THE TWO-ANYON PROBLEM WITH A MAGNETIC IMPURITY

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The system of two anyons (particles with fractional statistics in two dimensions) is considered in the presence of a magnetic impurity, i.e., a static magnetic flux. A numerical algorithm for finding the spectrum is worked out; the ground state is computed and is found to exhibit a nontrivial level crossing.

1. Introduction

It is widely recognized now that statistics in two dimensions can continuously interpolate between the bosonic and fermionic ones [1]; particles obeying such intermediate, or fractional statistics are called *anyons* [2]. The existence of anyons is possible because the two-dimensional space with an excluded point is multiply connected; a path taking one particle around another one is topologically nontrivial, and the statistical phase associated with such a path need not be equal to unity. Hence, an interchange of two particles may supply the wave function with a phase factor $\exp(\mathrm{i}\pi\alpha)$, where α, usually called *statistics parameter*, can be fractional (hence the term “fractional statistics”).

A physical incarnation of such statistics is possible due to the Aharonov—Bohm effect [3], whereby the phase comes from the coupling of a charge to the topologically nontrivial electromagnetic potential of a flux tube. “Charge-flux composites” in two dimensions are anyons. (It is also worth noting that, in the so-called Chern–Simons field model emerging as an effective model under certain conditions as a result of dimensional reduction in gauge theories, charges themselves generate a magnetic field, so that any point charge is, at the same time, a point flux. In that model, all charges are effectively anyons [4].)

The multianyon problem is not believed to be exactly solvable even in the absence of interaction. The reason is that, under the nontrivial interchange conditions, the usual representation of a multiparticle wave function (which is now a multivalued function of its complex arguments) in terms of products of single-particle functions is no longer possible. It is only the two-body case that has been solved [1,2], and much is known about the three-anyon problem (see [5,6] and references therein); for many anyons, there exists a class of exact analytic solutions [7], but rather little is known beyond that.

In recent years, a new class of systems has been considered: particles in the presence of *magnetic impurities* [8, 9]. The latter are static point fluxes—essentially, anyons with infinite mass. Like the standard multianyon system, this one is of interest because of its relevance to the fractional quantum Hall effect [10], where the elementary excitations are believed to be anyonic. Another motivation is the relevance of the system at hand to the problem of winding number distribution of random paths [11]. Essentially, the partition function of a particle in the presence of N impurities is a Fourier transform of the sequence $P_{m_1 m_2 \ldots m_N}$ of probabilities that a random closed path winds m_k times around point k for all $k = 1, \ldots, N$.

Up to now, only $(1+N)$-body systems (one particle plus N impurities) have been considered. For a random Poissonian distribution of magnetic impurities, calculations of the density of states averaged over the positions of the impurities have been performed [8], with some interesting qualitative conclusions. Also, a numerical treatment of the (1+2)-body system has been carried out [9], with low-lying quantum states being found and their dependence on the strengths of the impurities (the values of their magnetic fluxes) and on the distance between them elucidated.

In this paper, we proceed to solve the (2+1)-body problem—in other words, the standard two-anyon problem with one impurity. The presence of the impurity renders the system nontrivial: because of the impurity-induced boundary conditions, the relative motion is no longer independent of the center-of-mass motion. In fact, the complexity is about the same as that with the three-anyon problem: the topological properties of the wave function are the same, and the fact that the third particle is pinned down to a fixed point does not make it simpler in any crucial way.

We first demonstrate that there exists a set of exact eigenfunctions in the (2+1)-body system under consideration, which are special cases of the known exact solutions of the N-anyon problem [7]. The remainder of the spectrum, apparently, can only be evaluated numerically. To that end, we search for the wave function in terms of products of monomials in transformed single-particle coordinates. The boundary conditions imply a homogeneous system of linear equations for the coefficients, and the condition of its solvability yields the energy levels. The ground state is found numerically and is discovered to exhibit a nontrivial level crossing at a certain relationship between the statistics of the particles and the impurity magnetic flux.

