HADRON-NUCLEUS SCATTERING IN STOCHASTIC NUCLEAR OPTICS

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Basing upon a deep analogy between light optics and the physics of nuclear scattering, the application of methods of the theory of stochastic processes to calculating the parameters of elastic collisions between nuclei has been substantiated. Starting from the model of total absorption with near-surface spin-orbit interaction, the amplitude of the nucleon elastic scattering by atomic zero-spin nuclei, which takes into account the fluctuation character of the nucleon-nucleus interaction, has been built in the diffraction approximation. At small scattering angles, an agreement with experiment concerning both the differential cross-sections (DCSs) and the angular dependences of the scattered nucleon polarization has been achieved. At large angles, calculations are in agreement with experimental data concerning DCSs, if zero oscillations of the nuclear surface are taken into account. The results of calculations are in a qualitative agreement with polarization data. It has been shown that fluctuations of the nuclear density can stimulate fluctuations of the limiting angular momenta of orbital waves.

1. Application of Methods of the Theory of Stochastic Processes for Calculating the Characteristics of Hadron-nucleus Scattering

The wave nature of physical objects is a fundamental basis of light optics and nuclear physics. It reveals itself, e.g., in rotating the plane of polarization and birefringence in the nuclear optics of polarized media [1], in nuclear rainbow scattering [2, 3], in diffraction nuclear processes, which, contrary to light optics, can be inelastic [4, 5]. The theory of light diffraction is essentially inseparable from the concept of stochastic processes, because instant realizations (microstates) of the diffraction field are not interesting as separate entities and practically do not repeat themselves. Concerning the probabilistic distributions and statistical momenta, they represent stochastic processes adequately as the persistent characteristics of a random field [6]. In this work, we would like to demonstrate the results of introduction of elementary stochastic assumptions into the calculations of the angular dependences of polarization and the DCS of the elastic diffraction scattering of hadrons by atomic nuclei.

Suppose that a particle beam, which is characterized by the Poisson distribution, strikes a point inertialess counter. The probability for m particles to be registered within the time interval t is determined [7–9] by the expression

$$P(m,t) = \frac{(\lambda t)^m}{m!} e^{-\lambda t}, \tag{1}$$

where the intensity of the process λ corresponds to the average constant number of particle registration (events) per unit time [7]. Now, let the counter register particles that have already interacted with a target, and the radius R of particle interaction with target nuclei is constant. Since the scattering angle θ of a hadron in the center-of-inertia frame cannot be foreseen beforehand from dynamic considerations, the ensemble of target nuclei represents a statistical sampling [7, 9] of events distributed according to the Poisson distribution (1). Therefore, the counter positioned at an angle θ will register a Poisson-like beam of particles [7, 9] with a certain constant intensity $\lambda = \lambda(\theta)$. Provided $R = R(t)$, where $R(t)$ is a determinate function, the intensity λ becomes a regular function of time: $\lambda = \lambda(\theta, t)$. Now, if one defines a reference point t_0 on the time axis, then [7, 9]

$$P(m,t_0+t,t_0)=\frac{1}{m!}\left\{\int\limits_{t_0}^{t_0+t}\lambda(t')dt'\right\}^m\times$$

$$\times\exp\left\{-\int\limits_{t_0}^{t_0+t}\lambda(t')dt'\right\}\equiv\frac{U^m}{m!}\,e^{-U},\qquad(2)$$

where U is the integral intensity [9, 10]. At last, let the radius R fluctuate, i.e. depend on the parameter t randomly. Then, the intensity $\lambda(t)$ is a sample function (trajectory) of a definite, not necessarily Poisson-like, stochastic process $\Lambda(t)$, that corresponds to statistical properties of both the ensemble of beam particles and the ensemble of target nuclei. This means that the process, which is studied in the hadron-nucleus scattering experiment, turns out to belong to a class of double-stochastic Poisson processes introduced by Cox and studied in details by Bartlett [9].

In statistical light optics, it became a tradition to apply the theory of stationary stochastic processes to the problems similar to ours, and, assuming their ergodicity, to equate light fields averaged over time to those averaged over an ensemble. For example, in photocounting statistics [9, 10], in case of a fluctuating light flux that strikes a photocell, the Poisson distribution (2) of counts $P(m,t_0+t,t_0)$ is habitually averaged over the stochastic ensemble of integral intensities U with a distribution $W(U)$, which constitutes the essence of the Mandel formula [10]

$$\langle P(m,t_0+t,t_0)\rangle=\int\limits_0^\infty\frac{U^m}{m!}\,e^{-U}\,W(U)dU.\qquad(3)$$

