nonminimally coupled scalar field with nonlinear potential, and perfect fluid with the vacuum equation of state have been obtained and analyzed in the case of an arbitrary coupling constant ξ.

For the models with de Sitter-like asymptotics, when the Hubble's constant may grow up to Planck's values, the restriction on ξ have been found.

In [7] in the framework of the one-torsion ECT, the general exact solution for the spatially flat cosmological model with a stiff fluid, when the torsion is generated by a perfect fluid with the vacuum equation of state, has been obtained. It has been shown that the following asymptotics are true:

$$a|_{t\to \pm 0} \sim (\pm t)^{1/3}, \qquad a|_{t\to \pm\infty} \sim e^{\pm\sigma t}.$$

Thus, the analysis of the exact solutions that have been obtained in this article shows that, as compared with similar models in the ECT, the presence of a scalar-torsion field leads to
— the slow–down of the initial cosmological evolution: $a|_{t\to +0} \sim t^{1/2}$ for $\alpha_s = -1$, $\xi > 0$, $D_1 = 0$, $D_2 > 1$ ($n = +1$).
— the slow-down of the late stages of the cosmological evolution:
- — $a|_{t\to +\infty} \sim t^{4/3}$ for $\alpha_s = +1$, $\xi > 0$, $D_1 \geq 0$ and $\alpha_s = -1$, $\xi < 0$.
- — $a|_{t\to +\infty} \sim \exp(Ht)$, where $H < \sigma$, for $\alpha_s = +1$, $\xi > 0$ ($\xi \ll 1$), $D_1 = 0$, $D_2 > 1$ ($n = \pm 1$) and $D_2 < 1$ ($n = -1$).

— the creation of the effect similar to a curvature for $\alpha_s = -1$, $\xi > 0$, $D_1 = 0$, $D_2 \geq 1$ $(n = -1)$ and $D_1 \neq 32\pi^2 C_2$.
— the speeding-up of the late stages of the cosmological evolution: $a|_{t\to+\infty} \sim \exp(Ht)$, where $H > \sigma$, for $\alpha_s = +1$, $\xi > 0$ $(\xi \gg 1)$, $D_1 = 0$, $D_2 > 1$ and $\xi < 0$ $(\forall\xi)$; for $\alpha_s = -1$, $\xi > 0$ $(\xi \ll 1)$, $D_1 = 0$, $D_2 > 1$ $(n = +1)$ and $D_2 < 1$ $(n = -1)$.
— the removal of the initial singularity for $\alpha_s = +1$, $\xi > 0$, $D_1 = 0$, $D_2 > 1$ $(n = +1)$ and $D_1 > 0$; for $\alpha_s \pm 1$, $\xi < 0$. — the increase of the number of variants of the cosmological evolution for $\alpha_s = \pm 1$.

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ІНТЕГРОВНІ КОСМОЛОГІЧНІ МОДЕЛІ У ТЕОРІЇ ЕЙНШТЕЙНА—КАРТАНА З ДВОМА ДЖЕРЕЛАМИ КРУЧЕННЯ

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Р е з ю м е

Досліджуються просторово плоскі космологічні модеді з немінімально зв'язаним скалярним полем, ідеальною рідиною та рідиною "жорсткого" типу у двоторсійній теорії Енштейна—Картана. Одержано точні частинні розв'язки для довільних значень сталої зв'язку. Показано, що в залежності від типу скалярного поля можливі сингулярні та несингулярні моделі. Для отриманих розв'язків знайдено обмеження на сталу зв'язку. Обговорюються деякі наслідки двох джерел кручення.