HIGH-ENERGY APPROXIMATIONS FOR TWO-NUCLEON SCATTERING BASED ON THE DIRAC EQUATION WITH A POTENTIAL INTERACTION

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UDC 530

Based on the Dirac equation with direct potential interaction, the scattering of two nucleons in a singlet spin state is investigated within the Born and high-energy quasiclassical approximations. For the Yukawa potential, it is shown that the expansion of the total scattering amplitude in a series of perturbation theory with respect to the effective potential is not applicable. For potentials finite at zero, it is found that the Born scattering singlet S-phase has a non-zero high-energy asymptotics. For the potentials with Coulomb's singularity at small distances, it is obtained that the Born phase increases with energy logarithmically. For the Yukawa potential and a number of other interaction potentials, analytic expressions for perturbation terms of the second order to the Born S-phase are found. A qualitative analysis of the accuracy of the Born and eikonal high-energy approximations for the singlet S-scattering of nucleons is given.

1. Introduction

A relatively simple description of two-nucleon processes including relativistic effects can be developed with the use of the two-particle Dirac equation with direct interaction potential. This approach allows a transparent interpretation of scattering parameters in terms of potential functions [1, 2]. With this approach which takes into account relativistic effects in two-nucleon systems, it is possible to describe two-nucleon processes in singlet and triplet spin states within the same formalism. Notice also that the relativistic Dirac equations with direct potential interaction have interesting symmetry properties, namely, such equations possess, apart from standard, specific integrals of motion that are absent for non-relativistic equations [3—6].

From the Dirac equation for spatial components of wave functions of two particles, one can derive an equation of the Schrödinger—Breit type with a modified kinetic energy operator and effective potentials that depend on distance and the total energy of two particles. For the relatively simple singlet state of the two particles, when we have one spatial equation of the Schrödinger—Breit type, it is possible to carry out a detailed analysis of solutions, to obtain non-trivial results, and to describe the behavior of the singlet scattering S-phase of nucleons in a wide range of energies without the use of the standard strong repulsion core [2]. In the case of a triplet state of two nucleons (these equations are much more complicated), the relativistic equations include central, spin-orbital and tensor interactions (the tensor interaction is represented in terms of a momentum operator). Nevertheless, such triplet equations can be solved analytically for rectangular "potentials", which made it possible to create the model of a deuteron and to give a consistent explanation of its main experimental characteristics [7].

The principal feature of our approach is the use of the Dirac equation (instead of the Schrödinger equation) for the description of nucleon scattering at high energies and the application of approximate methods to this equation, such as the Born and high-energy quasiclassical approximations. In this work we consider the application of these methods to an equation of the Schrödinger—Breit type in order to describe the scattering of two nucleons in a singlet spin state. For the case of interaction in the form of the Yukawa potential, we consider the first and second Born approximations to the total scattering amplitude and the S and P scattering phases. For the interaction potentials that allow the analytic description of the S-phase of nucleon scattering, we consider the Born and quasiclassical approximations.

2. Two-particle Dirac Equation with Direct Interaction

As is known, the Dirac equation for two electrons with a direct potential interaction was introduced for the first time by Breit. In this equation, the direct two-electron interaction was taken into account as the lowest order approximation with respect to the fine structure constant. Later, these ideas were applied not only to electrons, but were also extended to relativistic two-nucleon nuclear systems, although only at the phenomenological level. The steady-state Dirac equation for two particles can be represented as a sum of two Dirac Hamiltonians of free motion for each particle and the direct interaction potential V. For the relativistic two-particle model (consider for simplicity only the central interaction), we can write the Dirac equation in the system of the center of inertia and in the equal-mass approximation (here, we use the system of units, in which $c = \hbar = 1$) as

$$\Big[(\vec{\alpha}_1 - \vec{\alpha}_2)\vec{p} + (\beta_1 + \beta_2)m + V\Big]\Psi = E\Psi, \qquad (1)$$

where E is the total energy of the system of two particles. The direct interaction (central) potential V of two particles is constructed with the Dirac matrices and comprises four spherically symmetric potential functions (a vector function $V_V(r)$, scalar one $V_S(r)$, pseudoscalar one $V_P(r)$ and $V_0(r)$, where $r = |\vec{r}_1 - \vec{r}_2|$):

$$V = (1 - \vec{\alpha}_1\vec{\alpha}_2)V_V + \beta_1\beta_2 V_S + \vec{\alpha}_1\beta_1\vec{\alpha}_2\beta_2 V_P + V_0. \qquad (2)$$

