LIGHT PRESSURE ON ATOMS IN THE FIELD OF COUNTERPROPAGATING WAVES WITH STOCHASTIC AMPLITUDES

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The numerical simulation of the light pressure on atoms in the field of counterpropagating waves with stochastic amplitudes has been carried out. It has been shown that, similarly to the light pressure exerted in the field of two counterpropagating bichromatic waves, the light pressure on an atom in counterpropagating stochastic waves, with one of them repeating the other with some delay, may considerably exceed the pressure exerted in the field of a single running wave.

1. Introduction

The study of the phenomenon of light pressure on atoms during last decades has brought about the development of new methods for manipulating the motion of atoms, which are based on the interaction of atoms with counterpropagating light waves [1—20]. The appropriate choice of the law of modulation of counterpropagating waves allows the process of atom-light interaction to be organized in such a manner that the atom should absorb a photon from one wave and emit it into the other one, with the atom momentum changing by $2\hbar k$, where $\hbar k$ is the momentum of a photon. The time interval needed for the atom momentum to be changed by this value is defined by the rate of the stimulated transitions, which is proportional to the electric field strength of light waves, and the modulation period of counterpropagating waves, and can be significantly smaller than the time of emission from the excited atom state. As a result, the light pressure caused by stimulated transitions, or the stimulated light pressure, can considerably exceed the pressure on an atom in the field of a single running wave $F_{\text{sp}} = \frac{1}{2}\hbar k\gamma$, where γ is the inverse lifetime of the atom in an excited state. The condition for the stimulated light pressure to emerge is, in general, the presence of a correlation of the amplitudes or phases (or simultaneously both amplitudes and phases) of counterpropagating waves. This pressure can also arise in the fields with stochastic modulation of the amplitude or phase, as was pointed out in works [11, 12], if, for example, the field of one of the counterpropagating waves repeats the field of the other wave with a certain time delay.

In work [20], the stimulated light pressure with stochastic modulation of the frequency (the phase diffusion) was theoretically studied. It was shown that the stochastic modulation of the phase gives an effect close to that caused by sinusoidal modulation. At the same time, the case of the stochastic amplitude modulation remains not studied for today; only the weak stochastic amplitude modulation of counterpropagating waves has been examined. The interest to stochastic modulation is connected with the opportunity to apply a powerful, but close to stochastic, emission of multimode lasers for manipulating the motion of atoms.

In this work, the numerical study of the light pressure on atoms in the field of counterpropagating waves has been carried within the stochastic field model which describes the laser emission with multiple non-correlated modes [21].

2. Model

Let us consider a two-level atom interacting with the field of two counterpropagating waves with identical frequencies, which coincide with that of the atomic transition ω_0 between the ground $|1\rangle$ and excited $|2\rangle$

states of the atom. The system is supposed to be closed, i.e. the atom comes back after a spontaneous emission from an excited state into the ground state. In the quasiclassical approximation, the approximate equations for the density matrix in the atom reference system look like

$$\frac{\partial}{\partial t}\rho_{11} = \frac{iE}{\hbar}(d_{12}\,\rho_{21} - d_{21}\,\rho_{12}) + \gamma\rho_{22}\,,$$
$$\frac{\partial}{\partial t}\rho_{22} = \frac{iE}{\hbar}(d_{21}\,\rho_{12} - d_{12}\,\rho_{21}) - \gamma\rho_{22}\,,$$
$$\frac{\partial}{\partial t}\rho_{12} = \frac{iE}{\hbar}(d_{12}\,\rho_{22} - d_{12}\,\rho_{11}) + i\omega_0\rho_{12} - \frac{\gamma}{2}\rho_{12}\,,$$
$$\frac{\partial}{\partial t}\rho_{21} = \frac{iE}{\hbar}(d_{21}\,\rho_{11} - d_{21}\,\rho_{22}) - i\omega_0\rho_{21} - \frac{\gamma}{2}\rho_{21}\,. \quad (1)$$

Here, $d_{12} = d_{21}^*$ are the matrix elements of the dipole moment, and E is the electric field strength at the point where the atom is located.

Now, let us write down the electric field strength in the atom reference system. Owing to the Doppler effect, the frequencies of counterpropagating waves felt by a movable atom differ by kv from the frequency ω_0, where v is the projection of the atom velocity onto the wave propagation direction, so that $\omega_1 = \omega_0 + kv$ and $\omega_2 = \omega_0 - kv$. Therefore,

$$E = \frac{1}{2}\left(E_1(t)e^{-i\omega_0 t - ikvt + ikz} + E_1^*(t)e^{i\omega_0 t + ikvt - ikz}\right) +$$

$$+\frac{1}{2}\left(E_2(t)e^{-i\omega_0 t + ikvt - ikz} + E_2^*(t)e^{i\omega_0 t - ikvt + ikz}\right)\,. \quad (2)$$