Similarly to the case of diffraction hadron-nucleus scattering, where it is possible to assume the presence of quasiclassical fluctuations of the interaction region boundaries and to use quantum-mechanical amplitudes of the scattering process, the double-stochastic Poisson process in photocounting statistics reflects both quantum-mechanical uncertainties, which are connected with the photoeffect, and classical fluctuations of an incident light flux. Therefore, having brought the ensemble of $R(t)$-values or, equivalently, the stochastic ensemble of corresponding values of the angular momentum $L(t)$ of incident particles in correspondence with the stochastic ensemble U in Eq. (3), we obtain the opportunity to compare our DCS of elastic diffraction scattering of hadrons by atomic nuclei, which is the average over time, with the relevant DCS averaged over the ensemble of trajectories of the process $L(t)$ with a distribution $W(L)$:

$$\langle\sigma(\vartheta)\rangle_L=\int\limits_0^\infty\sigma(L,\vartheta)\;W(L)dL.\qquad(4)$$

Here, the DCS $\sigma(L,\vartheta)$ is connected, analogously to the Poisson probability of registration (1) in the Mandel formula (3), to the probability of registration of a particle that has been scattered at an angle θ.

None of the specific cases of hadron-nucleus scattering is restricted in any way by the speculations presented above. In order to estimate the efficiency of the proposed approach, let us consider the scattering of nucleons by nuclei. The stochasticity of the parameters of the relevant potential function is associated with fluctuations of the mean nuclear field. If one proceeds, say, from the model of the diffraction of nucleons by a black nucleus with a sharp (diffusiveless) surface [2–4], the radius of nuclear absorption (the radius of nucleon-nucleus interaction) $R_0=r_0A^{1/3}$ turns out a single dynamic parameter, and the parameter of nuclear density r_0 can be regarded as a stochastic variable.

2. Construction of a Diffraction Amplitude with Regard for the Fluctuation Nature of Nucleon-nucleus Interaction

Let us consider the elastic scattering of nucleons possessing the kinetic energy E within the interval $0.1-1.0$ GeV by heavy nuclei [2–4] in a reference frame connected to a target within the framework of a stochastic modification of the diffraction approximation of nuclear optics. At energies $E>50$ MeV, the analysis of nucleon-nucleus scattering on the basis of the optical model [4] becomes essentially complicated, especially if both the DCS and the angular dependences of the polarization of scattered nucleons are reproduced simultaneously. If the analysis is confined to the scattering into the forward hemisphere, a simple diffraction model of strong absorption turns out rather productive at energies $E>50$ MeV and up to the values, at which the relativistic effects and phenomena connected with internucleon distances in nuclei become noticeable. A nonuniformity of absorption in this model, which is, to a certain extent, a reduction of the full-scale optical approach, is usually taken into account through introducing a profile function $\omega(b)$, where b is an impact parameter for a nucleon—nucleus collision [2–4]. An obvious shortcoming of such an approach is typical

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diffraction dips, which are brought in correspondence, more or less successfully, with the measured DCSs by making allowance, as a rule, for nuclear refraction [2–4]. We suggest an alternative way for matching the calculated and measured DCSs in the vicinity of their minimal values, restricting ourselves to an idea of a spherical black nucleus with sharp surface. In the framework of this approach, the effects of diffusiveness of a nuclear surface and a nuclear refraction turn out a consequence of the stochastic nature of the parameter r_0.

Our initial idea was to represent the process of scattering of nucleons at intermediate energies as the nucleon motion in a predominantly absorbing medium. This motion is accompanied by insignificant perturbations, which are induced by the spin-orbit interaction and have a refractive character. In the region of total absorption, which is limited by the value of the orbital moment l ($0 \le l \le l_{\max} \equiv L$), the diagonal elements S_l of the nuclear part of the S-matrix were considered equal to zero. The limiting value L of the angular momentum is a discrete stochastic variable, the stochasticity of which is generated by fluctuations of the radius of the nuclear surface. Moreover, we should bear in mind that the uncertainty of the limiting value L is also connected with a quasiclassical character of localization of the region of total absorption, which is determined in the basis of the angular momentum with an accuracy of about unity [2]. We suppose that possible values of L are in the interval $L_0 - \Delta L \le L \le L_0$ and a certain probability density $W(L)$ corresponds to them. We also suppose that the spin-orbit interaction is realized in the surface layer of a nuclear substance, which is external with respect to the region of total absorption and whose width is characterized by the values l within the interval $L_0+1 \le l \le L_0+1+\Delta l \equiv L_S$. We assume that, in the external domain with $l > L_0$, the diagonal elements of the nuclear part of the S-matrix satisfy the condition $\left|S_l^+\right|^2 = \left|S_l^-\right|^2 = \left|S_j\right|^2 = 1$, where $j = l \pm \frac{1}{2}$ is the total angular momentum of an incident nucleon and $S_l^{\pm} \equiv S_{j=l\pm\frac{1}{2}}$. In other orbital channels with $l > L_0$, the nuclear interaction is absent, so that $S_l^{\pm} = 1$. Analogously to the case of the absorption region, we take into account the opportunity for the value of L_S to fluctuate within the limits of variation of the spin-orbit interaction bandwidth: $0 \le \Delta l \le (\Delta l)_{\max} \equiv \Delta L_S$. A near-surface character of the spin-orbit interaction allows the parametrization

$$S_l^{\pm} = S \pm i\Delta S, \tag{5}$$

where $S = (1-(\Delta S)^2)^{1/2}$, $S = \mathrm{Re}S$, and $\Delta S = \mathrm{Re}(\Delta S)$, to be used.

