THE MECHANISMS OF FORMATION OF VORTICES IN OPTICS AND HYDRODYNAMICS

V.N. GORSHKOV, A.N. KHOROSHUN[1], M.S. SOSKIN

Institute of Physics, Nat. Acad. Sci. of Ukraine
(46, Nauky Prosp., Kyiv 03028, Ukraine),

[1]East-Ukrainian National University
(20a, Molodezhnyi Block, Lugansk 9100, Ukraine)

UDC 535.2

A theory of the nucleation of optical vortices after the asymmetric excitation in a beam with initially smooth wave fronts is built. The theory is based on the developed two-dimensional mathematical model of diffraction, as a diffusion of transverse perturbation waves. Two possible mechanisms of the nucleation of optical vortices in combined Gaussian beams and the diffraction of a plane wave by the arc are represented. The nucleation of vortices is considered as an analog of the well-known birth of vortices due to "the magnetohydrodynamic instability of neutral current layer" and "the hydrodynamic instability of tangential disruption of the current velocity". The conditions for their realization and evolution in space are analyzed. All the known cases of the linear and nonlinear nucleations of optical vortices can be described in the frame of the developed mechanisms.

1. Introduction

Phase singularities or wave-front disruptions were introduced as a new concept in wave theory in [1]. It was shown that they are zero-amplitude lines in space with indefinite phase. Most nontrivial are the screw wave-front dislocations or "optical vortices" [2]. The phase change after the circumference of a zero-amplitude line is equal to $\Delta\Phi = 2m\pi$, where m is an integer topological charge. Optical vortices affect all the properties of the light field [3, 4]. It has been found that optical vortices are modes of quantum noise [5]. What's more, it was marked recently that an optical vortex and its orbital angular momentum play the essential role in the fundamental properties of light and must be taken into account in the modern theory of photons [6]. All this allowed one to claim that phase singularities appear as the most essential features of solutions of the Maxwell equations [7]. The investigations of the fundamental properties of singular light beams and their applications [8] are of great significance and have formed *singular optics* last years as a new chapter of modern optics [3, 4, 9 — 11]. But the feeling that the fundamentals of singular optics are completely established is deceptive. Generally, the problem of the nucleation of optical vortices is far from the solution. The main goal of this paper is to make attempt to proceed in this direction.

As an introduction to the problem, we overview shortly the known methods of the creation of singular beams. It is known that optical vortices belong to the family of the Laguerre—Gauss modes [4]. Therefore, it is natural that they were realized firstly by the help of a laser with special cavity to suppress the usual generation on the Hermite—Gaussian modes [12]. Due to the subject of our consideration, we are interested in the out-of-laser methods of synthesis of singular beams from coherent beams with an initially smooth wave front. All the known methods can be divided into two next groups: (i) the use of mode converters [14], computer-synthesized holograms [15 — 17], spiral phase plate [18], and nonlinear resonator [19], (ii) vortex nucleation due to scattering, diffraction, or interaction with nonlinear media. The nucleation of a circular edge dislocation or a quadruple of optical vortices after a nonlinear Gaussian-like lens [24 — 26] is an example of the second-group methods. We will show that the *natural* (spontaneous) nucleation of phase singularities can be realized during the single-pass free propagation of the light field with an initially smooth wave front.

Three levels of optical singularities exist [13]: (i) ray optics caustics, (ii) phase singularities of scalar, i.e. linearly polarized, light fields, and (iii) polarization singularities of vector light fields. The vortex nucleation can be realized for all these levels. It was shown that caustics are decorated by optical vortices due to a finite value of the light wavelength [20, 21]. Light scattering by a rough surface produces not only speckle-fields, but simultaneously the system of optical vortices (on the average, one vortex per speckle [22, 23]). The diffraction of a plane wave by a circular aperture produces Airy rings in the far field, which represent themselves as circular edge dislocations [3, 27]. The diffraction of a screened optical vortex leads to its self-restoration [28] and the nucleation of a system of secondary vortices [29]. It was shown that the combined effect of diffraction and Poynting vector walk-off in the second-harmonic generation by a singular pump beam is accompanied by the nucleation of multiple vortex pairs quasi-aligned in vortex streets [30]. At last, polarization singularities appear in a polarization speckle-field [31]. In all cases, optical vortices nucleate in pairs with opposite topological charges. The transformation of optical vortices ("topological reactions") were investigated also in [32, 33].

2. Mathematical Model

We will investigate the process of optical vortex nucleation in the paraxial approximation [1]. It describes the variety of singular optics events good enough, when the criteria of its validity are fulfilled. We will seek for a solution of the wave equation for the vector potential $\hat{\mathbf{A}} = \mathbf{n}U(x,y,z)$ of a linearly polarized beam ($\mathbf{n}$ is a unit vector in the light polarization direction) in the form:

$$U = A(x,y,z)\exp\left[i(\omega t - kz + \Phi(x,y,z))\right] =$$

$$= (u + iv)\exp\left[i(\omega t - kz)\right]. \tag{1}$$

Here, ω is the beam frequency, k is the wave vector, $\Phi(x,y,z)$ is the phase of the beam propagating along the z axis. This function defines the shape of a wave front in the vicinity of a small region $z \approx z_0$ by the equation $z = \frac{1}{k}\Phi(x,y,z_0)$ to within some constant. The complex amplitude $A(x,y,z) = u(x,y,z) + \mathrm{i}v(x,y,z)$ satisfies the well-known Leontovich parabolic equation [2]

$$\frac{\partial A}{\partial z} = \frac{1}{2ik}\Delta A, \tag{2}$$

where Δ is the Laplace operator in the plane xy.