2. The Problem and Exact Solutions

As is commonly done, we place the anyons into a harmonic potential, and the impurity is put at the origin. Like between the anyons, there is no classical interaction force between an anyon and the impurity; the presence of the latter manifests itself only in the boundary conditions on the wave function. Therefore, the Hamiltonian is simply a sum of two single-particle ones,

$$H = -2\frac{\partial^2}{\partial z_1 \partial \bar{z}_1} + \frac{z_1\bar{z}_1}{2} - 2\frac{\partial^2}{\partial z_2 \partial \bar{z}_2} + \frac{z_2\bar{z}_2}{2} \qquad (1)$$

(z_i being the complex particle coordinates; the mass and harmonic frequency are scaled to unity). The presence of the impurity does not break rotational invariance, and the kinetic angular momentum

$$L = z_1\frac{\partial}{\partial z_1} - \bar{z}_1\frac{\partial}{\partial \bar{z}_1} + z_2\frac{\partial}{\partial z_2} - \bar{z}_2\frac{\partial}{\partial \bar{z}_2} \qquad (2)$$

commutes with the Hamiltonian. The boundary conditions on the two-particle wave function are:

(i) anyon interchange condition: when particles 1 and 2 are continuously interchanged anticlockwise (so that the relative vector $z_1 - z_2$ rotates by an angle of π) in such a way that the impurity is not encircled in the process, the wave function acquires a phase factor of $\exp(\mathrm{i}\pi\alpha)$;

(ii) impurity encircling condition: when a particle encircles the impurity anticlockwise (so that the vector z_i rotates by an angle of 2π) without encircling the other particle, the wave function acquires a phase factor of $\exp(2\mathrm{i}\pi\gamma)$.

Here, γ is the "strength of the impurity", equal to the flux it carries in the units of the flux quantum: $\gamma = \phi/\phi_0$, where $\phi_0 = 2\pi/e$.

Like in the N-anyon problem [7], there exists a class of exact solutions for the system at hand. The key observation is that a Jastrow—Laughlin-type phase factor for N anyons, $\prod_{jk}(z_j - z_k)^\alpha$, satisfies all the pairwise anyon interchange conditions; and it is possible to obtain some eigenfunctions of H by means of multiplying this factor by a symmetric function of z_j's. A different factor, $\prod_{jk}(\bar{z}_j - \bar{z}_k)^{-\alpha}$, is as good, except that with it, the symmetric function must vanish fast enough at $z_j - z_k \to 0$ for the total function to be nonsingular. A generalization of those factors to nonidentical particles is completely straightforward [12]. The very same procedure applies here, with the constraint that the coordinate of one of the particles—the impurity—is fixed at zero. The state which, at $\alpha = \gamma = 0$, becomes the ground state of two bosons,

$$\psi = (z_1 - z_2)^\alpha z_1^\gamma z_2^\gamma \exp\left(-\frac{z_1\bar{z}_1 + z_2\bar{z}_2}{2}\right), \qquad (3)$$

is an eigenstate of both H and L, with the eigenvalues

$$E = 2 + \alpha + 2\gamma\,, \qquad L = \alpha + 2\gamma\,, \qquad (4)$$

respectively, and satisfies both boundary conditions. Generally, each two-anyon state with a positive relative angular momentum and energy $E = E_0 + \alpha$ maps (for a positive γ) onto a state with energy $E = E_0 + \alpha + 2\gamma$. [Note that this is only possible with the impurity sitting at the origin. A factor $(z_1 - z_2)^\alpha (z_1 - z_0)^\gamma (z_2 - z_0)^\gamma$ with $z_0 \neq 0$ would satisfy the boundary conditions, but the corresponding wave function would not be an eigenfunction of the Hamiltonian. Therefore, in particular, the (1+N)-body problem is nontrivial for any $N \geq 2$.]

This class of exact states, however, does not exhaust the whole spectrum. The two-fermion ground level is twice degenerate ($E = 3$, $L = \pm 1$); the ground state of two anyons, $(z_1 - z_2)^\alpha$ (the exponential damping factor omitted) connects the two-boson ground state ($E = 2$, $L = 0$) to the first of the two two-fermion ground states, and a different state, $(\bar{z}_1 - \bar{z}_2)^{2-\alpha}$, connects an excited two-boson state to the second of them. There does not exist a straightforward mapping of this state onto a state with nonzero γ, like in Eq. (3): of the two possible variants, $(\bar{z}_1 - \bar{z}_2)^{2-\alpha} z_1^\gamma z_2^\gamma$ is not an eigenstate of the Hamiltonian, while $(\bar{z}_1 - \bar{z}_2)^{2-\alpha} \bar{z}_1^{-\gamma} \bar{z}_2^{-\gamma}$ is singular. The reason is, again, the same as in the N-anyon system: States in which all pairwise relative angular momenta do not have the same sign cannot be obtained exactly. A numerical approach is in order.

