EFFICIENCY OF ENERGY CONVERSION BY A BROWNIAN MOTOR WITH A FLUCTUATING DOUBLE-WELL POTENTIAL

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The efficiency of a Brownian motor with a periodic double-well potential, which undergoes the random shifting by half a period, has been analyzed in the framework of the thermodynamics of nonequilibrium processes, which provided the energy conversion from one form into another. An introduced measure of coupling between corresponding processes has been determined in terms of the difference between the potential barrier heights. The larger the difference, the stronger the coupling between processes, with the efficiency of the Brownian motor approaching unity.

A common feature of all devices that convert energy from one form into another is that they couple two processes, one of which supplies energy to a device, and the other withdraws it. The higher the degree of coupling q between these processes, the higher the efficiency of the device η, which is defined as the ratio of the useful work executed by a device to the spent energy. In the classical work of Kedem and Caplan [1], every such process is characterized by two thermodynamically conjugated quantities, the generalized force and current. In the thermodynamics of nonequilibrium processes, generalized currents are assumed to be linear combinations of generalized forces [2]. The parameter q can be expressed in terms of the kinetic coefficients of those linear combinations in such a way that its domain of variation will be an interval from zero to unity, with $\eta \to 1$ for $q \to 1$ [1]. Then, the phenomenological approach allows one to introduce a quantitative measure of coupling between two processes and to determine the efficiency of conversion of the corresponding energies as a function of this parameter.

The rapid development of nanotechnology has not also bypassed the miniaturization of energy converters. “Brownian motors” is a general name for “nanomachines” that convert various kinds of energy into a mechanical one, for systems that segregate nanoparticles, for molecular pumps that function on the energy of lysis of adenosine triphosphate, and so on. For today, a number of models have been proposed which describe the functioning of Brownian motors, in particular, the emergence of an ordered motion of particles in asymmetric potentials that fluctuate owing to arbitrary external factors, e.g., chemical reactions [3–5]. The mechanism of high efficiency that is inherent to such models was explained in work [6]. Its elementary realization is based on the consideration of a periodic double-well potential, which undergoes the random shifting by half a period with a certain frequency [7]. The assumption about a linear dependence between the generalized currents and forces is not necessary in this model for its high efficiency to be obtained. Nevertheless, as will be shown in this work, it is this assumption that ensures the highest efficiency of the motor. Therefore, it is reasonable to carry out the relevant consideration in a linear approximation, which allows the phenomenologically induced parameters of work [1] to be connected with the parameters of model [7], the latter having a clear physical meaning.

The Brownian motor couples two processes, the first one being responsible for supplying the motor with energy and the second for the production of a useful energy by the motor. Let X_1 and X_2 designate the generalized thermodynamic driving forces of those processes, while let J_1 and J_2 be the corresponding generalized currents. The production of the entropy dS/dt by those two processes and its connection to the efficiency of the motor η are determined by the expressions [1]

$$\frac{dS}{dt} = J_1X_1 + J_2X_2 = J_2X_2\left(1-\eta\right), \quad \eta = -\frac{J_1X_1}{J_2X_2}, \quad (1)$$

where J_2X_2 is the positive power that the motor is supplied with and J_1X_1 is the power generated by the motor and considered negative (hereafter, all energy quantities are measured in terms of $k_{\mathrm{B}}T$ units, where k_{B} is the Boltzmann constant and T is the absolute temperature). The state of thermodynamic equilibrium is determined by the conditions $X_2 = X_1 = 0$, under which $J_2 = J_1 = 0$. Therefore, expansions of the generalized currents in terms of small generalized forces are valid near the equilibrium point. Supposing that

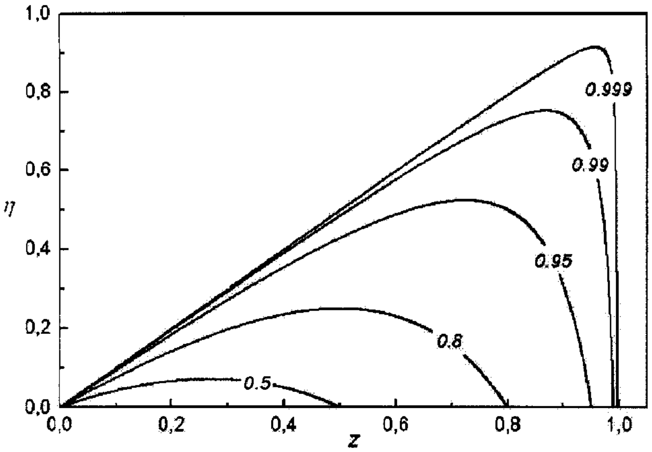

Fig. 1. Dependences of the efficiency of a Brownian motor η on the z variable that characterizes the ratio between the generalized forces for various values of the measure q of the coupling between the processes of energy supply and energy generation. The values of q are indicated on the curves