It is easy to find the evolution of the beam field $U(x,y,z=0)$ during its propagation by application of the integral valid for $z >> \sqrt{x^2+y^2}$:

$$U(x,y,z) = \frac{ik}{2\pi z}\iint U(x',y',0)\times$$

$$\times\exp[-ik\frac{(x-x')^2+(y-y')^2}{2z}]dx'dy'. \tag{3}$$

This relation is consistent with the calculation of the interference picture from the multitude of point sources located in the plane $z = 0$. It is possible to find also the zero-amplitude lines $U(x,y,z) = 0$ or the trajectories of vortices. Optical vortices nucleate as a result of deep processes in the whole light field. The goal of this paper is to investigate the evolution of the beam as a whole and to find, in this way, factors which define the vortex nucleation in a propagating beam with an arbitrary wave front, including the smooth wave front without singularities. To do this, it is necessary to introduce new notions into the mathematical description of the propagating wave beam.

In terms of real variables, we define

$$U(x,y,z) =$$

$$= u(x,y,z)\cos(\omega t - kz) - v(x,y,z)\sin(\omega t - kz) \equiv$$

$$\equiv U_0(x,y,z)\exp[\omega t - kz + \Phi(x,y,z)], \tag{4}$$

where $U_0 = \sqrt{u^2+v^2}$, $\Phi = \mathrm{Arctan}\frac{v}{u}$, $u = U_0\cos\Phi$, $v = U_0\sin\Phi$. All this is well known, but we will enlarge its physical interpretation. The temporal changes in the light field represent themselves as a superposition of vibrations with the $\pi/2$ phase-shifted amplitudes $u(x,y,z)$ and $v(x,y,z)$. These excitations propagate along and across the z axis according to the given laws. The wave-front structure $z = \frac{1}{k}\Phi(x,y,z_0)$ and a Poynting vector orientation are caused by the space inhomogeneity of the $v(x,y,z)/u(u,x,y,z)$ ratio. Therefore, we will concentrate on the evolution of the field components $u(x,y,z)$and $v(x,y,z)$, which will give us the key to the understanding of the mechanisms of the nucleation of vortices and their following evolution. This evolution can be followed by some imaginary observer who is moving along the $z$axis by the light velocity c.

The system of equations for u and v,

$$\frac{\partial u}{\partial \tau} = \frac{1}{4}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right), \quad \frac{\partial v}{\partial \tau} = -\frac{1}{4}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{5}$$

follows from (2) with the dimensionless variables $\tau \Rightarrow z/L_R$, $x,y \Rightarrow x/r_0, y/r_0$. The transverse coordinates are normalized to the beam waist $r_0(z=0)$, and the longitudinal coordinate is normalized to the Rayleigh range $L_R = kr_0^2/2$.

Equation (5) describes the *dynamics* of the u and v components on the XY plane (the beam broadening) with "time" $\tau \sim z = ct$ under the initial conditions

$$u = u(x,y,0), \quad v = v(x,y,0). \tag{6}$$

The spreading of the beam possesses a peculiar character, because it is defined by the u and v excitations in the regions of their pronounced space inhomogeneity. System (5) for excitations of the types $u = u_0\exp[i(\hat{\omega}\tau -$

 ISSN 0503-1265. Ukr. J. Phys. 2005. V. 50, N 4

$k_\perp x)]$, $v = v_0 \exp[i(\hat{\omega}\tau - k_\perp x)]$ leads to the next dispersion relation in the one-dimensional case:

$$\hat{\omega} = \frac{1}{4}k_\perp^2. \tag{7}$$

It fits the wavelength-dependent phase velocity of excitations:

$$\hat{v}_{\text{ph}} = \frac{\hat{\omega}}{k_\perp} = \frac{k_\perp}{4}. \tag{8}$$

The parameter $k_\perp$ defines the period of oscillations of the u and v functions in the plane. It follows from relations (5) and (7) that $u_0 = iv_0$, i.e. the v-component waves are $\pi/2$-phase-delayed relative to the waves u or are shifted in space by the wave quarter $\lambda_\perp = 2\pi/k_\perp$.

The characteristics of the considered perturbation waves can be found from the set of interferograms of the considered wave and a coaxial reference plane wave for various t, i.e. for the transverse planes with various z coordinates. The periodicity of the interference extrema on these planes defines the wave vector $k_\perp$ of perturbations from relation (7), which is characteristic of the given value of t. The values of the maxima (minima) shifts under a variation of z ("time" τ) characterize the perturbation of the phase velocity of waves in the dynamic problem (5) under consideration. Of course, relation (7) has to be corrected in the two-dimensional case. But its main consequence, namely the growth of $\hat{v}_{\text{ph}}$ with $k_\perp$, remains unchanged. Therefore, our qualitative analysis of the vortex nucleation mechanisms can be based on this conclusion.

The initial distribution (6) is asymmetric in the general case of inhomogeneous beams and axial beams after a sharp obstacle (diffraction). Therefore, the perturbation flows will be also asymmetric and will differ in the wave vector $k_\perp$. The perturbations with a shorter wavelength propagate faster and possess greater changes of $\Phi(x, y, \tau)$ along the flow (across the z axis). As a result, they will perturb the initial multiconnected sequence of smooth wavefronts. Moreover, the zones of anomalies will appear. They will represent themselves as various types of folds and shifts of one piece of the wave front to another one along the z axis. If such an anomaly is neatly pronounced in any narrow region and the fold deepness reaches $\lambda/2$, the wave front *disrupts*, the neighbor phase sheets switch together, and one single-connected phase surface makes up. This event corresponds to the nucleation (generation) of a pair of vortices with opposite topological charges. So, if we want to nucleate vortices, we need to realize such initial condition (6) that the transverse perturbations of the u and v flows be strongly asymmetric and strong enough.

We will show that the vortex nucleation mechanisms in the region of smooth wave-front anomalies appear themselves as the direct analogies with the creation of the structures of vortices on a neutral current layer in magnethydrodynamics [2] or on the layers with tangential disruption of the flow velocity in usual hydrodynamics [3].