3. The Numerical Algorithm

We will employ a scheme similar to the one used for the 3-body [5] and (1+2)-body [9] problems. Upon introducing, instead of z_1 and z_2, four real coordinates: an absolute distance r, a relative scale factor q, and two angles, φ_1 and φ_2, as

$$z_1 = \frac{rq\mathrm{e}^{\mathrm{i}\varphi_1}}{\sqrt{1+q^2}}\,, \qquad z_2 = \frac{r\mathrm{e}^{\mathrm{i}\varphi_2}}{\sqrt{1+q^2}}\,, \tag{5}$$

(cf. [5]), the Hamiltonian becomes

$$H = -\frac{1}{2m}\left[\frac{\partial^2}{\partial r^2} + \frac{3}{r}\frac{\partial}{\partial r} + \frac{1+q^2}{r^2}\times \right.$$
$$\left. \times\left(\frac{1+q^2}{q}\frac{\partial}{\partial q}q\frac{\partial}{\partial q} + \frac{1}{q^2}\frac{\partial^2}{\partial\varphi_1^2} + \frac{\partial^2}{\partial\varphi_2^2}\right)\right] + \frac{r^2}{2}. \tag{6}$$

Searching for its eigenfunction, $H\psi = E\psi$, in a separated form (disregarding the boundary conditions for now),

$$\psi(r,q,\varphi_1,\varphi_2) = f(r)g(q)\mathrm{e}^{\mathrm{i}(j\varphi_1+k\varphi_2)}\,, \tag{7}$$

yields the following equations for $f(r)$ and $g(q)$:

$$-\frac{1}{2}\frac{d^2f(r)}{dr^2} - \frac{3}{2r}\frac{df(r)}{dr} + \left(\frac{\mu(\mu+2)}{2r^2} + \frac{r^2}{2}\right)f(r) =$$
$$= Ef(r), \tag{8}$$

$$(1+q^2)\left(\frac{1+q^2}{q}\frac{d}{dq}q\frac{d}{dq} + \frac{j^2}{q^2} + k^2\right)g(q) =$$
$$= \mu(\mu+2)g(q). \tag{9}$$

The eigenvalues of the radial equation are

$$E = \mu + 2n + 2, \tag{10}$$

with the eigenfunctions

$$f_{n,\mu}(r) = r^\mu L_n^{1+\mu}(r^2)\mathrm{e}^{-r^2/2}. \tag{11}$$

This degree of freedom completely separates from the others and provides a "tower structure" of levels with step 2, just like in the three-anyon spectrum [13].

The solution of the q equation which is nonsingular at $q=0$ is

$$g(j,k,\mu;q) = q^{|j|}(1+q^2)^{-\mu/2}\times$$
$$\times {}_2F_1\left(\frac{|j|+|k|-\mu}{2}, \frac{|j|-|k|-\mu}{2}; 1+|j|; -q^2\right). \tag{12}$$

The initial problem being invariant with respect to $q \leftrightarrow 1/q$, $\varphi_1 \leftrightarrow \varphi_2$, there is another solution, $g(k,j,\mu;1/q)$, which is nonsingular at $1/q = 0$.

Now the boundary conditions have to be taken into account. It is impossible, in general, to satisfy these conditions with one function of a separated form (7); a linear combination of such functions has to be taken instead.

For $q<1$, rotating particle 1 around the origin, i.e., increasing φ_1 by 2π, means encircling the impurity only; whereas, rotating particle 2 around the origin leads to encircling both particle 1 and the impurity, generating a phase factor of $\exp[2\mathrm{i}\pi(\alpha+\gamma)]$. Consequently, one should have $j = m' + \gamma$, $k = p' + \alpha + \gamma$, where m' and p' are some arbitrary integers. The eigenvalue L of the angular momentum operator $L = -\mathrm{i}(\partial/\partial\varphi_1 + \partial/\partial\varphi_2)$ can be quantized together with E. For a wave function (7), $L = j + k$, or

$$L = L_0 + \alpha + 2\gamma, \tag{13}$$

where $L_0 = m'+p'$, an integer, is the angular momentum of the state in question in the absence of magnetic fluxes. The constraint on $j+k$ implies that two independent numbers, m' and p', get replaced with one integer, m, such that

$$j = \frac{L_0 - L_0 \bmod 2}{2} - m + \gamma, \tag{14}$$

$$k = \frac{L_0 + L_0 \bmod 2}{2} + m + \alpha + \gamma. \tag{15}$$

Introducing the average and relative angles according to

$$\varphi_1 = \varphi + \frac{\xi}{2}, \quad \varphi_2 = \varphi - \frac{\xi}{2}, \tag{16}$$