Here, $\vec{\alpha}$, β are the Dirac matrices:

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}, \ \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

In Eq. (1), the full wave function Ψ is represented by a direct product of bi-spinors 4×4 (16 functions) and depends on particle-antiparticle states, their spins, and distances between particles. In the general case, Eq. (1) is a system of sixteen differential equations of the first order.

In what follows, we consider only a singlet spin state ($S = 0$) of two nucleons. In this state, from system (1) with a direct potential interaction, we can obtain [2] one scalar wave equation of the Schrödinger—Breit type for one spinor component Ψ which we denote as ψ:

$$\vec{p}\,\frac{4}{E - V_3}\,\vec{p}\,\psi + \frac{4m^2}{E - V_2}\,\psi = (E - V_1)\psi. \qquad (3)$$

Here, we introduce, for brevity, new central potential functions V_1, V_2, and V_3 that describe the interaction in the singlet spin state and are constructed as a linear combination of the initial potentials (2):

$$V_1 \equiv V_0 + 4V_V + V_S - 3V_P,$$

$$V_2 \equiv V_0 - 2V_V + V_S + 3V_P,$$

$$V_3 \equiv V_0 - V_S - V_P.$$

It should be pointed out that the effective interaction of two particles in the relativistic equation (3) depends, apart from the relative distance, also on the total energy E through the fractional rational functions. Moreover, the modification of the kinetic energy operator, which depends on the total energy and the relative distance between particles through potential $V_3(r)$, indicates that the interaction between particles becomes effectively non-static and depending on system dynamics. Such a structure of the relativistic equation is due to the reduction of the system of equations (1) for the singlet spin state to one differential equation. It is interesting to note that the wave equation (3) is a differential equation of the second order, although it is a corollary of the system of differential equations of the total fourth order. In the non-relativistic limit, Eq. (3) is reduced to the standard Schrödinger equation with a potential that is a half-sum of the first and second potential functions. In the non-relativistic limit, the potential function $V_3(r)$ vanishes, which indicates its essentially relativistic nature. The function $V_3(r)$ is responsible for relativistic effects and starts to reveal itself at medium and high kinetic energies. In such a relativistic model, namely this potential function allows us [2] to explain the behavior of the two-nucleon singlet S-phase of scattering at medium and high energies without the use of the strong repulsive core at small distances.

3. Born Approximation to the Scattering Amplitude of Relativistic Particles

For asymptotically high energies, the effective interaction between particles in the singlet equation (3) becomes a linear function of the total energy and the sum of the first and third potentials. Namely at high energies, it is interesting to consider the Born and eikonal approximations. Without loss in generality, consider a version of Eq. (3) in which only one potential $V(r) = V_1(r) \neq 0$ is taken into account and others vanish, $V_2(r) = V_3(r) = 0$. In this case, we can obtain analytical solutions for some simple potential functions. Let us rewrite Eq. (3) with a non-zero first potential in the form of the standard Schrödinger equation with an effective interaction potential depending on the total energy:

$$-\triangle\psi(r) + \frac{1}{2}\sqrt{k^2 + m^2}\,V(r)\psi(r) = k^2\psi(r), \qquad (4)$$

where $4k^2 = E^2 - 4m^2$, and k is the wave vector. In this equation, the potential function $V(r)$ has a factor of the total energy, which can lead to non-standard results that are not encountered in the non-relativistic case. It should be pointed out that the dependence of the effective interaction on energy in the relativistic equation (4) is also due to dynamical reasons [such as the superposition of the kinetic and potential energies in (3)].