Let us now move to the application of the above-presented approach to the simple, but still unknown nontrivial examples.

3. Nucleation of Vortices by an Optical Analog of the Magnetohydrodynamic Instability of the "Neutral Current Layer"

Let us consider a combined beam with two noncoaxial Gaussian beams without any singularities. The axis of the first beam coincides with the z axis. The amplitude decreases exponentially with radius $r = \sqrt{x^2 + y^2}$:

$$u_1(x, y, 0) = U_{01} \exp(-r^2/r_{01}^2), \quad v_1(x, y, 0) = 0. \tag{9}$$

The second beam possesses the same amplitude, the lesser waist radius $r_{02} < r_{01}$ and the axis shifted along the y axis by a distance y_{12}:

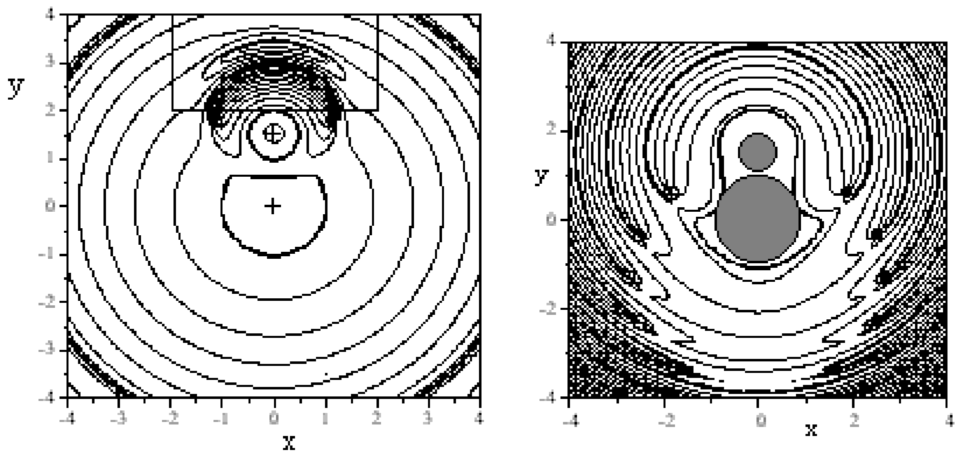
$$u_2(x, y, 0) = U_{01} \exp\left(-\frac{x^2 + (y - y_{12})^2}{r_{02}^2}\right),$$

$$v_2(x, y, 0) = 0. \tag{10}$$

It was presumed that $r_{02} = 0.4r_{01}$. All dimensionless variables were defined through r_{01}.

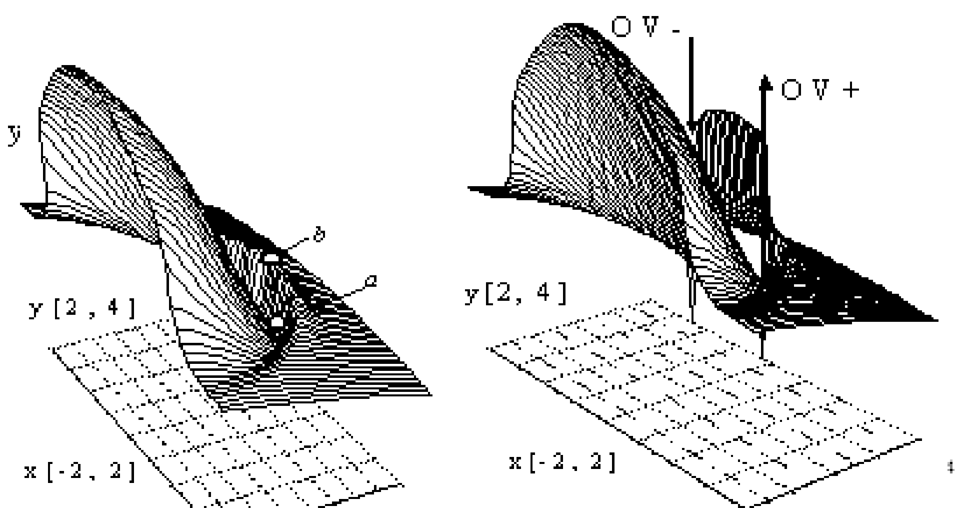
Which physical assumptions were in the basis of such a choice? It is clear that a narrower beam will produce the transverse perturbation waves u_2 and v_2 with a wavelength of the order of r_{02} ($\lambda_\perp \approx 0.4$) at $\tau > 0$. These relatively fast perturbations will propagate against the immovable background created by the broader second beam according to the evaluation by (8). On the other hand, the perturbations u_2 and v_2 possess relatively small amplitudes and will appear on the broader periphery only. Due to the chosen geometry, these waves will diminish essentially the phase $\Phi(x, y, \tau)$ of the combined beam in the narrow zone along the y axis. As a result, the phase folds have to appear on the smooth wave front and will manifest themselves as the regions of wave-front disruptions and the nucleation of vortex dipoles.

The designed characteristics of the combined beam presented in Fig. 1 support the foregoing assumptions. One vortex pair nucleated yet to the moment $\tau = 0.17$.



a **b**

Fig. 1. Phase map $\Phi(x, y, \tau)$ of the combined beam at $\tau = 0.17(a)$ and $0.7(b)$. Filled (open) circles are the single-charge positive (negative) optical vortices. Crests (a) are the centers of the Gaussian beams. The shadow disks with $r_i = r_{0i}$ (b) are the localization zones of the beams at $\tau = 0$. The bold lines correspond to the phase lines which differ by 2π



a **b**

Fig. 2. a — the shape of the wave front $z = \Phi(x, y, \tau)/k$ of the combined beam in the fold area, $\tau = 0.17$; b — the initiation of the wave-front disruption and nucleation of vortex dipole, $\tau = 0.175$

We draw the readers' attention to the region in the upper part of Fig. 1,a confined by the rectangular frame. The phase fold is formed there (its 3D structure is shown in Fig. 2,a). It is seen that the transverse components n_y of the normal $\mathbf{n}$ to the wave front at $x \approx 0$, y are oppositely directed. Therefore, two *oppositely directed* transverse energy flows appear in the combined beam. This key moment allows us to make a direct analogy of this optical effect with the well-known "neutral current layer" in magnetohydrodynamics [5]. The plasma contrary movement in such a layer results in switching on the magnetic power lines and the creation of the structures of vortices.

