PIEZO-OPTICAL SURFACES

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We present a method of construction of piezo-optical surfaces (POS) which consists in the finding of the difference of position vectors of the optical indicatrix perturbed by a mechanical stress and a nonperturbed one. It is proved that the method of construction of the indicative surfaces (IS) of the longitudinal and transversal piezo-optical effects (POE) is a partial case of the offered method. The examples of POS are adduced for cubic crystals BaF_2 and KBr which concern to the symmetry class m3m. It is shown that the transformation law of components of the tensor of piezo-optical coefficients (POC) upon a rotation of the coordinate system is identical to the difference of position vectors of the optical indicatrix perturbed by the tensor of POCs and those of a nonperturbed one. This statement is spread on other physical effects induced in crystals by external fields.

The anisotropy of a physical property of a crystal is most completely represented by the spatial distribution (the indicative surface) of the value of the appropriate effect [1,2]. Mathematically, the IS corresponds to a transformation law of components of the tensor of a physical quantity upon a rotation of the coordinate system. For the 4-rank tensor of POC, this law looks like

$$\pi'_{ijkl} = \alpha_{im}\alpha_{jn}\alpha_{kp}\alpha_{lo}\pi_{mnpo}, \tag{1}$$

where π'_{ijkl} is one of the components of the tensor of POC in the new coordinate system, π_{mnpo} are all the components of the tensor of POC in the old (crystal-physical) coordinate system X_1, X_2, X_3, and $\alpha_{im}, ..., \alpha_{lo}$ are the direction cosines between the axes of the new and old coordinate systems.

Let us take advantage of record (1) for cubic crystals [3] of the highest symmetry $m3m$ in the matrix denotations of POC [4,5]. The matrix of POC of these crystals has only three non-zero independent coefficients π_{11}, π_{12}, and π_{44}. Therefore, if the directions of the pressure action and the light polarization coincide, formula (1) for the longitudinal POE transforms to the simple relation for the IS of the POE

$$\pi'_{11} = \pi_{11} + 2(\pi_{12}\, \pi_{11} + \pi_{44})\times$$

$$\times(\alpha_{11}^2\alpha_{12}^2 + \alpha_{11}^2\alpha_{13}^2 + \alpha_{12}^2\alpha_{13}^2), \tag{2}$$

where α_{11}, α_{12}, and α_{13} are direction cosines between the axis X'_1 of a mobile coordinate system, with which the position vector **R** describing the surface coincides, and the axes X_1, X_2, and X_3 of the old coordinate system.

If we pass to a spherical coordinate system, in which the direction cosines are given by the known formulas (see, for example, [3])

$$\alpha_{11} = \sin\theta\cos\phi, \quad \alpha_{12} = \sin\theta\sin\phi, \quad \alpha_{13} = \cos\theta, \tag{3}$$

then we get a formula for the IS of the longitudinal POE:

$$R(\theta,\phi) = \pi'_{11} = \pi_{11} + 2(\pi_{12} - \pi_{11} + \pi_{44})\times$$

$$\times(\sin^4\theta\sin^2\phi\cos^2\phi + \sin^2\theta\cos^2\theta), \tag{4}$$

where R is the length of the position vector **R** which is given by the spherical coordinates θ, ϕ and coincides with the direction of light polarization; θ is the angle between a position vector **R** and axis X_3, ϕ is the angle between the projection of a position vector **R** on the plane X_1, X_2 and the axis X_1.

In works [3, 6—10], the method of construction of IS for the longitudinal and transversal POE is surveyed in detail. In the present work, we develop other method of the spatial description of POE which allows one to find the POE value at arbitrary mutual orientations of the light polarization vector **i** and the pressure action vector **m**. The essence of our method consists in the finding of the difference of values of the position vectors (indices of refraction) of the optical indicatrix of a crystal perturbed by a mechanical stress σ_m and a free one.

Let us write the equation for an optical indicatrix [4,5] as

$$a_1 x_1^2 + a_2 x_2^2 + a_3 x_3^2 = 1, \quad (5)$$

where $a_i = 1/n_i^2$ are the main polarization constants, n_i are the main indices of refraction (semi-axes of an optical indicatrix), and x_i are the Cartesian coordinates.