one has $j\varphi_1 + k\varphi_2 = L\varphi - (m + \alpha'/2)\xi$, where

$$\alpha' = \alpha + L_0 \bmod 2, \tag{17}$$

and a generic wave function has the form

$$\psi(q,\varphi,\xi) = \mathrm{e}^{\mathrm{i}L\varphi}\sum_m c_m\, g_m(q)\, e^{-\mathrm{i}(m+\frac{\alpha'}{2})\xi}\,, \tag{18}$$

with a shorthand notation $g_m(q) \equiv g(j,k,\mu;q)$, where j and k depend on m through Eqs. (14), (15).

The quantization of μ, yielding the energy levels through Eq. (10), stems from the remaining boundary condition associated with an exchange of the two particles. The condition relates the wave function at $q<1$ to the one at $q>1$, which is constructed in the same manner as above, with $(q,\varphi,\xi) \mapsto (1/q,\varphi,-\xi)$.

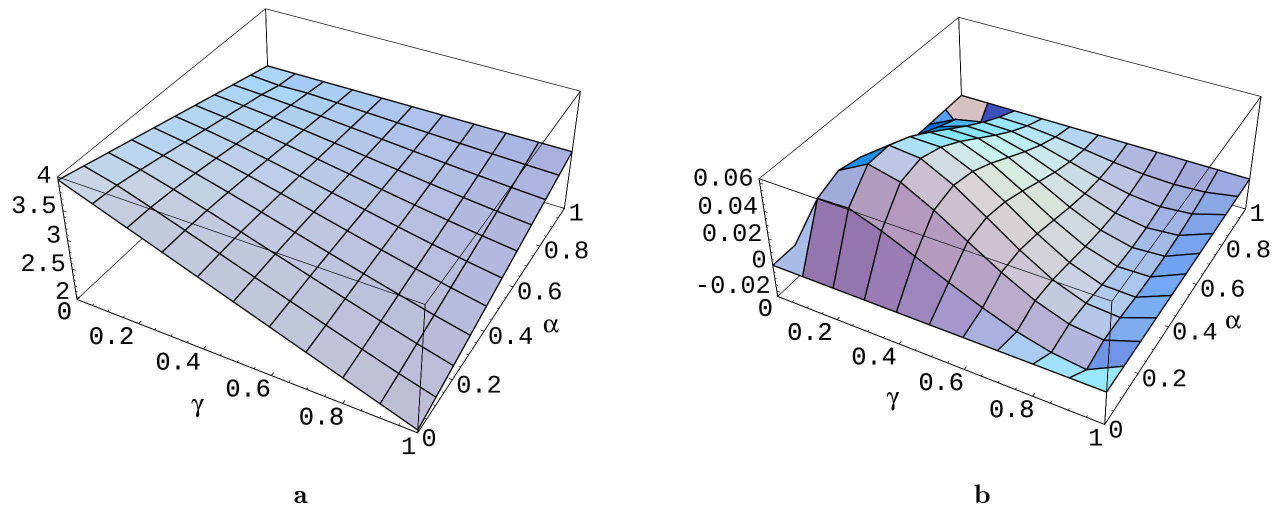

Fig. 1. The state which at $\gamma = 0$ connects to the fermion ground state: (a) $E(\alpha, \gamma)$; (b) $E(\alpha, \gamma) - \varepsilon(\alpha, \gamma)$, Eq. (23)

However, instead of a condition at an arbitrary q, it is enough to impose conditions on the wave function and its q derivative at $q = 1$. It is easy to see that turning $(1, \varphi, \xi)$ with $\xi > 0$ into $(1, \varphi, -\xi)$ in such a way that $q < 1$ in the process, corresponds to an anticlockwise interchange of particles 1 and 2. The interchange condition, therefore, is

$$\sum_m c_m g_m(1) \mathrm{e}^{\mathrm{i}(m+\frac{\alpha'}{2})\xi} = \mathrm{e}^{\mathrm{i}\pi\alpha} \sum_m c_m g_m(1) \mathrm{e}^{-\mathrm{i}(m+\frac{\alpha'}{2})\xi}. \quad (19)$$

Requiring the same to hold for q different from 1 by an infinitesimal value results in the condition on the derivative

$$\sum_m c_m g'_m(1)\, \mathrm{e}^{\mathrm{i}(m+\frac{\alpha'}{2})\xi} = -\mathrm{e}^{\mathrm{i}\pi\alpha} \sum_m c_m g'_m(1) \mathrm{e}^{-\mathrm{i}(m+\frac{\alpha'}{2})\xi}, \quad (20)$$

where $g'_m(q) \equiv dg_m(q)/dq$ (the minus sign comes from differentiating $1/q$ with respect to q).