The widening of the combined beam with growth in τ is attended by the enhancement of the transverse perturbation wave flows into the fold region and its "deepness" growth. When the distance between points a and b reaches the critical value $\lambda/2$, the wave-front disruptions appear and the pair of vortices nucleates (Fig. 2,b). The sheets of the neighbor wave fronts switch together, and a single-connected structure appears along the z axis by the translation of the profile in Fig. 2,b with the period λ.

The sign of born vortices can be defined directly from physical reasons even without help of the phase map. Actually, the fastest flows of transverse perturbations take place in the vicinity of the y axis. For the vortices located in the $x > 0$ area, the energy flow is higher leftwards than rightwards. Due to this the energy circulates clockwise. Therefore, the phase on the XY plane has to grow in the opposite direction, which corresponds to $m = +1$.

The next widening of the region with u and v perturbations leads to the automatic drift of vortices to the lower half-plane. The picture of the "hydrodynamic flow" round the broader "stationary" beam by the fast perturbations of the narrow beam is formed, and two vortex streets appear like the Karman vortex streets in hydrodynamics [6].

The spectrum of the transverse perturbation waves produced by the narrow beam undergoes the "red shift" with growth in t, and the process of vortex nucleation terminates. What's more, the family of vortex dipoles shown in Fig 1, b annihilates due to the approach of vortices in the pair to each other. It remains only one vortex dipole nearest to the beam center during some limited period of "time".

A detailed analysis of the space evolution of vortices will be done in future. We will concentrate now on the establishment of a mechanism of nucleation of optical vortices and their analogs in other branches of physics. The following forecast can be given on this subject. It is known that the Gaussian beam waist radius grows as $r_i = r_{0i}\sqrt{1+\tau^2/r_{0i}^4}$. Due to this, an initially narrower beam becomes more broad than an initially broader beam. It happens at $\tau^2 >> 1$, when $r_2/r_1 \approx r_{01}/r_{02}$. Due to these transformations, a new generation of optical vortex dipoles is born. Fig. 3 substantiates this prediction.

As a summary of this section, we note that the creation of asymmetric transverse perturbation flows in the combined beam with initially smooth wave fronts initiates the system of optical vortex dipoles. We emphasize that the optical "neutral current layers" appear due to the interaction of faster perturbation flows with the relatively slow background from another beam. This interaction possesses a nonlinear character because the combined beam phase is not the sum of the separate beam phases.

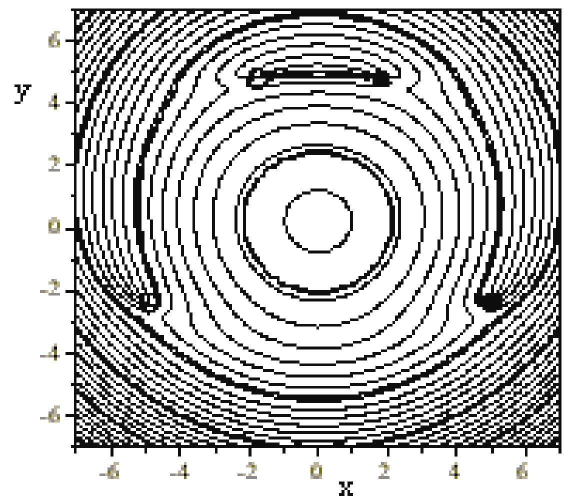


Fig. 3. Phase map of the combined light beam at $\tau = 3.5$. The new-generation vortex dipole has nucleated in the upper half-plane

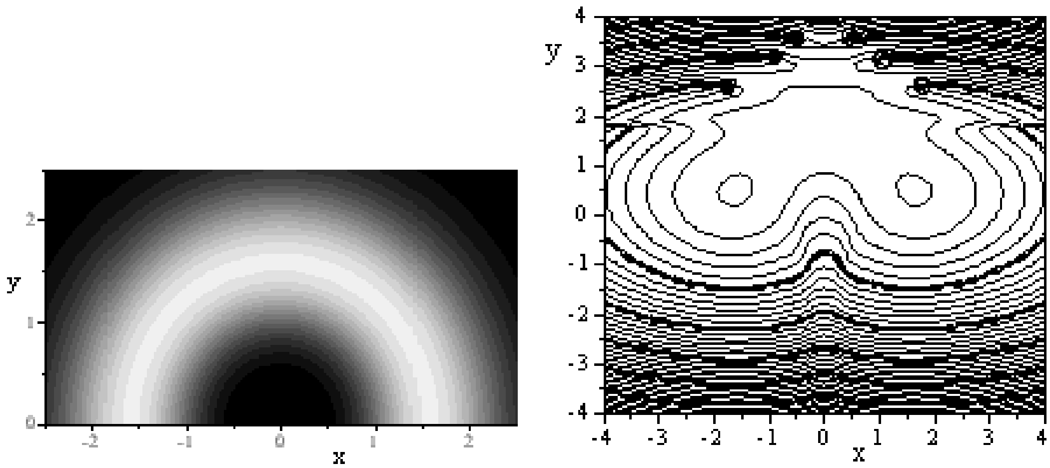
4. Nucleation of Vortices by an Optical Analog of the Hydrodynamic Instability of "Tangential Disruption of the Current Velocity"

We have found above that vortices nucleate in the combined light beam with initially smooth wave front of its components. Let us try now to answer the next natural question: Is the vortex nucleation in a *single* light beam with initially smooth wave front possible? To check this possibility, let us consider the diffraction of a ring-shaped light beam with smooth wave front which is half-screened by the half-plane (Fig. 4). We assume that the initial amplitude distribution (with $r_0 = 1$) is

$$u(r,0) \sim r^5 \exp(-r^2), \quad v(r,0) = 0. \tag{11}$$

The amplitude maximum is reached at $r_{\max} = 1.58$.