In a spherical coordinate system, if $x_1 = r\sin\theta\cos\phi$, $x_2 = r\sin\theta\sin\phi$, $x_3 = r\cos\theta$, Eq. (5) looks as

$$a_1 r^2 \sin^2\theta\cos^2\phi + a_2 r^2 \sin^2\theta\sin^2\phi + a_3 r^2\cos^2\theta = 1, \quad (6)$$

where r is the length of a position vector r which describes a characteristic surface of the tensor of polarization constants a_i (the optical indicatrix). Therefore, r has the sense of the refraction index of a crystal at the light polarization in an arbitrary space direction which is determined by the polar coordinates θ and ϕ.

The equation for an optical indicatrix perturbed by an external field in a Cartesian coordinate system looks like [11,12]

$$(a_1 + \delta a_1)x_1^2 + (a_2 + \delta a_2)x_2^2 + (a_3 + \delta a_3)x_3^2 +$$
$$+ 2\delta a_4 x_2 x_3 + 2\delta a_5 x_1 x_3 + 2\delta a_6 x_1 x_2 = 1 \quad (7)$$

and, in a spherical coordinate system respectively

$$(a_1 + \delta a_1) r_z^2 \sin^2\theta\cos^2\phi +$$
$$+ (a_2 + \delta a_2) r_z^2 \sin^2\theta\sin^2\phi + (a_3 + \delta a_3) r_z^2\cos^2\theta +$$
$$+ \delta a_4 r_z^2 \sin 2\theta\sin\phi + \delta a_5 r_z^2 \sin 2\theta\cos\phi +$$
$$+ \delta a_6 r^2 \sin^2\theta \sin 2\phi = 1. \quad (8)$$

Here, r_z is the length of a position vector $\mathbf{r}_z$ (the index of refraction) of a perturbed indicatrix; δa_i (i =1,2,3) are changes of the main polarization constants which determine changes of the main indices of refraction [see the comment to (5)], and δa_i (i =4,5,6) describes rotations of the optical indicatrix around the axes X_1, X_2, and X_3, respectively.

Having defined values of r and r_z from (6) and (8) and having subtracted them, we get the function which describes the spatial surface of values of the refraction index δn (ϕ, θ) which is defined by changes δa_i (i =1,2..., 6) of components of the tensor of polarization constants:

$$\delta n(\theta,\phi) = r_z - r = -\big[(\delta a_1\cos^2\phi + \delta a_2\sin^2\phi)\sin^2\theta +$$
$$+ \delta a_3\cos^2\theta + (\delta a_4\sin\phi + \delta a_5\cos\phi)\sin 2\theta +$$
$$+ \delta a_6\sin^2\theta\sin 2\phi\big] / \big(2(a_1\sin^2\theta\cos^2\phi +$$
$$+ a_2\sin^2\theta\sin^2\phi + a_3\cos^2\theta)^{3/2}\big). \quad (9)$$

We note that relation (9) was obtained by using one of the postulates of piezo-optics: $\delta a_i << a_i$. This allows us to simplify the expression in the denominator of (9) which is a cube of the refraction index of a crystal in an arbitrary direction (θ, ϕ). This is easy to verify by substituting, for example for a cubic crystal, the condition $a_1 = a_2 = a_3 = 1/n^2$ in the denominator.

The changes of δa_i which enter (9) depend, in turn, on the value and direction of the action of an external field on the crystal. For POE, δa_i are induced by the mechanical stress and correspond to the basic law of POE

$$\delta a_i = \pi_{im}\sigma_m, \quad (10)$$

where π_{im} are POC (components of the tensor of POC), σ_m are components of the tensor of mechanical stresses, the indices i, m = 1,2, ..., 6 correspond to the directions of light polarization and action of uniaxial pressure, respectively.