Multiplying these equations by $\mathrm{e}^{-\mathrm{i}(n+\alpha'/2)\xi}$, integrating over ξ from 0 to 2π, and excluding c_n from the left-hand side yields

$$\sum_m D_{nm} c_m = 0, \quad (21)$$

where

$$D_{nm} = \frac{1 - \mathrm{e}^{2\mathrm{i}\pi\alpha}}{2\mathrm{i}\pi(n + m + \alpha')} \left[g'_n(1) g_m(1) + g_n(1) g'_m(1)\right]. \quad (22)$$

The g functions depend on μ, and the eigenvalues of μ, to which the energy levels are related through Eq. (10), are those for which the determinant of $||D_{nm}||$ vanishes.

4. Results and Discussion

In a numerical procedure, the infinite sum over m is necessarily truncated to a finite one over $m = -N, \ldots, N$. In order to improve precision, one should make the calculation for several values of N and extrapolate to $N \to \infty$. Convergence in N becomes better if, for a given finite N, the continuous Fourier transformation of Eqs. (19)—(20) is replaced with a $(2N + 1)$-point discrete transformation [6,9]. The factor $(n+m+\alpha')$ in the denominator of (22) then gets replaced with $2N \sin[(n + m + \alpha')/2N]$, with the correct $N \to \infty$ limit.

We have taken $N = 20, 40, 80, 160$ for the extrapolation, and the N dependence was found to be rather well described by a semi-empirical formula $E(N) = E_\infty + cN^{-2\alpha}$. The convergence rate depends on α crucially, because the main inaccuracy comes from the difficulty of representing a singular function, behaving like z^α as the relative distance z between the particles tends to zero, as a linear combination of regular functions.

The above-devised numerical scheme has been applied to finding the ground state of the system in question. First of all, from the boundary conditions, it is evident that there is periodicity in α with period 2 and in γ with period 1. Due to periodicity and P-symmetry, $(1+\alpha, \gamma) \leftrightarrow (-1+\alpha, \gamma) \leftrightarrow (1-\alpha, -\gamma) \leftrightarrow (1-\alpha, 1-\gamma)$; consequently, it is enough to restrict oneself to the square $\alpha, \gamma \in [0, 1]$. For small enough γ, by continuity, the ground state is (3), with energy (4). However, at least at $\gamma = 1$ it is certainly not the ground state anymore;

consequently, for some values of α and γ, a level crossing must occur.

The lowest state with a negative relative angular momentum is the one which, at $\gamma = 0$, connects to the fermion ground state, with energy $E = 4 - \alpha$ and angular momentum $L = -2 + \alpha$ (i.e., $L_0 = -2$). The numerically computed energy of the same state for nonzero γ is shown on Fig. 1,*a*. The behavior of the energy at the extreme values of α and γ can be explained semiclassically. For a single-particle state free of radial excitations, $E = 1 + |L|$, and the effect of the impurity is to add γ to L, thus increasing or decreasing the energy depending on the sign of the angular momentum. The two-boson ground state has $E = 2$ (the angular momentum of both particles is zero), whereas the two-fermion ground level, $E = 3$, is twice degenerate: one particle has angular momentum zero, the other one ± 1. For an integer α (bosons or fermions), the "good" quantum numbers are angular momenta of individual particles. At $\alpha = 0$, the state being discussed corresponds to $L_1 = L_2 = -1 + \gamma$, so that $E = 2(1 + |-1 + \gamma|) = 4 - 2\gamma$; at $\gamma = 1$, the two-boson ground state is recovered. At $\alpha = 1$, instead, one has $L_1 = -1 + \gamma$ and $L_2 = \gamma$; the state interpolates between the two two-fermion ground states with the energy remaining constant. Finally, at $\gamma = 1$, when the impurity has no effect, the state becomes the same as (4)—the ground state of two anyons. (For a fractional α, single-particle angular momenta are no longer good quantum numbers; the center-of-mass and relative momenta are.)