The dynamics of the beam evolution behind the screen can be found even without calculation of the interference integral (3). The flows of initial fast short-wavelength transverse perturbation waves are generated by the sharp intensity jumps at $y = 0$ in the interval $1 < |x| < 2$. They are directed up and down along the y axis because the perturbation wavelengths are much less than the size of the "generation region" along the x axis. In the propagation region, the forthcoming nucleation of vortices should occur with a low background according to the mechanism of "neutral current layer" as in the previous section (Fig. 4,b). The sign of born vortices is defined by the direction of perturbation flows. It is positive for the right direction and negative for the left



a **b**

Fig. 4. a — beam initial amplitude distribution, b — phase map for $\tau = 0.5$ found with the help of (3). Filled (open) circles correspond to optical vortices with $m = +1$ ($m = -1$)

one. The widening of wave-front disruptions with τ moves the "inner" vortices to the Y axis, which is seen in Fig. 4,b. The "external" vortices are pushed to the beam periphery.

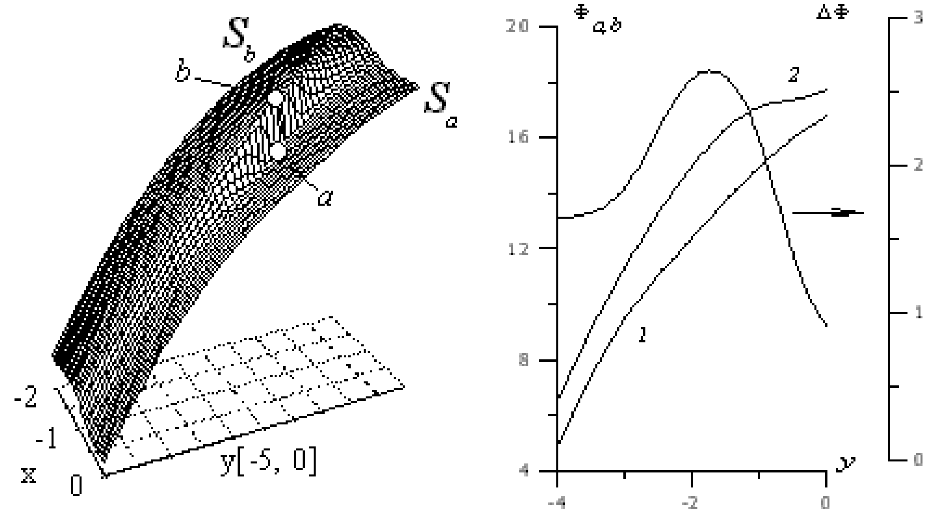
With increase in the "time" τ, the perturbation flows clean from the fast short-wavelength perturbations, and their spectrum undergoes "the red shift". As a result, the pairs of central vortices have to annihilate. But the next mechanism of vortex nucleation develops at this moment, and vortices nucleate in the shadow region ($y < 0$) where the immovable background is absent due to the new "hydrodynamic mechanism". Really, the perturbation flows on the right and left sides of the light beam diminish. The reverse process takes place in the central part of the light beam. The central part of the arc-shaped beam "radiates" the perturbation waves like a peculiar searchlight. Their wavelengths are of the order of the wave-front transverse dimensions. This slow preaxial flow penetrates into the shadow region. Nevertheless, its velocity is higher than the velocity of peripheral flows. As a result, an optical analog of the hydrodynamic instability of "tangential disruption of current velocity" is created.

Fig. 5 gives an insight into the structure of optical version of such mechanism. The difference between the rates of phase changes along flows in the XY plane leads to a shift of the neighbor competing sections of the wave front relative to each other (Fig. 5,a). Fig. 5,b gives a change of the phase $\Phi_{a,b}(x,y,1.5)$ along the straight lines located along dissimilar sites of the tangential disruption layer. The location of the maximal phase difference between these lines $\Delta\Phi_{\max}$ at $y \approx 1.7$ corresponds to the border of the domination of the flow of central perturbations.

The shift of wave-front regions between points a and b (Fig. 5,a) is of the order of $\Delta\Phi/k$. During the next "red shift" of the spectrum of perturbations, the phase shift reaches the critical value $\lambda/2$, which is followed by switching together regions S_a and S_b^- located under region S_b in the λ interval.

The wave-front disruptions and two nucleated dipoles are shown in Fig. 6. It is seen that the wave sheet in S_a switches to the lower sheet S_b^- to the right and left. Due to this, the vortices nearest to the coordinate origin will possess the charge $m = +1$ ($m = -1$) for x <0 (x >0). Such an arrangement of vortices is natural, when the phase velocity of perturbations for the central flow is higher than that for the lateral flows. The opposite-sign vortices born during these tangential velocity disruptions are alien in this sense and are pushed out to the light beam periphery. Fig. 7 gives the beam amplitude and phase structure for great t, i.e. on the final stage of evolution.