Therefore, (10) presents six equations for $\delta a_1, \delta a_2, \ldots \delta a_6$, each consists of six terms which correspond to six components of the tensor of mechanical stresses σ_1, σ_2, ... σ_6 [5, 13], should be substituted in (9). Then we obtain an expression for the surface $\delta n(\theta,\phi)$ by means of POCs π_{im}. In addition, components of the tensor σ_m should be written in the spherical coordinate system, namely:

$$\sigma_1 = \sigma\sin^2\beta\cos^2\alpha, \quad \sigma_4 = \frac{\sigma}{2}\sin 2\beta\sin\alpha,$$
$$\sigma_2 = \sigma\sin^2\beta\sin^2\alpha, \quad \sigma_5 = \frac{\sigma}{2}\sin 2\beta\cos\alpha,$$
$$\sigma_3 = \sigma\cos^2\beta, \qquad \sigma_6 = \frac{\sigma}{2}\sin^2\beta\sin 2\alpha. \quad (11)$$

Here, σ is the mechanical stress created by an uniaxial compression (stretching) in a direction which is set by the polar coordinates β, α, which are, respectively, the angles between the pressure action vector **m** and the X_3 axis and between the projection of the vector **m** on the plane X_1, X_2 and the X_1 axis.

It is easy to prove the validity of (11) if we use the following short form of the tensor of mechanical stresses σ_m in the Cartesian coordinate system [12, 14]:

$$\sigma_m = [\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6] = \sigma[a^2, b^2, c^2, bc, ac, ab], \quad (12)$$

where a, b, and c are cosines of the angles between the pressure action direction and the principal axes X_1, X_2, and X_3, and express the cosines a, b, c through the spherical coordinates β, α by analogy to (3).

Let us remark that the polar coordinates β, α of the pressure action vector should be distinguished from the polar coordinates θ, ϕ which set the polarization vector of light.

If we substitute (11) in (10) and (10) in (9), we get a function $\delta n(\theta, \phi, \beta, \alpha)$, which describes a change of the index of refraction δn in any spatial direction (θ, ϕ) depending on a value and a direction of the acting mechanical stress $\sigma(\beta, \alpha)$. The appropriate general relation is awkward, therefore we give an example for cubic crystals of a maximum symmetry, for which we have only three non-zero independent coefficients π_{11}, π_{12}, and π_{44} from 36 components of the matrix of POC:

$$-\frac{2}{\sigma n^3}\delta n(\theta, \phi, \alpha, \beta) = \pi'_{im} = \left[\pi_{11}\sin^2\beta\cos^2\alpha + \right.$$
$$\left. +\pi_{12}(\sin^2\beta\sin^2\alpha + \cos^2\beta)\right]\sin^2\theta\cos^2\phi +$$
$$+\left[\pi_{11}\sin^2\beta\sin^2\alpha + \pi_{12}(\sin^2\beta\cos^2\alpha + \cos^2\beta)\right]\times$$
$$\times\sin^2\theta\sin^2\phi + (\pi_{11}\cos^2\beta + \pi_{12}\sin^2\beta)\cos^2\theta +$$
$$+\frac{1}{2}\pi_{44}\left[\sin^2\theta\sin 2\phi\sin^2\beta\sin 2\alpha + \right.$$
$$\left. + \sin 2\theta\sin 2\beta(\sin\phi\sin\alpha + \cos\phi\cos\alpha)\right]. \quad (13)$$

In (13), we have transferred the factor $(-2/(\sigma n^3))$ from the right side to the left one and got the expression which designates the value of POE describing a change δn in the directions of the light polarization (θ, ϕ) undet the action of σ along the direction (β, α). To prove that the expression on the left side in (13) has the content of POE is possible by means of the differentiation of (10) with respect to one component of the tensor σ_m: $\delta a_i = \delta\frac{1}{n_i^2} = -\frac{2}{n_i^3}\delta n_i = \pi_{im}\sigma_m$. Whence we obtain the relation $\pi_{im} = -2\delta n_i/(n_i^3\sigma_m)$ similar to the left part of (13).

So, (13) describes the spatial distribution of POE. Therefore, there is a problem: By what does (13) differ from the indicative surface of POE (1) which also describes the spatial distribution of the effect.

Let us substitute the condition which correspond to the longitudinal POE in (13), if the direction of light polarization (θ, ϕ) coincides with the direction (β, α) of the action of a mechanical stress, i.e. $\theta = \beta$, $\phi = \alpha$. After transformations, we obtain the relation identical to (4) which is IS and corresponds to the transformation law of the tensor of POC (1) for cubic crystals of symmetry $m3m$ in the polar coordinates.