Below, we confine the consideration to the scattering of nucleons by nuclei with a zero spin. Then, as is well known, the amplitude of the process can be written down as

$$f_0(L, L_S, \vartheta) = f_{CN}(L, L_S, \vartheta) + (\vec{\sigma}\vec{n}) f_{L_\sigma}(L_S, \vartheta), \tag{6}$$

where $\vec{\sigma}$ is the Pauli spin vector, $\vec{n} = [\vec{k} \times \vec{k}']/kk'$ is the unit vector , $\vec{k}(\vec{k}')$ is the wave vector of an incident (scattered) nucleon, and $\hbar = c = 1$. Within the examined energy interval, the wave number $k = k'$ is determined [11], making allowance for relativistic corrections, as

$$k = E_A\{[E(E + 2E_N)]/[(E_N + E_A)^2+$$

$$+2E_A E]\}^{1/2}/197.3, \tag{7}$$

where k is measured in fm^{-1} units, if the kinetic energy E and the rest energy E_N (E_A) of a nucleon (of a target nucleus) are measured in MeV units. The amplitude $f_{CN} = f_C + f_L + f_{L_S}$ is a sum of the Coulomb and nuclear central amplitudes modified by a Coulomb field in the region of total absorption and in the near-surface layer:

$$f_C = -\frac{\eta}{2k(\sin\frac{\vartheta}{2})^2} \exp\left\{2i\left[\sigma_0 - \eta \ln\left(\sin\frac{\vartheta}{2}\right)\right]\right\}, \tag{8}$$

$$f_L = -\frac{1}{2ik}\sum_{l=0}^{L}(2l+1) \exp(2i\sigma_l)\, P_l(\cos\vartheta), \tag{9}$$

$$f_{L_S} = -\frac{1}{2ik}\sum_{l=L_0+1}^{L_S}[(2l+1)(1-S) - i\Delta S]\times$$

$$\times \exp(2i\sigma_l)\, P_l(\cos\vartheta). \tag{10}$$

Here, $\eta = Z_A e^2/V$ is the Sommerfeld parameter for a proton and a target nucleus with the atomic number Z_A, V is the relative velocity of the proton and the nucleus when the distance between them is infinitely large, and $\sigma_l = \arg\Gamma(l+1+i\eta)$ is the Coulomb phase shift. The amplitude of the spin-orbit interaction f_{L_σ} looks like

$$f_{L_\sigma} = -\frac{1}{ik}\Delta S \sum_{l=L_0+1}^{L_S} \exp(2i\sigma_l)\, P_l^1(\cos\vartheta), \tag{11}$$

where the associated Legendre polynomials $P_l^m(x)$ are defined according to work [12].

We make the simplest assumption concerning the distribution function of discrete stochastic values of angular momenta. Namely, let these values be distributed uniformly within the corresponding intervals of averaging. Such an assumption is made, e.g., when analyzing radio signals with unknown parameters in a white noise (see, e.g., [13]). In this case, the probability density $W(L)$ for a stochastic variable L can be represented [14] as

$$W(L) = \frac{1}{(1+\Delta L)} \sum_{L_k=(L_0-\Delta L)}^{L_0} \delta(L - L_k). \tag{12}$$

The forms of the probability densities $W(L_S)$ and $W(L_\sigma)$ are similar. The averaging of the amplitudes f_L, f_{L_S}, and f_{L_σ} according to formula (4) was carried out over the corresponding intervals of uncertainty of the limiting orbital momenta L, L_S, and L_σ, which gave the following results:

$$\langle f_L \rangle_L = \frac{1}{(1+\Delta L)} \sum_{L=L_0-\Delta L}^{L_0} f_L, \tag{13}$$

$$\langle f_{L_S} \rangle_{L_S} = \frac{1}{(1+\Delta L_S)} \sum_{L_S=L_0+1}^{L_0+1+\Delta L_S} f_{L_S}, \tag{14}$$

$$\langle f_{L_\sigma} \rangle_{L_S} = \frac{1}{(1+\Delta L_S)} \sum_{L_\sigma=L_0+1}^{L_0+1+\Delta L_S} f_{L_\sigma}. \tag{15}$$