Let us analyze the total amplitude of two-nucleon scattering within the first and second Born approximations on the basis of the relativistic equation (4). The validity conditions of the Born approximation for the mean value of the potential $\overline{V}$ with characteristic radius R are as follows (here, for convenience, we again use the Planck constant $\hbar$)

$$\overline{V} \ll \frac{4\hbar^2 kR}{\sqrt{(\hbar k)^2 + m^2R^2}} \approx \frac{4\hbar}{R}\left\{1 - \frac{1}{2}\left(\frac{m}{\hbar k}\right)^2\right\},$$

$$kR \gg 1. \tag{5}$$

In order to drop the second term in the braces, it is necessary that the condition

$$k \gg \frac{m}{\hbar}, \tag{6}$$

be fulfilled, and, in the case of nucleons with characteristic interaction radii $R \sim 1$ fm, the condition $kR \gg 1$ is fulfilled automatically. Thus, Eq. (5) can be rewritten in the form

$$\overline{V} \ll \frac{4\hbar}{R}, \quad kR \gg 1. \tag{7}$$

Consider the total scattering amplitude in the first and second Born approximations for the case of the Yukawa potential

$$V(r) = \gamma\, r^{-1} e^{-r/R}. \tag{8}$$

For the total scattering amplitude, the Born approximation for Eq. (8) with the Yukawa potential takes the form

$$f_{\mathrm{B}}(k,q) = -\frac{\gamma\sqrt{k^2+m^2}}{2(R^{-2}+q^2)} = -\frac{\gamma E}{4(R^{-2}+q^2)}, \tag{9}$$

where $q = |\vec{p}\,' - \vec{p}\,| = 2k\sin(\Theta/2)$ is the transferred momentum which differs by the factor E from the standard Born approximation in the non-relativistic case. In the Born approximation, the amplitude depends on the total energy not only through the transferred momentum q, but also through the linear dependence of the effective potential on energy. At small-angle scattering in the cone $k\Theta \ll 1$, the Born amplitude (9) in the relativistic case becomes proportional to the total energy:

$$f_{\mathrm{B}}(k,0) = -\frac{\gamma R^2}{2}\sqrt{k^2+m^2} = -\frac{\gamma R^2}{4}E. \tag{10}$$

At the same time, it is known that the Born amplitude for the small-angle scattering is independent of energy in the non-relativistic case. For asymptotically high energies, the Born approximation to a relativistic amplitude is a function inversely proportional to k ($f_{\mathrm{B}}(k) = f_{\mathrm{B}}(k,q)$),

$$|f_{\mathrm{B}}(k)| \sim \frac{1}{k}, \quad k \to \infty. \tag{11}$$

In the relativistic case, the second Born approximation to the total scattering amplitude obtained on the basis of Eq. (4) with the Yukawa potential has the form

$$f_2(\vec{p},\vec{p}\,') = -\frac{\pi^2}{2}\left\langle \vec{p}\,'\middle| V\frac{k^2+m^2}{k^2-\vec{p}^{\,2}+i0}V\middle|\vec{p}\right\rangle =$$

$$= \frac{\gamma^2(k^2+m^2)}{8Aq}\left(2\ \mathrm{arctg}\,\frac{q}{2AR} + i\ln\frac{A+kq}{A-kq}\right), \tag{12}$$

where $A^2 = R^{-4}+4R^{-2}k^2+k^2q^2$. If we take into account that, at high energies (here, $k \to \infty$), the coefficient A is a quadric function of k,

$$A \approx kq + 2R^{-2}kq^{-1},$$

then, for the second Born approximation, relation (12) yields the following asymptotic behavior:

$$|f_2(k)| \sim \frac{\ln(k)}{k}, \quad k \to \infty. \tag{13}$$

The main contribution to asymptotics (13) comes from the imaginary term in (12). At small-angle scattering, $k\Theta \ll 1$, the second Born approximation takes the form

$$f_2(k,0) = \frac{\gamma^2 R(k^2+m^2)}{8(R^{-2}+4k^2)}(1+2i\,kR). \tag{14}$$

For asymptotically high energies, the second Born approximation for the scattering cone $k\Theta \ll 1$ remains a complex value with the main contribution from the imaginary term which is proportional to the total energy.