Let us check up this conclusion for the transversal POE, when the directions of the light polarization and the action of a pressure are mutually perpendicular. If IS is described by a position vector $\mathbf{R}$ which coincides with the light polarization and if the direction of a pressure changes in the plane X_1, X_2 (these conditions for IS are given in [3]), it is obvious that the polar coordinates of the directions of light polarization and the direction of a pressure are connected as follows: $\beta = 90°$, $\alpha = \phi + 90°$.

Having substituted the indicated values of β and α in in (13), we get the expression

$$\pi'^{(i)}_{12} = \pi_{12} + 2(\pi_{11} - \pi_{12} - \pi_{44})\sin^2\theta\sin^2\phi\cos^2\phi, \quad (14)$$

which is identical to the relation for IS of light polarization written in [3] on the basis of a transformation law of the tensor of POC (1). That is, expression (14) obtained as the difference of position vectors $\mathbf{r}_z - \mathbf{r}$ of a perturbed indicatrix and a nonperturbed one is also the transformation law of the tensor of POC in the polar coordinates.

We can also obtain the relation identical to (14) if we substitute the conditions $\theta = 90°$, $\phi = \alpha + 90°$ in (13). These conditions correspond to IS of the transversal POE which is described in space by a position vector $\mathbf{R}$ conterminous with the direction of action of the pressure, and the light polarization direction changes in the plane X_1, X_2 (the definitions of such IS are also formulated in [3]).

So, the differences of the position vectors $\mathbf{r}_z - \mathbf{r}$ of perturbed and nonperturbed optical indicatrices (9) and (13) in partial cases, which correspond to IS of the transversal and longitudinal POEs, are just the indicative surfaces of POE. The last correspond, in turn, to the transformation law of components of the tensor of POC upon a rotation of the coordinate system. The analysis, which has been carried out on the basis of the equations of indicative surfaces of cubic, three- and tetragonal crystals in [3, 6—10], proves that this conclusion is valid for crystals of any symmetry.

Therefore, the following statement is true: the difference of the position vectors $\mathbf{r}_z - \mathbf{r}$ of an indicatrix perturbed by the tensor of POC (owing to the action of mechanical stress) and a nonperturbed indicatrix is just the transformation law of the tensor of POC upon a rotation of the coordinate system.

If we substitute the appropriate direction cosines on the basis of (3) and (11) in the relations such as (13) instead of the polar coordinates $\theta, \phi, \beta, \alpha$, we get the transformation laws for components of the tensor in the ordinary form such as in (2). Clearly, this statement is true not only for the 4-rank tensor of POC, but also for the tensors for other effects and of other ranks, e.g. for those tensors which perturb a characteristic surface of a material 2-rank tensor.

Let us pass to the analysis of formula (13) for cubic crystals of the highest symmetry.

1. Formula (13) and similar formulas for the crystals of other symmetry classes are the general expressions of POS, whereas the IS formulated and investigated in [3, 6—10] are partial cases of POS.

Surface (13) enables us to estimate a spatial distribution of POE for any conditions of a mutual orientation of the vectors of light polarization $\mathbf{i}$ and pressure action $\mathbf{m}$ and to construct not only the surfaces of changes of the optical indicatrix upon the action of a pressure in a fixed direction, but also the surfaces of changes of the refraction index in a fixed direction under a change in the pressure action direction in the whole space.

2. It is of interest, for example, how the optical indicatrix will change if the pressure acts along one of the principal axes X_1, X_2, X_3. Having substituted the appropriate condition in (13) in the case, for example, where $\mathbf{m}||X_1$, that is, $\beta = 90°$, $\alpha = 0°$, we get the appropriate expression for the surface $\delta n(\theta, \phi)$ which we call as POS of polarization:

$$\delta n(\theta,\phi) = -\frac{\sigma}{2} n^3 \left[\pi_{11} \sin^2\theta \cos^2\phi + \right.$$

$$\left. + \pi_{12}(\sin^2\theta \sin^2\phi + \cos^2\theta)\right]. \tag{15}$$

In Figure, the exterior of such piezo-optical surface is shown. Let us remark that all the POS shown in Figure are given in the format of POE by transferring the factor $(-\sigma n^3/2)$ from the right side of equations such as (13), (15) to the left one.