Here, we neglect the difference of the wave vectors of counterpropagating waves in the atom reference system, taking into account a very small value of the ratio kv/ω_0. The field of a wave is supposed to repeat the field of the other one with a time delay τ,

$$E_2(t) = E_1(t-\tau) \quad (3)$$

(for example, wave 2 is obtained by reflecting wave 1 from a mirror). The phase shift $\omega_0\tau - kv\tau$ which corresponds to that delay was not introduced into the term of Eq. (2), because it can be compensated by a proper choice of the origin of the z-coordinate, the averaging over which will be carried out further. The real and imaginary parts of $E_n(t)$ $(n = 1, 2)$ are described by independent Gaussian fluctuations (the stochastic field model) [21]. We introduce the Rabi frequencies $\Omega_n(t) = d_{12}E_n(t)/\hbar$. For $\Omega_1(t)$, we have

$$\langle\Omega_1(t)\rangle = 0, \qquad \langle\mathrm{Re}(\Omega_1(t))\,\mathrm{Im}(\Omega_1(t'))\rangle = 0,$$

$$\langle\mathrm{Re}(\Omega_1(t))\,\mathrm{Re}\,\Omega_1(t')\rangle = \frac{1}{2}\Omega_0^2\exp\left(-G|t-t'|\right),$$

$$\langle\mathrm{Im}(\Omega_1(t))\,\mathrm{Im}\,\Omega_1(t')\rangle = \frac{1}{2}\Omega_0^2\exp\left(-G|t-t'|\right) \quad (4)$$

(the Ornstein—Uhlenbeck process).

The force that acts on the atom is calculated according to the formula [22, 23]

$$F = d(\rho_{12} + \rho_{21})\frac{\partial E}{\partial z} \quad (5)$$

and is averaged over the atom coordinate.

3. Numerical Simulation

Two series of values of a random variable $\xi(t_j)$, which simulate the real and imaginary parts of $\Omega_1(t_j)$ at the time moments $t_j = t_{j-1} + \Delta t$, were generated making use of the colored noise simulation algorithm proposed in [24, 25]:

$$\xi(t_{j+1}) = \xi(t_j)\exp\left(-G\Delta t\right) + h(t_j), \quad (6)$$

with $h(t_j)$ being distributed according to the Gaussian law with the first momentum equal zero, and

$$\left\langle h(t_j)^2\right\rangle = \frac{1}{2}\Omega_0^2\left(1 - e^{-2G\Delta t}\right). \quad (7)$$

For generating the $h(t_j)$-series, the standard Matlab function *randn* was used.

The average value of the force exerted on the atom was calculated as the ratio between the momentum got by the atom during the time of averaging T and the time of force action, with a further averaging over the atom coordinate. The calculations were carried out in the "heavy"-atom approximation, neglecting the variation of the Doppler frequency shift during averaging. In order to diminish the influence of the initial stage of interaction on the results of averaging, the latter has been carried out starting from the time moment $t = 7/\gamma$ for a time interval $T = 30/\gamma$.

4. Results of Numerical Simulations

At first sight, the light pressure on an atom might seem to arise only in the case of time delays $\tau \sim 1/G$, because, for longer delays, the fields of counterpropagating waves are no more correlated, and one may expect that the result of their mechanical action on the atom is nil.

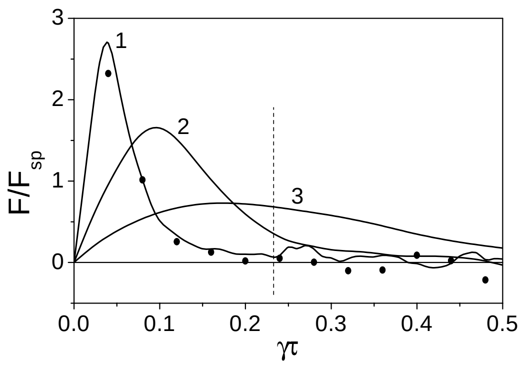

Fig. 1. Dependences of the light pressure in the field of counterpropagating stochastic waves in units of the light pressure in the field of a single running monochromatic wave $F_{\text{sp}} = \frac{1}{2}\hbar k\gamma$ on the time delay between the waves in units of $1/\gamma$. Parameters: $G = 10\gamma$ (for all the curves), $\Omega_0 = 25\gamma$ (*1*), 10γ (*2*), and 5γ (*3*). Circles correspond to the same parameters as curve *1* but for another realization of the stochastic process

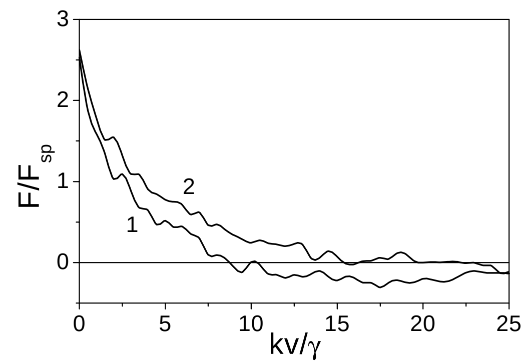

Fig. 2. Dependences of the light pressure in the field of counterpropagating stochastic waves in units of the light pressure in the field of a single running monochromatic wave $F_{\text{sp}} = \frac{1}{2}\hbar k\gamma$ on the atom velocity in units of γ/k. Parameters: $\tau = \Omega_0^{-1}$ for all the curves, which corresponds to the maximal light pressure on the atoms with a zero velocity component along the direction of the laser beam propagation; $\Omega_0 = 25\gamma$, $G = 10\gamma$ (*1*); $\Omega_0 = 25\gamma$, $G = 20\gamma$ (*2*)

Nevertheless, it should be taken into account that the atom "remembers" the field which acted on it for the time interval $\tau \sim 1/\gamma$. Therefore, the interaction between the atom and two fields with a large, as compared to the correlation time $1/G$, time delay between them can lead to appreciable effects connected to the correlation of those fields [20, 26].

