SYMMETRIES OF AN 8-COMPONENT EQUATION OF THE DIRAC—KÄHLER TYPE

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S u m m a r y

A complex analog of the Dirac—Kähler equation (CDK) as a system of 8 (but not 16) equations for 8 independent complex components with nonzero mass $m = \sqrt{\kappa_1 \kappa_2}$ is proposed. This equation is written in three Bose (2.11), (2.13), (2.14) and two Fermi (2.19), (2.24) forms. It is shown (Theorem 3) that, irrespective of $m \neq 0$, the CDK equation is invariant relative to the algebra $\tilde{A}_8$ of purely matrix transformations, whose 8×8-matrices are constructed from 4×4-matrices of the Pauli—Gürsey invariance algebra A_8 for the massless Dirac equation $\gamma \partial \psi = 0$. Six generators of the algebra $\tilde{A}_8$ generate the internal symmetry group for the CDK equation which can be identified with the isospin group SU(2) of the compound-field $\Psi = (\psi_1, \psi_2)$. It is shown (Theorems 4, 5) that the CDK equation (in any form) is invariant relative to two *nonequivalent representations* $\mathcal{P}^{\mathrm{S}}$ and $\mathcal{P}^{\mathrm{TSV}}$ of the Poincare group $\mathcal{P} \supset \mathcal{L}$ which are generated by the spinor $2\mathcal{L}^{\mathrm{S}}$ (3.20) and, respectively, tensor-scalar-vector $\mathcal{L}^{\mathrm{TSV}}$ (3.29) matrix representations of the Lorentz group $\mathcal{L}$. The operator connecting the Bose and Fermi forms of the CDK equation is found: by the action of this operator, the Fermi compound-field $\Psi = (\psi_1, \psi_2)$ is expressed through the system $\mathcal{F} = (\mathcal{B}^{\mu\nu}, \phi, V^{\mu})$ of three $\mathcal{P}$-irreducible Bose fields. An equation of the CDK type is given (without any discussion) in the 5-dimensional Minkowski space.