Interesting and unexpected is the fact that the POS which describes a change of the index of refraction along one of the principal axes, for example X_1, upon a change of the direction $\mathbf{m}$ of pressure action in the whole space is identical to (15). We call it as the POS of a mechanical stress. It is easy to be verified by substituting the conditions for the direction of polarization $\mathbf{i}||X_1$, i.e. $\theta = 90°$, $\phi = 0$, in (13).

Moreover, on the basis of (13), it is possible to prove that POS of a polarization and POS of a mechanical stress are identical for any arbitrary direction of the pressure action (β, α) or a change of the index of refraction (θ, ϕ). This conclusion follows from the fact that the factors which consist of trigonometric functions and stand at POC π_{11}, π_{12}, and π_{44}, are identical relative to the coordinates θ, ϕ and β, α. If the identity of the factors of π_{11} and π_{44} is obvious in (13), it is possible to prove a similar identity of the factors of the coefficient π_{12} by multiplying the term $\pi_{12} \sin^2\beta$ in the second line of (13) by the sum $(\sin^2\alpha + \cos^2\alpha)$ equal to 1.

The analysis shows that POS of a polarization and POS of a mechanical stress for crystals of the lowest symmetry will be different.

3. In Figure, *a* we show surface (15) for crystal BaF_2 (the symmetry class $m3m$). It is seen that, under the action of a pressure along one of the principal axes, the maximum value of POE is observed in the plane perpendicular to the pressure action axis, and the POE is mainly positive. The negative effect is characteristic only of a narrow spatial "lobe" prolated along the pressure action axis. In the directions which form a cone around the pressure action axis with a solid angle

$$\omega = \text{arctg}\sqrt{-\pi_{11}/\pi_{12}} = 27.5°, \tag{16}$$

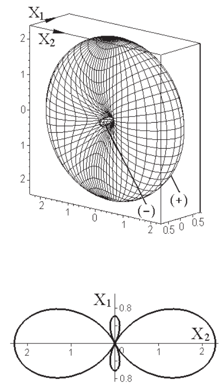
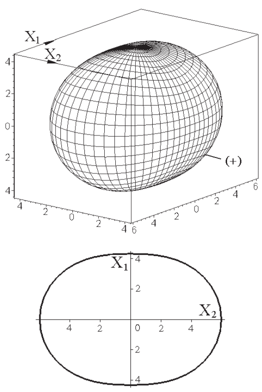
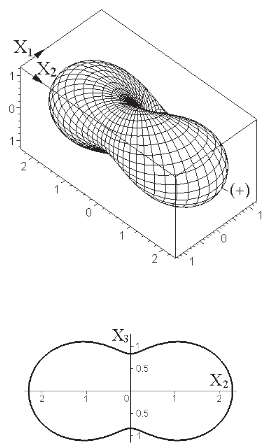
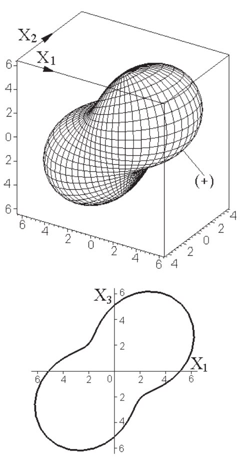
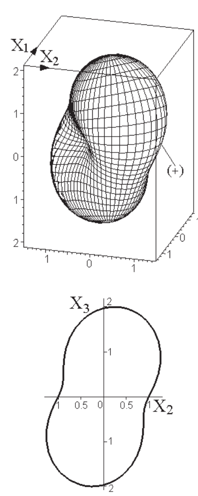
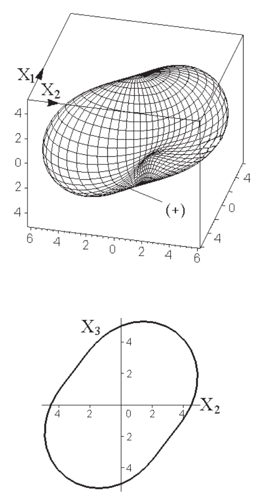
the POE is equal to zero.