The variance of a complex-valued stochastic function of a real argument $Z(\tau)$ is defined, according to [8], as the mathematical expectation of the function $|Z(\tau) - \langle Z(\tau)\rangle|^2$. Then, the general expression for the variances of the amplitudes $f_i = f_L$, f_{L_S}, and f_{L_σ} looks like

$$Df_i = \sum_{i=i_{\min}}^{i_{\max}} (f_i f_i^* - \langle f_i \rangle \langle f_i \rangle^*)/(i_{\max} - i_{\min}). \tag{16}$$

From all the stated above, it follows that the observable polarization of scattered nucleons $\vec{P}(\vartheta)$ and the DCS of elastic scattering $\sigma_0(L, L_S, \vartheta)$ are determined as

$$\langle \vec{P}(\vartheta) \rangle_{L,L_S} = \vec{n}\, \frac{2\mathrm{Re}[\langle f_{CN}(L, L_S, \vartheta)\rangle_{L,L_S} \langle f_{L_\sigma}(L_S, \vartheta)\rangle^*_{L_S}]}{\langle \sigma_0(L, L_S, \vartheta)\rangle_{L,L_S}}, \tag{17}$$

where

$$\langle f_{CN}(L, L_S, \vartheta)\rangle_{L,L_S} = \langle f_L(L, \vartheta)\rangle_L + \langle f_{L_S}(L_S, \vartheta)\rangle_{L_S} + f_C(\vartheta), \tag{18}$$

and

$$\langle \sigma_0(L, L_S, \vartheta)\rangle_{L,L_S} = \left\langle \frac{1}{2} S_p\, f_0(L, L_0, \vartheta)\; f_0^{\dagger}(L, L_0, \vartheta) \right\rangle_{L,L_S} = |\langle f_{CN}(L, L_S, \vartheta)\rangle_{L,L_S}|^2 + |\langle f_{L_\sigma}(L_S, \vartheta)\rangle_{L_S}|^2 + Df_L(\vartheta) + Df_{L_S}(\vartheta) + Df_{L_\sigma}(\vartheta). \tag{19}$$

Up to this point, the sharp nuclear surface with a stochastically fluctuating radius has been supposed to possess a spherical form. Hereafter, we take also into account the presence of the zero oscillations of the nucleus surface. According to [15], making allowance for these multipole pulsations results in renormalizing the amplitude of scattering:

$$f(L, L_S, \vartheta) = f_0(L, L_S, \vartheta)\; F_d(\vartheta), \tag{20}$$

where

$$F_d(\vartheta) = \exp(-(kd\vartheta)^2/2) \tag{21}$$

is the damping factor, $d^2 = R_0^2 \sum_\lambda \beta_\lambda^2$, and β_λ is the total amplitude of the zero oscillations with multipolarities $\lambda \geq 2$. Therefore, the multipole zero oscillations of the nuclear surface induce an additional exponential reduction of the DCS

$$\sigma(L, L_S, \vartheta) = \sigma_0(L, L_S, \vartheta) F_d^2(\vartheta) \tag{22}$$

as the scattering angle increases, but do not change expression (17) for the polarization of scattered nucleons.

3. Results of Calculations of the DCS and the Angular Dependences of Polarization for Proton Scattering by ^{208}Pb Nuclei

We share the doubts of the authors of work [16] concerning the frequently used procedure of summation of partial scattering amplitudes, which is based on the continuous approximation typical of the traditional diffraction model or on the Poisson formula with the following substitution of the Legendre polynomials in the integrand by a Bessel function [2, 3]. Taking into account the Mehler—Rayleigh theorem [17],

$$\lim_{n\to\infty} P_n(\cos\frac{\rho}{n}) = J_0(\rho), \tag{23}$$

the substitution of the Bessel function for the Legendre polynomials is a mathematically correct operation. By no means the same can be asserted if the matter concerns a transition from the summation of series over orbital waves to the integration over the relevant continuous