Finally, we evaluate the nucleation "time" τ_c of the vortices under consideration in the shadow region, i.e. behind the half-plane screen. The inner front width of the initial amplitude distribution (11) for the perturbation length $\lambda_\perp$ is of the order of 0.7 according to (11). Then, according to formula (8) for the phase velocity of perturbations, the wave field penetrates into the shadow half-plane by the distance $r_{\max}$during the time $\tau_c \approx 4r_{\max}\,\lambda_\perp/\pi = 1.4$. This evaluation is in full



a **b**

Fig. 5. a — structure of the wave front in the area of the tangential velocity disruption of the perturbation velocity; b — the wave front $\Phi_{a,b}$ $(x, y,\ 1.5)$ phase shift at $x_a = -0.3$ (*1*) and $x_b = -1.2$ (*2*) (left scale). The phase shift $\Delta\Phi = \Phi_b - \Phi_a$ (bold curve) between curves *2* and *1* (right scale). Both scales are given in radians

accordance with the results of calculations. It is important only to define the directions of the asymmetry of perturbation flows and the time evolution of their spectrum values $\lambda_\perp$.

5. Discussion

First, we discuss a heuristic sense of the elaborated mechanisms in the general approach to the problem of the nucleation of vortices in optical fields with an arbitrarily given structure including fields with or without initial phase singularities. It is clear that the diffraction integral (3) allows one to obtain all the desired results including the nucleation of optical vortices. On the other hand, the final result did not answer the questions about which mechanism provides for this result and which is the totality of light beam parameters allowing this process. The answer to these questions is of principal importance for the nucleation of vortices due to the accumulation of topological changes in the amplitude-phase structure of a light beam during the free propagation, interaction with nonlinear media, and diffraction by some obstacle.

The vortex nucleation itself is a result of the wave front π-disruption. We have seen it proceeds by smooth changes of a wave-front shape. It is clear from the topological point of view that these changes can possess the fold or ledge in the direction of propagation. The analysis shows that the two developed mechanisms

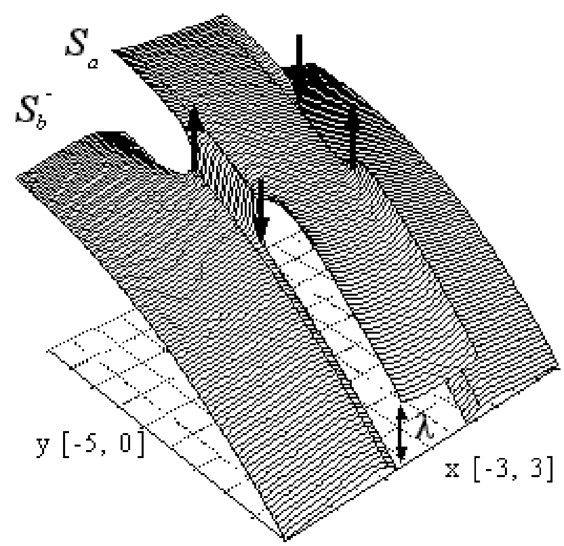
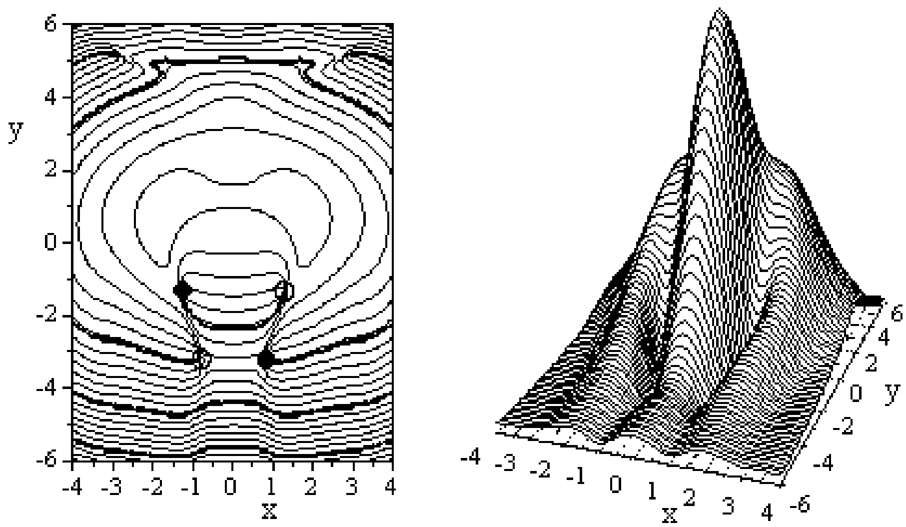


Fig. 6. Structure of wave-front fragments after the switching together phase sheets and the nucleation of the dipoles of vortices. The arrows directed above and down give the location of positive and negative single-charge vortices

correspond just to these variants. What's more, we believe that they exhaust all possible variants of the optical vortex nucleation on the whole. Actually, the phase fold occurs before vortex nucleation by the mechanism of neutral current layer instability (see Fig. 2,a). This fold is realized due to the propagation of



a b

Fig. 7. Phase map of a propagating beam and the amplitude distribution at $\tau = 5$

transverse perturbation waves along the initially existing immovable background. As opposed to this case, the phase ledge appears when the faster current of transverse perturbation waves competes with the slower lateral current (the instability of tangential disruption of the current velocity). This occurs, for example, when the light field penetrates into the shadow region due to the diffraction on a screen (Fig. 6). Common for both cases is the presence of a sharp asymmetry of the initial field of the corresponding transverse excitation waves. Its appearance depends on all actual parameters of the considered system. So, such parameters for two nonaxial Gaussian beams are the distance between beam axes and the ratio of their amplitudes and waists. It is important to note that the needed asymmetry value can be obtained for each given system only as a result of the computer modeling, as done in this paper.