In Fig. 1, the dependences of the light pressure on an atom in units of the maximal light pressure exerted in the field of a single running wave $F_{\text{sp}} = \frac{1}{2}\hbar k\gamma$ [22, 23] on the time delay between the waves in units of γ^{-1} are presented for various ratios between G and Ω_0. As the Rabi frequency increases, the amplitude of the maximum considerably grows and exceeds unity (curve *1*), which evidences for an opportunity of exceeding the light pressure in the field of a single running wave by that in the field of two stochastic counterpropagating waves. To estimate the accuracy of the presented results, the data obtained for the same parameters as in curve *1* but for another realization of the stochastic process are also depicted. The maxima of the curves are defined by the Rabi frequency, provided that $\Omega_0 \gg G$ (they are obtained if the time delay $\tau \sim \Omega_0^{-1}$). As the Rabi frequency decreases, the optimum time of delay is governed by the correlation time G^{-1}; if the inequality $\Omega_0 \ll G$ holds true, it can be seen from the analytical expression for the light pressure on the atom, averaged over the coordinate and the time, which can be obtained by solving the equations for the density matrix (1) using the method of perturbation theory and considering the field strength as a small parameter:

$$F = \hbar k \frac{\Omega_0^4}{G^2\gamma}\left[\exp\left(-\gamma\tau\right) - \exp\left(-\frac{1}{2}\gamma\tau - G\tau\right)\right]. \qquad (8)$$

The light pressure is maximal at $\tau = -G^{-1}\ln(g/G)$. In Fig. 1, this value is shown by a vertical dashed line for $G = 10\gamma$. The maximum of curve *3* is close to this value, though for the corresponding parameters of the curve do not fulfill the conditions for the perturbation theory eligibility.

It is worth pointing out the analogy of the obtained results with the dependence of the light pressure on atoms in a bichromatic field of counterpropagating waves on the phase difference between these waves. In this case, the frequency and phase differences of these waves play the role, respectively, of a reciprocal correlation time and a time delay. Similarly to the considered case of the atom interaction with stochastic counterpropagating waves, if the Rabi frequency is large as compared with the modulation one, the optimal phase difference between the waves is reciprocal to the Rabi frequency or, otherwise, it is close to $\pi/4$, which corresponds to $\frac{1}{8}$ of the modulation period [4].

The dependence of the light pressure in the bichromatic field of two standing waves (or, which is equivalent, in a field of amplitude-modulated counterpropagating waves) on the velocity projection in the wave propagation direction is known to be nonmonotonous [4]; in particular, the structure with a characteristic width of the order of γ/k is observed. At the velocity $v \sim \Omega_m/2k$, where Ω_m is the modulation frequency, the force direction becomes opposite. Fig. 2 illustrates the similar dependence obtained in the framework of the stochastic field model. The calculations of both the curves were carried out for the same realization of the stochastic process $\Omega_1(t)$. One can see that this dependence has a similar view in the case of stochastic modulation as well. The only difference is that the role of a modulation frequency is played by the doubled reciprocal correlation time $2G$ of counterpropagating waves.

5. Conclusions

We have shown that, in the field of counterpropagating stochastic waves, one of which repeats the other wave with some time delay, the light pressure on an atom, averaged over the coordinate and time, may considerably exceed the pressure exerted in the field of a single running wave. In the case of the large intensities of counterpropagating waves, where the Rabi frequency exceeds the reciprocal correlation time, the optimal delay time between waves is equal to the reciprocal Rabi frequency. This light pressure changes its sign, if the velocity projection onto the direction of wave propagation is close to G/k. The considered model of a laser field is close to the real fields of multimode lasers, which allows the latter to be used in experiments dealing with monitoring the light pressure action on atoms and to be applied in optical control systems for manipulating the atom motion.

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ТИСК СВІТЛА НА АТОМИ У ПОЛІ ЗУСТРІЧНИХ ХВИЛЬ ЗІ СТОХАСТИЧНОЮ АМПЛІТУДОЮ

В.І.Романенко

Р е з ю м е

Проведено чисельне моделювання світлового тиску на атоми в полі зустрічних хвиль зі стохастичною амплітудою. Показано, що сила світлового тиску на атом у зустрічних стохастичних хвилях, одна з яких повторює іншу з деякою затримкою, подібно до сили світлового тиску в полі двох зустрічних біхроматичних хвиль може значно перевищувати силу тиску в полі однієї біжучої хвилі.