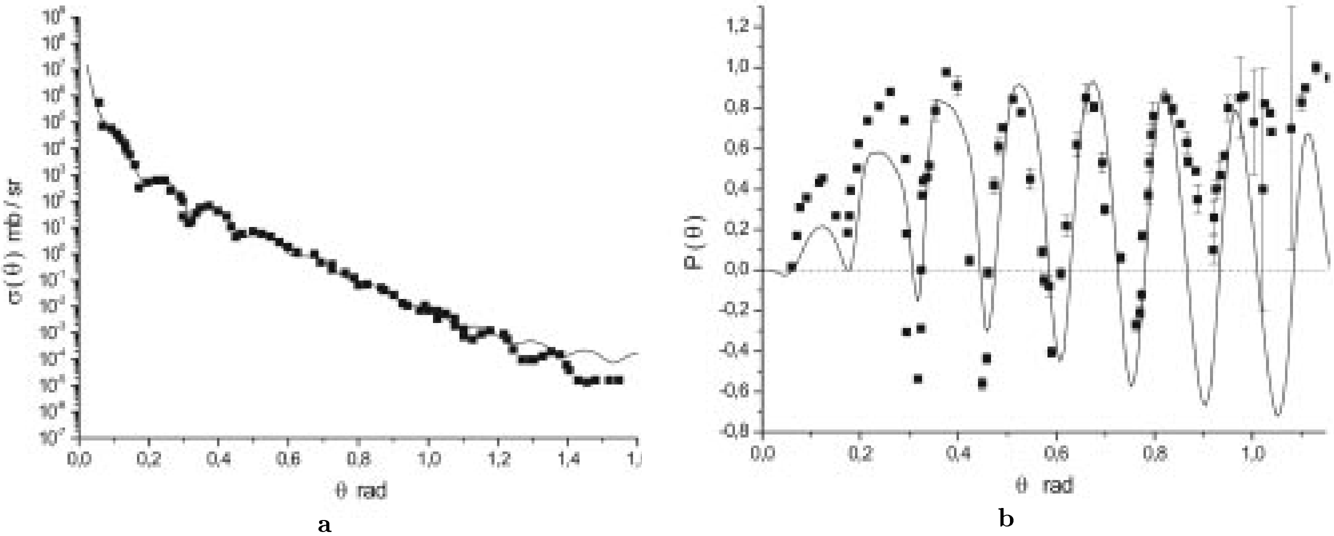


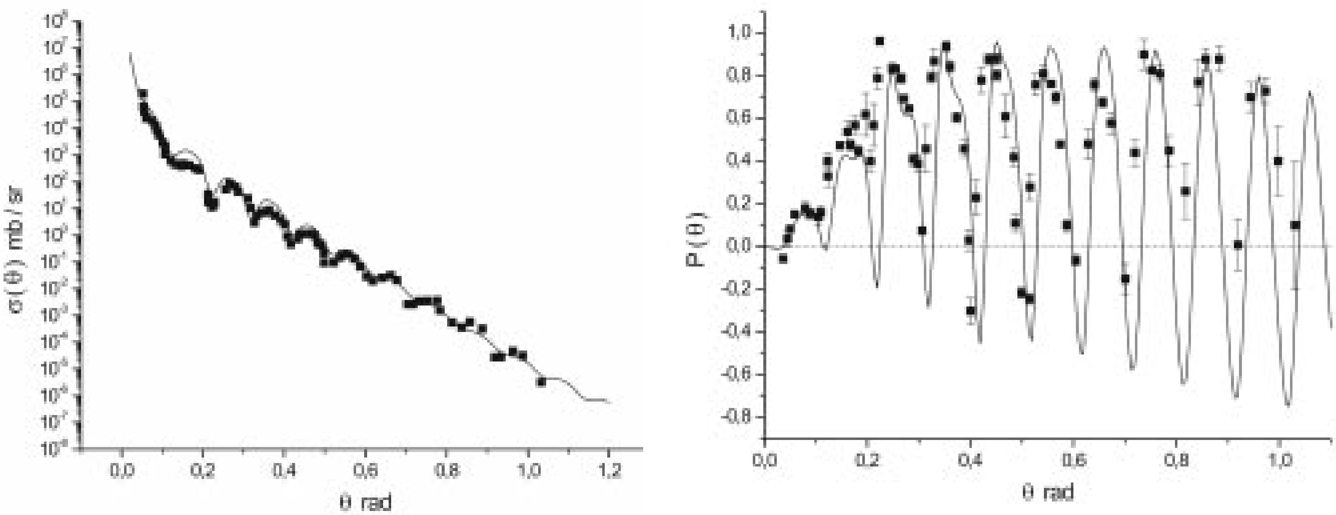
Fig. 1. DCS (a) and the polarization (b) of elastic scattering of protons with the energy $E = 200$ MeV by ^{208}Pb nuclei. The points correspond to the experimental data from [19] and the curve to the result of calculations according to the proposed model of stochastic diffraction. The parameters of calculation are quoted in the table

variable [16]. In work [16], the elastic scattering of charged particles by strongly absorbing atomic nuclei with either sharp or diffusive surfaces is responsible for amplitudes which are constructed without use of the Poisson formula, valid in a wide range of angles θ, and composed of the Fresnel and Fraunhofer parts, the relation between them being analyzed carefully. The comparison and discussion of the calculated angular dependences of the ratio $\sigma(\vartheta)/\sigma_R(\vartheta)$, where $\sigma_R(\vartheta)$ is the Rutherford DCS, up to the angles $\vartheta \simeq 120°$ and for various variants of the amplitude of scattering by a nucleus with a sharp surface were carried out in work [16]. The authors of [16] emphasized that, in a dark region [3, 18], i.e. at $\vartheta > \vartheta_C$, where ϑ_C is the critical angle [2] that demarcates the regions of Coulomb and nuclear scattering, the positive and negative Fraunhofer amplitude branches [16] begin to make approximately equal contributions to the amplitude, thus generating the Fraunhofer mode of diffraction. Conclusions of work [16] have substantiated to a certain extent and render a comparison of the results of our calculations with experimental data obtained beyond the range $\vartheta < (kR)^{-1}$ [2] to be meaningful. By the way, such a comparison has been done in a number of known researches [2–4, 16, 18]. The circumstance that the analytical approach was checked in work [16] by the numerical summation of partial amplitudes is also extremely important for us. Such a monitoring evidences for an impressive coincidence between the results of analytical and numerical calculations of the amplitude of elastic scattering of charged particles by a black nucleus with sharp surface within the whole interval of angles $0 < \theta < 120°$. Basing on this result and on a community of starting points of the diffraction theory in this work and in work [16], namely, on the assumption that the S-matrix can be represented as