The developed approaches admit, in our opinion, to forecast both the vortex nucleation for the initial light field structure and the actual mechanism of nucleation. From this point of view, we will analyze shortly all the cases of vortex nucleation known up to date. Let us start from the initially smooth light fields. It seems that the multitude of vortices, which decorate caustics (the so-called "diffraction catastrophe" [20, 21]), appears due to the mechanism of neutral current layer. Vortices appear in the region where the phase gradient of the intersecting transverse excitation waves achieves the threshold π value. The nonlinear diffraction catastrophe with the asteroid-shape caustic was obtained due to the self-defocusing of an elliptical Gaussian beam with the aspect ratio 2:1 [34]. Two counterpropagating systems of transverse waves were excited due to the incident beam asymmetry. Their intersection inside the asteroid results in the appearance of a system of interferometric intensity minima. But the wave-front disruptions occur inside the sharp cusps only, and a vortex quadruple nucleates. The threshold character of this event was demonstrated also in [35]. In our opinion, vortices nucleate by "the instability of neutral current layer" due to the sufficient asymmetry of the incident light beam.

The next example is the circular edge dislocation [3] which appears in the far field of a smooth wave diffracted by the round aperture (Airy rings) [3, 27, 33]. The birth of such a dislocation in the common waist of two out-of-phase Gaussian beams with unequal waist parameters (1:10) and amplitudes (2:1) was recently investigated. It was shown that the phase ledges are created in the vicinity before and after the waist plane, and their transformation into a circular wave-front disruption in the waist plane was demonstrated. It easy to see that such two-beam system is axially symmetric, but radially asymmetric. As a result, the instability of the circular-shape neutral current layer is the cause for the circular edge dislocation birth. It could be believed that such a dislocation is created also if the interacting beams are in-phase. But this will happen at such a distance from the common waist plane, where the height of the phase ledge achieves $\lambda/2$.

The same mechanism of circular edge dislocation nucleation was realized also due to the creation of a nonlinear Gaussian-like lens by a Gaussian laser beam self-action [26, 37]. When the nonlinear medium is anisotropic, a circular edge dislocation transforms to a quadruple of optical vortices.

The family of vortices nucleates during the diffraction of a screened circular beam [38, 39]. The propagation and shadow regions have to be distinguished. The optical analog of the instability of a neutral current layer is realized in the propagation region, and the analog of the instability of tangential disruption of the current velocity appears in the shadow region, where the immovable background is absent.

Few papers are devoted to nonlinear transformations of singular beams. The mismatch of the phase condition causes a walk-off of the born beams (see, for example [38, 39]). This leads immediately to a linear arrangement of the born vortices up to the creation of a vortex street [30]. As we have seen, the same two-fold vortex street was obtained in our case (see Fig. 6). Therefore, it is the reason to believe the nonlinear vortex street is created due to the optical analog of the instability of tangential disruption of the current veloccity.

To conclude, the nucleation of optical vortices is possible both in initially smooth and singular beams. In all the known cases, the reason for the vortex nucleation was a sufficient asymmetry of the transverse excitation wave.

6. Conclusions

We have studied thoroughly the problem of the nucleation of optical vortices in the light beams with both initially smooth wave front and pronounced transverse inhomogeneity. The tool for such an analysis was the developed model based on the space evolution of the $\pi/2$-shifted u and v components of a light field. It was shown that the elaborated model allows one to establish the mechanisms of the nucleation of optical vortices.

In brief, we have obtained the following results:

(i) It is shown that the formation of a wave-front fold or ledge, i.e. the shift of neighbor wave-front pieces along the axis of propagation, precedes the wave-front disruption and the nucleation of phase singularities.

(ii) According to the possible forms of a phase anomaly (a fold or ledge), the nucleation of optical vortices can be realized due to two next mechanisms: (a) an analog of the known “instability of a neutral current layer” in magnetohydrodynamics, which takes place against the immovable nonzero background, (b) an analog of the hydrodynamic “instability of tangential disruption of the current velocity” in continuos media which takes place during the penetration of a light field into the shadow area.

(iii) We have shown the possibility of the nucleation of a system of optical vortices in the light beam formed by two noncoaxial coherent Gaussian beams with unequal waists via the instability mechanism for a neutral current layer. The born vortices flow round the initial broader beam and form a two-fold Karman vortex street well-known in hydrodynamics. They annihilate later, and the “second-generation” vortices nucleate.

(iv) The nucleation of optical vortices is possible in a *single* light beam with the half-ring shape both via the mechanism of neutral current layer (in the propagation region) and the instability mechanism of transverse disruption of the current velocity (in the shadow region).

(v) The performed analysis allows us to make the next general conclusion: the nucleation of optical vortices occurs in a single or combined light beam with smooth wave front if the current of transverse excitation waves possesses a pronounced asymmetry.

(vi) We have analyzed the known cases of the nucleation of optical vortices during the propagation of a light beam in free space and/or in nonlinear systems. It is shown that all of them fit in two proposed mechanisms.

This research was supported by STCU Partner Project P051.

1. *Nye J.F., Berry M.*// Proc. Roy. Soc. (London), Ser A. **336**, 165 (1974).
2. *Coullet P., Gil L., RoccaF.*// Opt. Communs. **73**, 403 (1989).
3. *Nye J.F.*// Natural Focusing and Fine Structure of Light: Caustics and Wave Dislocations (Bristol: Institute of Physics, 1999).
4. *Soskin M., Vasnetsov M.*// Photonics Sci. News **4**, is. 4, 21 (1999).
5. *Trapani P.Di, Berzanskis A., Morandi S. et al.*//Phys. Rev. Lett. **81**, 5133 (1998).
6. *Tiwari S.C.*// J. Mod. Optics **46**, 1721 (1999).
7. *Berry M.V.* Physics of Defects/ Ed. by R. Balian, M. Kléman, and J.P. Poirier. — Amsterdam: North Holland, 1981. — P.453.
8. *He H., Heckenberg N.R., Rubinzstein-Dunlo H.*// J. Mod. Optics **42**, 217 (1995).
9. *Proc.* Intern. Conf. on Singular Optics/ Ed. by M. Soskin. — SPIE, **3487** (1998).
10. *Horizons* of World Physics. — Vol. 228: Optical Vortices /Ed. by K. Staliunas and M.V. Vasnetsov. — New York: Nova Sci. Publ., 1999.
11. *Proc.* 2nd Intern. Conf. on Singular Optics/ Ed. by M. Soskin and M. Vasnetsov. SPIE, **4403**, 2001.