those expansions are linear, we have

$$J_i = \sum_{j=1}^{2} L_{ij} X_j, \quad i = 1,\ 2, \tag{2}$$

where the expansion factors L_{ij} satisfy the Onsager symmetry relation $L_{12} = L_{21}$ and the inequalities $L_{11} > 0$, $L_{22} > 0$, and $L_{12}^2 \leq L_{11} L_{22}$, which ensure the nonnegativeness of the quadric form, $dS/dt \geq 0$.

If one introduces the variable

$$z \equiv \sqrt{L_{11}/L_{22}}\,(-X_1/X_2) > 0, \tag{3}$$

which is proportional to the ratio between the generalized forces, and defines the parameter

$$q \equiv L_{12} \Big/ \sqrt{L_{11} L_{22}} \quad (0 \leq q \leq 1), \tag{4}$$

then the efficiency of the motor can be expressed as the function of z:

$$\eta\,(z) = \frac{z\,(q - z)}{1 - qz}. \tag{5}$$

The plots of this function for various values of q are shown in Fig. 1. One can see that the maximum η_m of the function $\eta(z)$ approaches the ideal limit $\eta = 1$ as $q \to 1$. Therefore, the parameter q, which is defined by relation (4), can be considered as a quantitative measure of coupling between two considered processes. In this case,

$$\eta_m = q^2 \Big/ \left(1 + \sqrt{1 - q^2}\right)^2, \quad z_m = q \Big/ \left(1 + \sqrt{1 - q^2}\right). \tag{6}$$

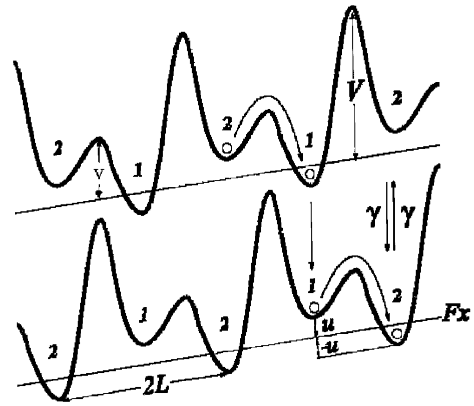

Fig. 2. Fluctuating double-well potentials which ensure the functioning of a Brownian motor

If $q \to 1$, the value of η_m approaches unity by the law $\eta_m \to 1 - 2\sqrt{1 - q^2}$.

The presented relations describe the efficiency of energy conversion formally, in terms of the generalized currents and forces. They acquire a clear physical meaning in the framework of the Brownian motor model, where the generation of unidirectional motion occurs due to the random shifting of a periodic double-well potential by half a period with the frequency γ [7]. In Fig. 2, the model of such a motor is depicted, the parameters of the fluctuating potential profile and the linear potential Fx of the load force being also indicated. In the framework of this model, the power $-J_1 X_1$ produced by the motor is determined as the product of the average velocity of the unidirectional motion $2LJ_1$, where $2L$ is the period of the potential, and the applied force F. This yields that the generalized force is the quantity $-2FL$. The explicit expression for the current J_1 looks like [7]

$$J_1 = \frac{1}{2}\gamma\zeta \times$$

$$\times \frac{\sinh\,(u - f) - e^{-V+v}[\sinh\,(u + f) + 2\zeta \sinh\,(2f)]}{1 + \zeta[\cosh\,(u - f) + e^{-V+v}\cosh\,(u + f)]}. \tag{7}$$

Here, $2u$ is the difference between the potential well minima, $V - v$ is the difference between the potential barrier heights, $f = FL/2$, and the parameter ζ is equal to the ratio between the characteristic frequency of the thermally activated overcoming of the small barrier $k_0 \exp(-v)$, k_0 being the frequency of collisions with this barrier, and the frequency of potential switching γ. The expressions for the supplied power $J_2 X_2$ were obtained in work [7]. They depend on the distribution functions