A simple bilinear expression encompasses all of these four special cases:

$$\varepsilon(\alpha, \gamma) = 3 + (1 - \alpha)(1 - 2\gamma). \tag{23}$$

By looking at the numerical data, one finds the energy of the state in question to be described by this formula rather well, but not exactly (Fig. 1,*b*). The numerical error from the N extrapolation, which can be estimated by looking at the values of energy obtained at different values of N, does not surpass 0.01 (at $\alpha = 0.1$, where the convergence is worst) and becomes negligible at $\alpha > 0.5$.

As mentioned before, there are two types of pairwise factors with the correct anyonic interchange properties, $(z_j - z_k)^\alpha$ and $(\bar{z}_j - \bar{z}_k)^{-\alpha}$. The exactly solvable N-anyon states are those where each pair of particles contributes a factor of the same type (two classes of states correspond to two types). Semiclassically, this means that all $N(N-1)/2$ pairs rotate in one and the same direction. In the 2-body problem, there being only one pair, all the states are exact. In the 3-body problem, 1/3 of them are [14]. In the (2+1)-body problem at hand, 1/2 of them are: Since the impurity-induced factor can only be z_j^γ (lest the wave function becomes singular), it is only the $(z_1 - z_2)^\alpha$ ones that are exact. That is why the state analyzed above is not exact. One can, however, use Eq. (23) as a good analytic approximation in order to determine the structure of the ground state. Equating $\varepsilon(\alpha, \gamma) = 2 + \alpha + 2\gamma$ yields

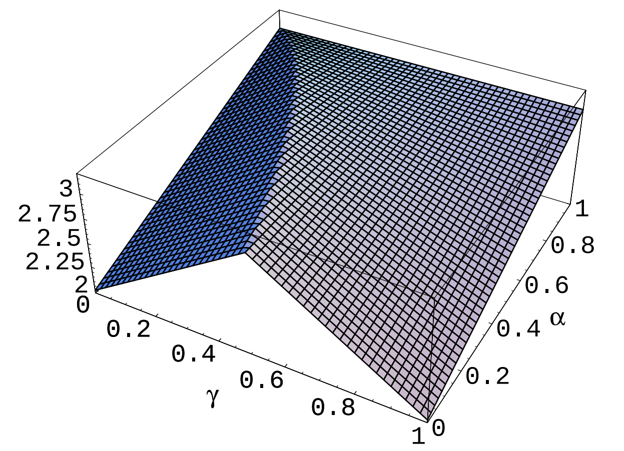

Fig. 2. The ground state. In the approximation (23), the level crossing is at $\gamma = (1 - \alpha)/(2 - \alpha)$

$$\gamma = \frac{1 - \alpha}{2 - \alpha}. \tag{24}$$

This is an (approximate) interrelation between the impurity strength and the anyon statistics parameter at which there is a level crossing. At γ smaller than the value above, the ground state is the continuation of the two-anyon ground state; when it is larger, it is the continuation of the lowest two-anyon state with a negative angular momentum. Evidently, this is a true crossing, since the symmetry of the two states involved is different. It is to some extent reminiscent of the level crossing in the ground state of the three-anyon problem; there, too, the configuration stemming from the boson ground state is the most energetically favorable at small enough values of the statistics parameter, but another one takes over—and leads to the fermion ground state—as the statistics interaction increases.

5. Conclusion

We have shown that there exists a class of exactly solvable states in the (2+1)-body problem of two anyons in the presence of a magnetic impurity. Further, we have devised a numerical algorithm to find the states that are not exactly solvable, and demonstrated its validity by computing the energy of the ground state. The latter belongs to the exactly solvable class for certain relations between the anyon statistics parameter and the impurity strength, and, for other values of the parameters, its energy turns out to be rather well described by a simple analytic formula (which, in turn, yields a simple expression for the condition of a level crossing). It would be interesting to see whether this result can be obtained within some analytic approximation, e.g., with a variational wave function.

The next logical step would be to calculate enough of the spectrum to be able to infer the qualitative properties of the nonsolvable states, as well as to evaluate the influence of the impurity on the second virial coefficient of anyons. We plan to address these issues in a future work.

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ЗАДАЧА ДВОХ ЕНІОНІВ З МАГНИТНОЮ ДОМІШКОЮ

С. Машкевич

Р е з ю м е

Розглянуто задачу двох еніонів (частинок із дробовою статистикою у двовимірному просторі) в присутності магнітної домішки, тобто статичного магнітного потоку. Розроблено чисельний алгоритм для знаходження спектра; обчислено основний стан та знайдено нетривіальний перетин рівнів у ньому.