The comparison of the asymptotics of the first and second Born approximations to the total amplitude gives

$$\left|\frac{f_2(k)}{f_{\mathrm{B}}(k)}\right| \sim \ln(k), \quad k \to \infty. \tag{15}$$

In the relativistic case at high energies, the second Born approximation to the total amplitude becomes larger in absolute value than the first Born approximation. At the same time in the non-relativistic case, we have the relation

$$\left|\frac{f_2(\varepsilon)}{f_{\mathrm{B}}(\varepsilon)}\right| \sim \frac{\ln(\varepsilon)}{\sqrt{\varepsilon}}, \quad \varepsilon \to \infty, \tag{16}$$

where ε is the kinetic energy. It can be seen that the relativistic Born approximation to the scattering amplitude for the Yukawa potential does not approach the total amplitude (for the asymptotically high energies). With increase in energy, the second term in the perturbation series with respect to the effective potential becomes even more important than the first Born approximation. This statement remains valid also for the next members of the series, because all the terms of the perturbation theory series become equally important.

It is known that, in the two-particle non-relativistic model with any interaction potential, there is always an energy, for which the Born approximation is valid, and this approximation becomes even better with increase in energy. In contrast, in the relativistic case for potential functions that do not satisfy condition (7), the Born approximation cannot be used at any energy. If the potential satisfies this condition, then we can possibly find such a range of energies, in which the Born approximation will be valid. However, with increase in energy, the Born approximation stops to be valid. In the general case of the relativistic approach considered here [based on (3)], the Born approximation (more precisely, the perturbation theory with respect to the effective interaction potential) is not applicable for the total scattering amplitude.

4. High-energy Scattering

For the non-relativistic Schrödinger equation, there is a region of high enough energies, in which the Born approximation is not valid:

$$1 \ll \frac{\varepsilon}{\overline{V}} \leq \overline{V} / \frac{\hbar^2}{mR^2},$$

Such a region can exist if the average value of the potential satisfies the condition (see, for example, [8])

$$\overline{V} \gg \frac{\hbar^2}{mR^2}.$$

For nucleons, this condition can be rewritten in the form $\overline{V} \gg 40$ MeV. For the investigation of the scattering with such potentials, it is common to apply the high-energy eikonal approximation. However, the relativistic effects for nucleons become apparent at energies of order of 250 MeV [1,2]. Therefore, the applicability of the high-energy eikonal approximation becomes questionable. At the same time, the application of the high-energy approximation to the total elastic scattering amplitude described by the relativistic equation (4) should be more justified than in the the case of a non-relativistic equation.

At high energies, when the potential in Eq. (4) satisfies the condition $V \ll 4k^2/E$, the plane wave acquires an additional phase, and the dependence of the wave function on coordinates should be eikonal-like (analogously to [8]):

$$\psi(\vec{r}) = \exp\left(ikz - \frac{iE}{8k}\int\limits_{-\infty}^{z} dz\, V(x,y,z)\right). \tag{17}$$

A characteristic feature of this expression is that the second term of the phase does not disappear at high energies as it does in the non-relativistic case, but becomes a constant. Thus, if a correspondent non-relativistic wave function reduces at $k \to \infty$ to the wave function of free motion, the eikonal wave function (17) remains deformed in the relativistic case at high energies.

The total scattering amplitude that accounts for relativistic effects can be represented in the standard form

$$f(k,\Theta) = -ik\int\limits_0^{\infty} d\rho\, \rho J_0\left(2k\rho\sin\frac{\Theta}{2}\right)\omega(\rho), \tag{18}$$

where

$$\omega(\rho) = e^{2i\delta(\rho)} - 1,$$

$$\delta(\rho) = -\frac{1}{8}\sqrt{1+\frac{m^2}{k^2}}\int\limits_{-\infty}^{\infty} dz\, V(\rho,z),$$

$$J_0(x) = \frac{1}{2\pi}\int\limits_0^{2\pi} d\varphi\, e^{ix\cos\varphi},$$

are the profile function, the total phase function depending on the target parameter, and the Bessel function, respectively. In contrast to the non-relativistic case, here the total phase $\delta(\rho)$ does not tend to zero with increase in energy. As a consequence, the profile function $\omega(\rho)$ also tends to a constant (that is not zero). This can lead to the increase of the total scattering amplitude (18) with energy. According to the optical theorem, this leads to a non-zero high-energy asymptotics of the total scattering cross-section.