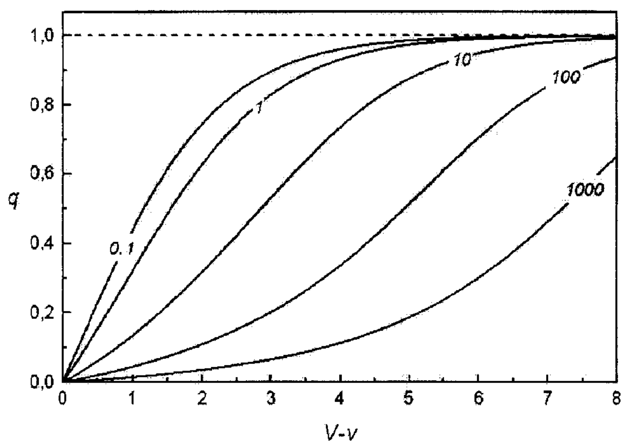

Fig. 3. Dependences of the measure of coupling q on the difference between the barrier heights for various values of the model parameter ζ. The values of ζ are indicated on the curves

which are presented there in the explicit form. Making use of those expressions, it is easy to show that the supplied power can be expressed as $4uJ_2$, where

$$J_2 = \frac{1}{2}\gamma\zeta\frac{\sinh(u-f) + e^{-V+v}\sinh(u+f)}{1+\zeta[\cosh(u-f) + e^{-V+v}\cosh(u+f)]}. \quad (8)$$

Therefore, the generalized forces in this model are the parameters $X_1 = -4f$ and $X_2 = 4u$.

The expansions of the currents J_1 and J_2 in terms of small f and u bring about the following expressions for the coefficients L_{ij} in formula (2):

$$L_{11} = \frac{1}{8}\gamma\zeta\frac{1+(1+4\zeta)\,e^{-V+v}}{1+\zeta\,(1+e^{-V+v})},$$

$$L_{22} = \frac{1}{8}\gamma\zeta\frac{1+e^{-V+v}}{1+\zeta\,(1+e^{-V+v})},$$

$$L_{12} = L_{21} = \frac{1}{8}\gamma\zeta\frac{1-e^{-V+v}}{1+\zeta\,(1+e^{-V+v})}. \quad (9)$$

As a result, the parameter of coupling (4) between the two processes equals

$$q = \frac{1-e^{-V+v}}{\sqrt{1+2\,(1+2\zeta)\,e^{-V+v} + (1+4\zeta)\,e^{-2(V-v)}}}. \quad (10)$$

The plots of the dependence of q on $V-v$ for various ζ are shown in Fig. 3. We note that, for $\zeta \ll 1$, relation (10) gives $q \approx \tanh[(V-v)/2]$. The parameter q tends to unity quickly when the difference between the barrier heights becomes large enough $[\exp(V-v) \gg 1]$:

$$q \approx 1 - 2\,(1+\zeta)\,e^{-V+v}. \quad (11)$$

In this case, according to Eq. (6), the maximal value of the efficiency approaches unity by the law

$$\eta_m \approx 1 - 4\sqrt{1+\zeta}e^{-(V-v)/2}. \quad (12)$$

This dependence is in agreement with the conclusions of works [6,7], according to which the deviation of the peak efficiency from unity diminishes by the law $\exp(-V/2)$ as the height V of the higher barrier, which blocks the reverse current, grows.

Thus, the functioning of the Brownian motor, where the unidirectional motion is induced by random shifts of the periodic double-well potential by half a period, brightly illustrates the general regularities that accompany the energy conversion [1]. In the model under investigation, the energy supplied to the motor is proportional to the energy difference between the well minima of the potentials, which are being switched, while the useful energy produced by the motor is equal to the work against the external load force. The measure of coupling between those power processes is the parameter q, which is defined by relation (10). It depends on the difference between the potential barrier heights. The larger this difference, the stronger the coupling between the processes and the closer the efficiency of a Brownian motor to unity.

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ЕФЕКТИВНІСТЬ ПЕРЕТВОРЕННЯ
ЕНЕРГІЇ БРОУНІВСЬКИМ МОТОРОМ
З ФЛУКТУЮЮЧИМ ДВОХ'ЯМНИМ ПОТЕНЦІАЛОМ

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Р е з ю м е

Ефективність броунівського мотора з періодичним двох'ямним потенціалом, який зазнає випадкових зсувів на півперіоду, проаналізована в рамках термодинаміки нерівноважних процесів, що забезпечують перетворення енергії з однієї форми в іншу. Введена міра зв'язку відповідних процесів визначається різницею висот потенціальних бар'єрів. Чим більша ця різниця, тим сильніше зв'язані процеси, а ефективність броунівського мотора наближається до одиниці.