$$S_l = H(l - L_0)\exp(2i\sigma_l), \tag{24}$$

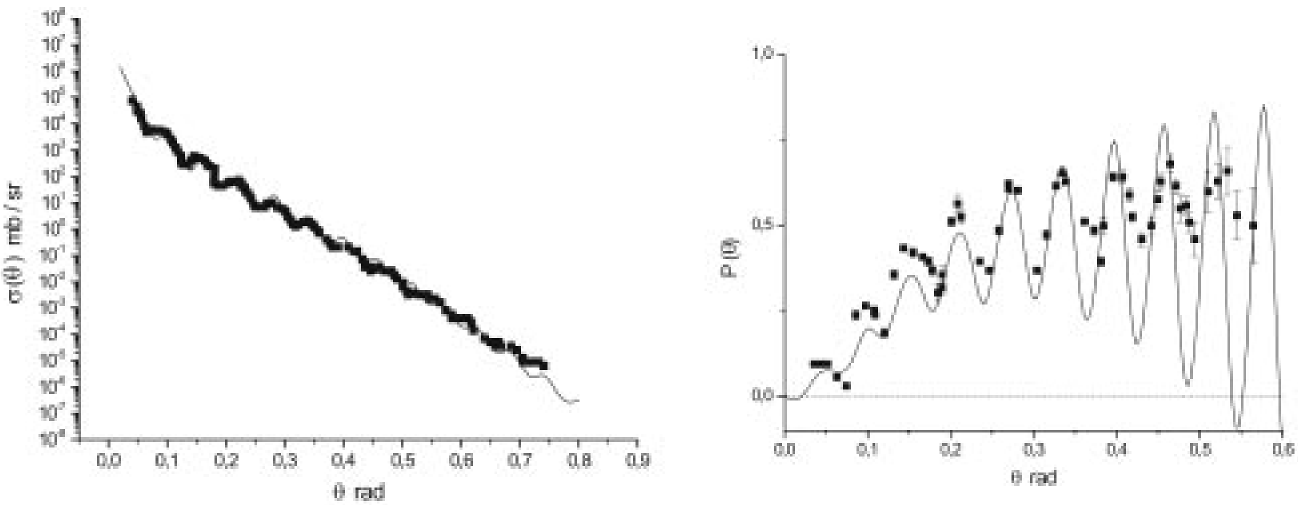
where $H(x)$ is the Heaviside function, we sum up the series of partial amplitudes also numerically, not using the Poisson formula. The upper limit of the orbital angular momentum values reached, as can be seen from the table, the value of 20 in expression (9) and 64 in (10) and (11).

The availability of a plenty of experimental data concerning proton–nucleus scattering allows one to test the features of the model developed here and the influence of the averaging over the fluctuating number of orbital waves that take part in the short-range nuclear interaction in detail. We have calculated the DCS and the polarizations for the elastic scattering of protons with the energies $E = 200$, 400, 800, 1000, and 1040 MeV by ^{208}Pb nuclei. The relevant parameters of the model are quoted in the table. The results of calculations are shown in Figs. 1—4 in comparison with existing experimental data [19–22] (the EXFOR library). Analyzing the figures, we may state that the developed model of the stochastic diffraction scattering of protons of intermediate energies by atomic nuclei results in a qualitative agreement with experimental data



a **b**

Fig. 2. The same as in Fig. 1 but for $E = 400$ MeV. The points correspond to the experimental data from [19]



a **b**

Fig. 3. The same as in Fig. 1 but for $E = 800$ MeV. The points correspond to the experimental data from [20]

concerning both the DCS and the polarizations of scattered protons. But experimental data are not always reproducible quantitatively. In particular, it is hard to remove deep dips in the angular dependences of the polarization at relatively large angles θ. For an agreement to be improved, an expansion of the physical foundations of the model may be demanded.