Consider a high-energy scattering for the case of a rectangular potential, which allows us to obtain expressions for the total amplitude and phase not only in the integral forms but also expressed in elementary functions. Consider the interaction potential in the form

$$V(r) = \begin{cases} V_0, & r < R, \\ 0, & r > R. \end{cases} \tag{19}$$

The total phase function for this potential can be found as

$$\delta(\rho) = \begin{cases} \nu\sqrt{1-\rho^2/R^2}, & \rho < R, \\ 0, & \rho > R, \end{cases} \tag{20}$$

where $\nu = -\frac{V_0 R}{4}\sqrt{1+\frac{m^2}{k^2}}$ is the analogy of the Born parameter in the relativistic case. Correspondingly, the total scattering amplitude for the zero angle is

$$f(k,0) = \frac{kR^2}{2}\left\{i - \frac{e^{2i\nu}}{\nu} - \frac{i}{2\nu^2}\left(e^{2i\nu}-1\right)\right\}. \tag{21}$$

According to the optical theorem, we obtain the total scattering cross-section as

$$\sigma(k) = \frac{4\pi}{k}\,\mathrm{Im} f(k,0) =$$

$$= 2\pi R^2\left\{1 + \frac{1}{2\nu^2} - \frac{\sin(2\nu)}{\nu} - \frac{\cos(2\nu)}{2\nu^2}\right\}. \tag{22}$$

For $k\to\infty$, the asymptotics of the total scattering cross-section is

$$\sigma_0 = 2\pi R^2\left\{1 + \frac{1}{2\nu_0^2} - \frac{\sin(2\nu_0)}{\nu_0} - \frac{\cos(2\nu_0)}{2\nu_0^2}\right\}, \tag{23}$$

where $\nu_0 = -V_0 R/4$.

It is useful to compare (23) with the total scattering cross-section in the Born approximation for potential (19). The Born approximation to the scattering amplitude takes the form

$$f_{\mathrm{B}}(\vec{p},\vec{p}\,') = -\pi^2\sqrt{k^2+m^2}\langle\,\vec{p}\,'|V|\vec{p}\,\rangle =$$

$$= -\frac{V_0\sqrt{k^2+m^2}}{2q^3}\left[\sin(qR) - qR\cos(qR)\right]. \tag{24}$$

Integrating the square of this expression over the angles, we obtain the total scattering cross-section in the Born approximation for the rectangular potential as

$$\sigma_{\mathrm{B}}(k) = \frac{\pi V_0^2 R^4}{8}\left(1+\frac{m^2}{k^2}\right)\times$$

$$\times\left\{1 - \frac{1}{(2kR)^2} + \frac{\sin(4kR)}{(2kR)^3} - \frac{\sin^2(2kR)}{(2kR)^4}\right\}. \tag{25}$$

Correspondingly, at high energies, $k\to\infty$, the asymptotic behavior of the total scattering cross-section in the Born approximation takes the form

$$\sigma_{\mathrm{B}} = \frac{\pi V_0^2 R^4}{8}\,. \tag{26}$$

At the same time, within the non-relativistic approach based on the Schrödinger equation, scattering cross-sections in the high-energy eikonal and Born approximations are identical at high energies ($kR \gg 1$). This is not the case, however, in the relativistic case, as can be seen by comparison of the scattering cross-section in the high-energy approximation (23) and the relativistic cross-section in the Born approximation (26) obtained for Eq. (4) with a rectangular potential. Thus, the Born approximation based on the relativistic equation (4) is not justified, and we have to apply the full expansion in effective interaction potential that depends on the total energy. At the same time, the high-energy approximation is more justified and natural within the relativistic approach.