12. *Vaughan J.M., Willets D.V.*// J. Opt. Soc. Amer. **73**, 1018 (1983).
13. *Berry M.V.*//Proc. Intern. Conf. on Singular Optics/ Ed. by M. Soskin. — of SPIE, **3487** (1998). — P. 1.
14. *Beijersbergen M.W., Allen L., Van der Veen H.E.L.O., Woerdman J.P.*// Opt. Communs. **96**, 123 (1994).
15. *Bryngdahl O.*// J. Opt. Soc. Amer. **63**, 1098 (1973).
16. *Bazhenov V.Yu., Soskin M.S., Vasnetsov M.V.*// JETP Lett. **52**, 429 (1990).
17. *Heckenberg N.R., McDuff R. Smith, C.P.*//Opt. Quantum Electron **24** S951 (1992)
18. *Beijersbergen M.W., Coerwinkel R.P.C., Kristiensen M., Woerdman J.P.*// Opt. Communs. **112**, 321 (1994).
19. *Lippi G.P., Ackemann T., Hoffer L.W. et al.*// Phys. Rev. A. **48**, R4043 (1993).
20. *Berry M.V.*//Prog. in Optics. **18**, 259 (1980).
21. *Wright F.J., Nye J. F.*// Phil. Trans. Roy. Soc. (London A. **305**, 339 (1982).
22. *Baranova N.B., Zel'dovich B.Ya., Mamaev A.V. et al.*// JETP Lett. **33**, 195 (1981); JETP **56**, 983 (1982); J. Opt. Soc. Amer. 73801, 525 (1983).
23. *Freund I., Shwartsman N., Freilicher V.*// Opt. Communs. **101**, 247 (1993); *Shwartsman N., Freund I.*// Phys. Rev. Lett. **72**, 1008 (1994); *Freund I.*// J. Opt. Soc. Amer. A. **11**, 1644 (1994).
24. *Vasnetsov M.V., Ilyenkov A.V., Soskin M.S.* // Techn. Digest of 5th Europ. Quant. Electron. Conf. Amsterdam, 1994. — P. 109; *Ilyenkov A.V., Khizhnyak A.I., Kreminskaya L.V. et al.*// Appl. Phys. B **62**, 465 (1996); *Ilyenkov A.V., Kreminskaya L.V., Soskin M.S., Vasnetsov M.V.*// J. Nonlinear Phys. and Math. **6**, 169 (1997).
25. *Ackemann T., Kriege E.E., Lange W.*// Opt. Communs.**115**, 339 (1995).
26. *Kreminskaya L.V., Soskin M.S., Khizhnyak A.I.*//Ibid. **145**, 377.
27. *Karman G.P., Beijersbergen M.V., Van Duijl A., Woerdman J.P.*// Opt. Lett. **22**, 1503 (1997).
28. *Vasnetsov M.V., Marienko I.G., Soskin M.S.*// JETP Lett. **71**, 130 (2000).
29. *Gorshkov V.N., Soskin M.S., Kononenko A.N.*// 2nd Intern. Conf. on Singular Optics/ Ed. by M. Soskin, M. Vasnetsov. — SPIE, **4403**, 128 (2001).
30. *Molina-Terriza G., Torner L., Petrov D.V.*// Opt. Communs. **24**, 899 (1999).
31. *Angelsky O.V., Mokhun I.I., Mokhun A.I., Soskin M.S.* //Proc. 2nd Intern. Conf. on Singular Optics / Ed. by M. Soskin, M. Vasnetsov. — SPIE, **4403**, 116 (2001).
32. *Berry M.V.* // J. Mod. Optics **45**, 1845 (1998).
33. *Karman G.P., Beijersbergen M.V., Van Duijl A. et al.*// J. Opt. Soc. Amer. A. **15**, 884 (1998).
34. *Deykoon A.M., Soskin M.S., Swartzlander G.A.*// Opt. Lett. **24**, 1224 (1999).
35. *Deykoon A.M., Soskin M.S., Swartzlander G.A.*// J. Opt. Soc. Amer. (to be published).
36. *Vasnetsov M.V., Gorshkov V.N., Marienko I.G., Soskin M.S.*// Opt. and Spektr. **88**, 260 (2000).
37. *Reznikov Yu.A., Soskin M.S., Slyussarenko S.S. et al.*// Mol. Cryst. Liq. Cryst. **324**, 25 (1998).
38. *Beržanskis A., Matijošius A., Piskarskas A. et al.*// Opt. Communs. **150**, 372 (1998).
39. *Petrov D.V., Molina-Terriza G., Torner L.*//Ibid. **162**, 357 (1999).

Received 24.06.04

МЕХАНІЗМИ ФОРМУВАННЯ ВИХОРІВ В ОПТИЦІ ТА ГІДРОДИНАМІЦІ

В.Н. Горшков, Г.М. Хорошун, М.С. Соскін

Р е з ю м е

Побудовано теорію виникнення оптичних вихорів в разі асиметричного збурення пучка з початково гладким хвильовим фронтом. Теорія грунтується на розвинутій двовимірній математичній моделі дифракції як дифузії поперечних хвиль збурення. Розглянуто два можливих механізми виникнення оптичних вихорів у комбінованих гауссових пучках і дифракції плоскої хвилі на дузі. Народження вихорів розглянуто як аналог добре відомого випадку народження вихорів внаслідок "магнітогідродинамічної нестабільності тангенціального розриву швидкості потоку". Проаналізовано умови її виникнення та еволюції у просторі. Всі відомі випадки лінійного та нелінійного народження вихорів можуть бути описані у рамках розглянутих механізмів.