Let us introduce the effective limiting orbital momentum L_{eff} and let its values be distributed uniformly within the interval $\langle L_{\text{eff}}\rangle - \Delta L_{\text{eff}} \le L_{\text{eff}} \le \langle L_{\text{eff}}\rangle + \Delta L_{\text{eff}}$, where $\Delta L_{\text{eff}} = (\Delta L + \Delta L_S + 1)/2$ is the quantity that is recognized as the degree of an orbital fluctuation. Since the lower limit of the interval of the

Parameters of calculation of the DCS and the polarizations at elastic scattering of protons by ^{208}Pb nuclei

Parameter	E, MeV				
	200	400	800	1000	1040
L_0	20	30	52	63	63
ΔL	0	0	2	4	8
ΔL_S	4	4	1	0	0
ΔS	0.5	0.5	0.7	1	1
d, fm	1	0.9	0.8	0.83	0.83

orbital momentum uncertainty is $L_0 - \Delta L$, whereas the upper limit is $L_0 + 1 + \Delta L_S$, ΔL_{eff} is just a half-interval between the lower and upper limits. If the stochastic variable L_{eff} is distributed uniformly, the mathematical expectation $\langle L_{\text{eff}} \rangle$ is equal to a half-sum of the upper and lower limits [14], i.e. $\langle L_{\text{eff}} \rangle = L_0 + (1 - \Delta L + \Delta L_S)/2$. The standard deviation δL_{eff} in this case amounts to [14]

$$\delta L_{\text{eff}} = \Delta L_{\text{eff}}/\sqrt{3}. \tag{25}$$

Following work [2], let us determine now the effective limiting radius

$$R_{\text{eff}} = L_{\text{eff}}/k = R_0(1 - 2\eta/kR_0)^{1/2}. \tag{26}$$

Within the considered energy interval, the values of $\langle L_{\text{eff}} \rangle$ correspond, on the average, to the parameter of nuclear density $r_0 = (1.17 \pm 0.03)$ fm that does not contradict the values of r_0 found when studying other nuclear processes. For example, $r_0^{\text{Coul}} \approx 1.2$ fm is a typical value of a radial parameter of charge distribution in nuclei. On the other hand, making use of the approximate formula $D[h(x)] \approx [h'[\langle x \rangle]]^2 D(x)$ and Eq. (26), we obtain the standard deviation

$$\delta r_0 = \frac{1}{kA^{1/3}} \frac{\langle L_{\text{eff}} \rangle \delta L_{\text{eff}}}{[\eta^2 + \langle L_{\text{eff}} \rangle^2]^{1/2}}. \tag{27}$$

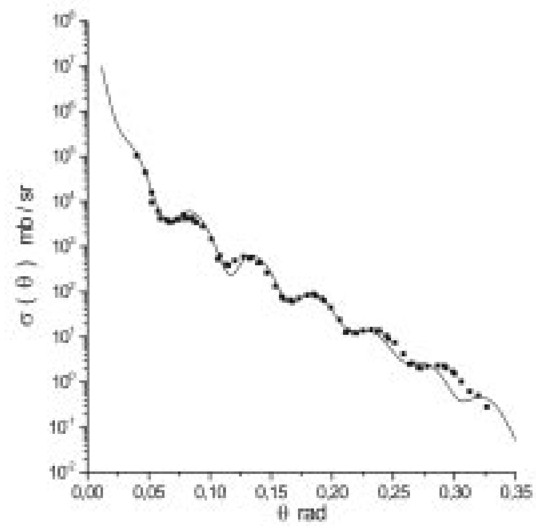
It turns out that the sampling average is equal to $\delta\bar{r}_0/\bar{r}_0 = (3.2 \pm 1.4)\%$ in the considered energy interval. Such an estimation of the relative variation of the parameter of nuclear density can be interpreted as a hint at the primary source of fluctuations of the limiting angular momentum, fluctuations of the nuclear density. The obtained value for $\delta\bar{r}_0/\bar{r}_0$ can be confronted with the data concerning the compressibility of nuclear substance. For example, when expanding the energy of a nucleus in a series and determining the compressibility modulus K, the approximate estimation

$$\left(\frac{\delta r_0}{r_0}\right)_{\text{theor}} \approx -2\frac{r_0}{KA}\left(\frac{\partial E_A}{\partial r}\right)_{r_0} \tag{28}$$

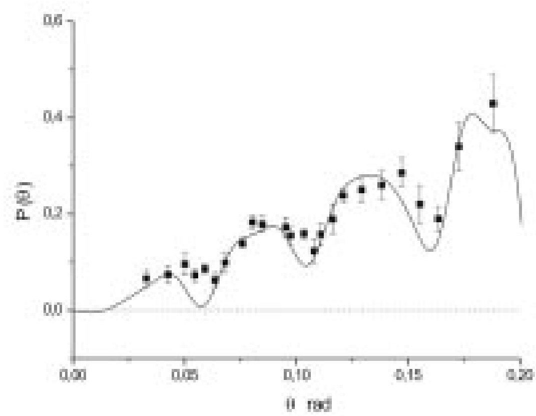
is obtained.