5. Born and Quasiclassical WKB Approximations to Partial Phases

The linear dependence of the effective interactive potential on the total energy in the relativistic equation (4) results in the non-standard asymptotic behavior of scattering partial phases. It was shown [1, 2] that the singlet scattering S-phase of Dirac relativistic particles can have a non-zero asymptotics for high energies. For two-nucleon scattering, it was possible to describe, within experimental errors, the singlet scattering S-phase in the full available energy range on the basis of the full relativistic equation (3) with the Yukawa-type potential. It was shown that the passage of the phase across zero at a certain energy is a relativistic effect. To explain this effect, it is no need to use a strong repulsive core. It would be interesting to investigate the possible phase behavior in the Born and quasiclassical approximations and to evaluate if these approximations are sufficient at medium and high energies.

Consider first the Born approximation to the singlet partial scattering phase using the method of phase functions on the basis of relativistic equation (4). In its structure, the phase equation formulation is the same as that in the non-relativistic case where the effective interaction potential is a linear function of the total energy. Within this approach, the first and second Born approximations to the phase function with respect to the effective potential can be represented by the following integrals:

$$\delta_l^{\mathrm{B}}(r) = -\frac{E}{4k}\int\limits_0^r dr'\; V(r') j_l^{\,2}(kr'),$$

$$\delta_l^{(2)}(r) = \frac{E}{2k}\int\limits_0^r dr'\; V(r') j_l(kr') n_l(kr') \delta_l^{\mathrm{B}}(r'), \qquad (27)$$

where $j_l(x)$, $n_l(x)$ are the Bessel and Neumann functions. These expressions differ from the standard non-relativistic approximations to the phase function by factor $E/4$, which results in the non-zero asymptotic behavior at high energies.

Let us use these expressions to find the first and second Born approximations to the singlet scattering S-phase for a few different interaction potentials, which allows analytical solutions. Such potentials were chosen in the form of a rectangular well (19), the exponential potential $V(r) = V_0 \exp(-r/R)$, and the Yukawa one (8) ($\gamma = V_0 R$):

a) rectangular well

$$\delta_0^{\mathrm{B}}(r) = -\frac{V_0 R}{4}\sqrt{1+\frac{m^2}{k^2}}\left(1-\frac{\sin(2kR)}{2kR}\right),$$

$$\delta_0^{(2)}(r) \approx \frac{(V_0 R)^2}{32}\left(1+\frac{m^2}{k^2}\right)\frac{2\cos(2kR)+1}{kR},$$

$$kR \gg 1; \qquad (28)$$

b) exponential potential

$$\delta_0^{\mathrm{B}}(r) = -V_0 R\sqrt{1+\frac{m^2}{k^2}}\frac{(kR)^2}{1+(2kR)^2},$$

$$\delta_0^{(2)}(r) \approx \frac{(V_0 R)^2}{64}\left(1+\frac{m^2}{k^2}\right)\frac{1}{kR}, \quad kR \gg 1; \qquad (29)$$

c) Yukawa potential

$$\delta_0^{\mathrm{B}}(r) = -\frac{V_0 R}{8}\sqrt{1+\frac{m^2}{k^2}}\,4\ln\left(1+(2kR)^2\right),$$

$$\delta_0^{(2)}(r) \approx \frac{(V_0 R)^2}{16}\left(1+\frac{m^2}{k^2}\right)\frac{\ln(kR)}{kR}, \quad kR \gg 1. \qquad (30)$$

It should be noted that, in contrast to the second Born approximation to the total amplitude, the second Born approximation to the phase tends to zero at $k \to \infty$ for all potentials. In the Born approximation and for the potentials finite at zero, the phase becomes constant at high energies and equals $V_0 R/4$. At the same time, the