We can easily get relation (16) if we cut, at first, POS (15) by one of the planes which pass through the pressure action axis X_1 (for example, by the plane perpendicular to X_3, which corresponds to the condition $\theta = 90°$) and equate the obtained expression for $\delta n(\theta = 90°, \phi)$ to zero.

In the other pressure action directions shown in Figure, the anisotropy of POE (a change of the optical indicatrix) is less pronounced. For example, the pressure action direction $\alpha = 0°, \beta = 45°$ (Figure,*b*), i.e. $\mathbf{m}$ makes an angle of 45° with the X_3 axis in the plane X_1, X_3, causes only the positive POE. Thus, the maximum absolute value of POE is observed along the X_2 axis.

We note that (16) contains the sign "−" in the radicand, which requires that the coefficients π_{11} and π_{12} have different signs (for BaF_2, π_{11} is negative, as wiil be indicate below). If the signs of these POCs are identical, POS of such a type will be one-sign and will not have zero values (see, for example, Figure,*a* for crystal KBr).

4. If we compare the POS of crystals BaF_2 and KBr (symmetry $m3m$), a large difference in the POE

α, β	BaF_2	KBr
a) $\alpha = 0°$, $\beta = 90°$		
b) $\alpha = 0°$, $\beta = 45°$		
c) $\alpha = 45°$, $\beta = 70°$		

Piezo-optical surfaces (POS) for crystals BaF_2 and KBr for different directions of the action of a pressure (α,β) and some cross-sections of these POS by the main planes

anisotropies of these crystals is characteristic of the pressure action along one of the principal axes only (Figure,a). For other selected conditions of POS (Figure,b,c), the anisotropy is weakly pronounced, which is seen, in particular, on the cross-sections of POS by the main planes.

The values of POCs needed for the construction of the surfaces of POE are borrowed from [15] (in units of 10^{-12} m^2/N = 1 Brewster). For $BaF_2 - \pi_{11} = -0.6$; $\pi_{12} = 2.3$; $\pi_{44} = 1.1$; and, for KBr, $\pi_{11} = 6.0$; $\pi_{12} = 4.3$; $\pi_{44} = -4.5$. For the estimation of a piezo-optical change of the refraction index $\delta n(\theta, \phi) = -\frac{1}{2}\pi' \sigma n^3$ [where π' is the value of POE in a selected direction of POS; see the expression in square brackets in (15)], we give also the refraction indices of these crystals [15]: n =1.473 for BaF_2 and n =1.556 for KBr.

Conclusion

We have proved that the difference of the position vectors of an optical indicatrix perturbed by the tensor of mechanical stresses and a nonperturbed one corresponds to the transformation law of the tensor of POC upon a rotation of the coordinate system. The offered way of the construction of the spatial distribution of POE enables one to study more fully the spatial anisotropy of POE, than the method of indicative surfaces of POE, and is, therefore, an essential addition to the technique of piezo-optics. The given method can be applied also to other physical effects which describe disturbances of the characteristic surface of the material second-rank tensor under the action of external fields.

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П'ЄЗООПТИЧНІ ПОВЕРХНІ

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Р е з ю м е

Запропоновано метод побудови п'єзооптичних поверхонь (ПОП), який полягає в знаходженні різниці радіус-векторів оптичних індикатрис, збуреної механічним напруженням і незбуреної. Доведено, що метод побудови вказівних поверхонь (ВП) поздовжнього і поперечного п'єзооптичного ефекту є частинним випадком пропонованого методу. Приклади ПОП наведено для кубічних кристалів BaF_2 і KBr, що належать класу симетрії $m3m$. Показано, що закон перетворення компонент тензора п'єзооптичних коефіцієнтів (ПОК) при повороті системи координат тотожний різниці радіус-векторів оптичних індикатрис, збуреної тензором ПОК і незбуреної. Це твердження поширене на інші фізичні ефекти, індуковані в кристалах зовнішніми полями.