For a ^{208}Pb nucleus, the value $K \approx 211$ MeV was determined from experimental data concerning the position of the giant monopole resonance [23]. Having adopted that the main contribution to the derivative is provided by the surface and Coulomb energies, we obtained, according to the Weizsäcker formula, that

$$\left(\frac{\partial E_A}{\partial r}\right)_{r_0} \approx [-2A^{2/3}b_{\text{sur}} +$$



a



b

Fig. 4. (*a*) The same as in Fig. 1,*a* but for $E = 1040$ MeV. The points correspond to the experimental data from [21]. (*b*) The same as in Fig. 1,*b* but for $E = 1000$ MeV. The points correspond to the experimental data from [22]

$$+ b_C Z_A^2 (1 - 0.76 Z_A^{-2/3}) A^{-1/3}]/r_0, \tag{29}$$

where $b_{\text{sur}} \approx 17$ MeV and $b_C \approx 0.70$ MeV [24]. Then, the expected value of the relative variation $\delta r_0/r_0 \approx 0.02$, which does not contradict our estimation. At last, our calculations led to the estimation of the parameter d, namely, $d \approx (0.87 \pm 0.08)$ fm, which, according to Eq. (21), can be confronted with the parameters of dynamic deformation of a ^{208}Pb nucleus. A collective low-frequency oscillation mode reveals itself in a ^{208}Pb nucleus through the excitation of its octupole state 3^- $(= J^\pi)$ and is characterized by the experimental value $\beta_3 \approx 0.12$ [25], wherefrom $d \approx 0.83$ in a good agreement with our estimation.

4. Conclusions

The results of this work have led to the following statements.

1. There exists a deep analogy between light optics and the physics of elastic and inelastic nuclear scattering, both being based on the wave nature of light and a nuclear (corpuscular) emission.

2. The application of methods of the theory of stochastic processes to calculating the characteristics of nuclear phenomena beyond the framework of the theory of counters has been substantiated. The attention was concentrated on the stochastic nature of the DCS measurements in nuclear physics, which is similar, to some extent, to the photocounting statistics.

3. Starting from the simplest model of diffraction scattering, i.e. the model of total absorption with the surface spin-orbit interaction, we have constructed the amplitude of the elastic scattering of nucleons by atomic nuclei with zero spin, which takes into account the fluctuation character of the nucleon-nucleus interaction.

4. As an example, the amplitude of scattering has been constructed assuming a uniform distribution of the angular momenta of partial orbital waves within the limits of their uncertainties. But nothing prevents from using other, more complicated distributions.

5. The DCS and the polarizations of scattered nucleons have been calculated making no use of the quasiclassical transition, typical of the diffraction approach, from the summation over orbital waves with discrete angular momenta to the integration over the relevant continuous variable.

6. At small scattering angles, which are inherent to the traditional diffraction approach, our approach allows one to reproduce adequately and simultaneously both the experimental DCSs and the polarizations of scattered nucleons, removing deep diffraction dips. But at large angles, the description of experimental data has a qualitative character, in particular, for polarizations.

7. To improve an agreement with experiment, we made allowance for zero oscillations of the nuclear surface which generate, according to a corresponding calculation, the exponential damping of a DCS at large scattering angles.

8. Fluctuations of the nuclear density can serve as a primary source of fluctuations of the limiting angular momenta (i.e. the radius of the nucleon-nucleus interaction). It is evidenced, in particular, by a coincidence of the parameter values calculated by us with those available in the literature.

Thus, the model of statistical diffraction, proposed by us, has a certain physical substantiation, illustrates the basic features of the DCS and polarization behavior at the scattering of protons of several hundreds of MeV in energies, and can be applied for the estimation of the relevant structural characteristics of nuclei. Making the model more complicated and using a huge body of knowledge produced by the theory of stochastic processes, we may expect for a better understanding of the physics of nuclear scattering which has much in common with stochastic light optics.

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АДРОН-ЯДЕРНЕ РОЗСІЯННЯ У СТОХАСТИЧНІЙ ЯДЕРНІЙ ОПТИЦІ

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Р е з ю м е

Спираючись на глибоку аналогію між світловою оптикою та фізикою ядерного розсіяння, обгрунтовано застосування методів теорії стохастичних процесів для розрахунків характеристик пружних ядерних зіткнень. Виходячи з моделі повного поглинання з приповерхневою спін-орбітальною взаємодією, у дифракційному наближенні побудовано амплітуду пружного розсіяння нуклонів атомними ядрами з нульовим спіном, яка враховує флуктуаційний характер нуклон-ядерної взаємодії. При невеликих кутах розсіяння досягається одночасне узгодження з експериментом як диференціальних перерізів, так і кутових залежностей поляризації розсіяних нуклонів. При великих кутах розрахунки узгоджуються з даними про диференціальні перерізи з врахуванням нульових коливань ядерної поверхні. Збіг результатів розрахунків із поляризаційними даними має якісний характер. Показано, що джерелом флуктуацій граничних кутових моментів орбітальних хвиль можуть бути флуктуації ядерної густини.