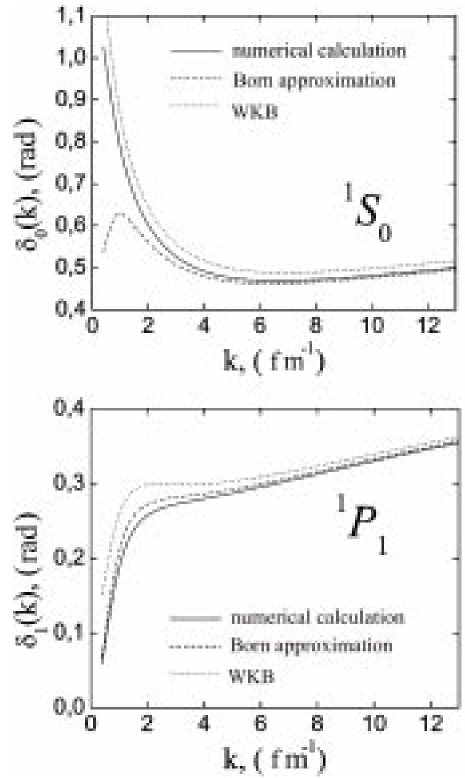

Fig. 1. Comparison of the full numerical solution of Eq. (4) with the Born and quasiclassical (WKB) approximations to the S and P scattering phases (for the Yukawa potential)

example of the Yukawa potential shows that if a potential has a weak singularity at zero, its Born approximation increases at high energies according to the logarithmic law.

In the quasiclassical approximation based on Eq. (4), the expression for partial phases can be presented in the standard form as

$$\delta_l(k) = \frac{\pi}{2}\left(l+\frac{1}{2}\right) - kr_0 +$$

$$+\int\limits_{r_0}^{\infty} dr\left\{\sqrt{k^2-\frac{(l+1/2)^2}{r^2}-\frac{EV(r)}{4}}-k\right\}, \qquad (31)$$

where the turning point r_0 is determined as a zero of the radicand. Here, the partial phase differs also from the phase in the non-relativistic approximation by the additional factor of the total energy under the direct interaction potential $V(r)$.

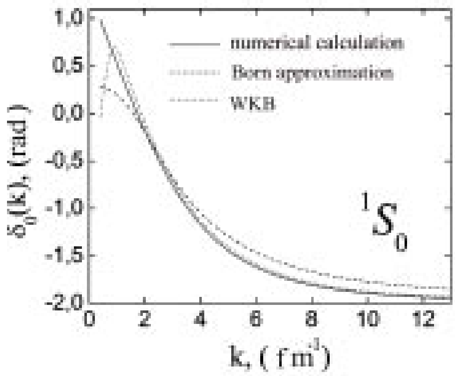

Fig. 2. Comparison of the full numerical solution of Eq. (3) with the Born and WKB approximations to the S-phase of nucleon scattering

Fig. 1 shows S and P scattering phases in the singlet spin state for the Yukawa potential (8) ($\gamma = V_0R$) in the Born and quasiclassical approximations in comparison with the result of a full numerical calculation of the phase (based on the phase equation equivalent to the relativistic equation (4)). Parameters for the Yukawa potential were chosen to belong to the low-energy region (the scattering length and the effective interaction radius) in order to correspond to the non-relativistic model of nucleon S-scattering:

$$V_0 = -87.428\,\text{MeV}, \quad R = 1.22\,\text{fm}. \tag{32}$$

For relatively low energies, the S phase in different approximations behaves in a standard way (although its values are numerically different) and this is the same behavior as that in the non-relativistic case: the phase first increases with energy, reaches its maximum, and then starts to decrease with a further increase in energy. However, starting approximately from $k = 7\,\text{fm}^{-1}$ (in the laboratory system, this is approximately 4 GeV), this similarity with the non-relativistic behavior ends and the S-phase starts to increase logarithmically. This makes the behavior of the relativistic phase different from the phase behavior predicted by the standard non-relativistic equation. For the P-phase, such an anomalous increase begins at smaller energies, $k = 3\,\text{fm}^{-1}$ (approximately 746 MeV). At these energies, both the WKB and Born approximations are numerically close to the results of the full numerical calculation. We recall that the increase of the phase with energy is related to the total energy factor of the direct interaction potential in Eq. (4). It should be noted that the quasiclassical approach for the S-phase correctly describes its behavior starting from the smaller energies (approximately at $1\,\text{fm}^{-1}$ or 83 MeV) than those in the Born approximation, and the accuracy of the quasiclassical approximation does not practically depend on energy. In contrast, the Born approach better describes both S and P phases and this approximation only improves with energy. However, it should be noted that, for this choice of the potential shape and its parameters (32), the P phase differs from the experimental data at all energies.

Fig. 2 shows the application of the Born and WKB approximations to the singlet S-phase description of nucleon scattering in the relativistic approach based on the full equation (3) where all three potential functions are non-zero.

In this calculation, the first V_1 and the second V_2 potentials were chosen in the forms of the Yukawa potentials, and the third potential V_3 was chosen in the form of the Woods-Saxon potential. In should be stressed that the Born approximation is obtained with respect to the full effective potential from the corresponding equation using the method of phase functions [2]. For comparison, we also present the numerical solution (taken from work [2]) of the phase equation that correctly describes the experimental behavior of the S-phase within experimental errors [9]. In contrast to the preceding example related to Eq. (4) with one Yukawa potential, the quasiclassical eikonal approximation based on Eq. (3) describes more accurately the S-phase than the Born approximation. It is possible that small deterioration of the S-phase description in the Born approximation is related to the potential function $V_3(r)$ in the "kinetic energy" operator (the first term) in Eq. (3). Nevertheless it should be noted that such high-energy approximations provide a quantitatively correct description of S-scattering of two nucleons at medium and high energies, and these approximations can be used for the investigation of the scattering phases with higher orbital moments.

6. Conclusions

The investigations of relativistic nucleons on the basis of the two-particle Dirac equation with a direct interaction potential, when closed equations of the Schrödinger—Breit type can be derived in the singlet and triplet spin states, can be carried out with the use of standard methods of quantum mechanics, which allows one to obtain the results in simple and descriptive forms. In this work, using the Yukawa potential and the rectangular potential as examples, we have demonstrated a failure of the Born approximation to the total scattering amplitude at high energies. The linear dependence of the effective interaction potential on the total energy results in the fact that, in this model, the Born scattering amplitude differs from the total amplitude even for asymptotically high energies, and all the terms of the Born expansion of the total amplitude become equally important with increase in energy. A specific form of the effective interaction leads also to the anomalous behavior of the scattering partial phases, although the Born expansion of partial phases can be carried out. An interesting fact is that in the Born approximation the singlet S scattering phase has a non-zero asymptotic behavior at high energies and tends to a constant for non-singular potentials, but it increases according to the logarithmic law for a weakly singular Yukawa potential. For a relativistic system of two nucleons, we obtained the validity conditions of the Born and high-energy WKB approximations for the description of the singlet scattering S-phase for potentials that correctly represent its experimental behavior in a wide range of energies.

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Received 06.08.04.
Translated from Ukrainian by M. Shendeleva

ВИСОКОЕНЕРГЕТИЧНІ НАБЛИЖЕННЯ В РОЗСІЯННІ ДВОХ НУКЛОНІВ НА ОСНОВІ РІВНЯННЯ ДІРАКА З ПОТЕНЦІАЛЬНОЮ ВЗАЄМОДІЄЮ

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Р е з ю м е

На основі рівняння Дірака з прямою потенціальною взаємодією досліджено розсіяння двох нуклонів в синглетному спіновому стані в борновому та високоенергетичному квазікласичному наближеннях. На прикладі потенціалу Юкави показано незастосовність розкладання повної амплітуди розсіяння в ряд теорії збурень за ефективним потенціалом. Встановлено, що борнова синглетна S-фаза розсіяння для скінченних в нулі потенціалів при великих енергіях має ненульову асимптотику. Для потенціалів з кулоновою особливістю на малих відстанях отримано зростання борнової фази за логарифмічним законом зі збільшенням енергії. Для потенціалу Юкави та низки інших потенціалів взаємодії знайдено аналітичні вирази поправок другого порядку теорії збурень до борнової S-фази. Проведено якісний аналіз точності опису синглетного S-розсіяння нуклонів в борновому та ейкональному наближеннях при високих енергіях.