# ON THE THEORY OF MODULATIONAL INSTABILITY OF STOKES WAVES

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We study the modulational instability of Stokes waves without the traditional assumption as to the movement of a mean flow with the group velocity of the first harmonic. We have shown that the well-known limit $kh = 1.363$, where $h$ is the fluid depth and $k$ is the wave number, for the transition between the states of modulationally stable and unstable fluids in the case of weakly nonlinear waves can be obtained without this assumption. This limit is shown to be shifted to greater $kh$, as the nonlinearity is strengthened. It is also shown that the disappearance of the modulational instability (restabilization) for small wave numbers of a perturbation, which was predicted on the basis of the Zakharov equations and numerical calculations of the exact equations, does not follow also from a weakly nonlinear theory, provided that the above assumption is not used.

## 1. Introduction

As well known, the envelope of a packet of rapidly oscillating waves on the surface of a layer of the ideal fluid is not stable against a small longitudinal harmonic perturbation if the product of a wave number of fast oscillations $k$ on the depth of a fluid $h$ is more than 1.363. This result is obtained theoretically in [1] by analyzing the interaction of a harmonic perturbation of the first harmonic with the second and zero harmonics, in [2] within the method of averaged Lagrangian, and in [3] firstly as a consequence of the application of the criterion for the existence of soliton solutions to a nonlinear Schrödinger equation (NSE) obtained in [3] and, secondly, by examining the instability increment of a perturbed Stokes wave as the solution of this NSE. All three works used the assumption (directly in [3] and indirectly in [1] and [2]) that the mean flow (zero harmonics) moves with the group velocity of the first harmonic.

In the present work, we have solved the problem without the above-mentioned assumption.

Until today, the experimental works (see, for example, [4, 5]) considered the case of a fluid of the infinite depth ($kh \to \infty$). They confirmed the conclusions of the theory [4, 6—8] about that the envelope of waves is really modulationally unstable at the infinite depth of a fluid. However, it is known that the average values induced by waves tend to zero at $kh \to \infty$, and the application of the above-mentioned assumption about the velocity of average values is not necessary. Recently, the first experimental works concerning intermediate depths appear [9, 10], and the physical mechanism of a "roque wave phenomenon" [11] is actively studied, in particular on the basis of the modulational instability and long-time evolution of the instable Benjamin — Feir mode [12]. Therefore, the development of the theory in the case of finite-depth fluid layers seems to be actual.

## 2. Study of the Modulational Instability of Stokes Waves without the Assumption as to the Mean Flow Motion with the Group Velocity

The envelope amplitude $A$ for the first harmonic of an elevation of the surface $\eta = \frac{1}{2}A\exp(i(kx-\omega t))+$c.c. and the amplitude $\Psi_1$ of the zero harmonics of the velocity potential in the approximation $O(\varepsilon^3)$, where $\varepsilon$ is a small parameter which characterizes both a smallness of the amplitudes $A$ and $\Psi_1$ and a slowness of their change in time and space, satisfy the system of coupled equations [3, 13, 14]

$$i(\frac{\partial A}{\partial t} + c_g\frac{\partial A}{\partial x}) + p\frac{\partial^2 A}{\partial x^2} + \tilde{q}A^2\,\overline{A} - -(k\frac{\partial \Psi_1}{\partial x} + \mu\frac{\partial \Psi_1}{\partial t})A = 0\,, \quad (1)$$

$$\frac{\partial^2\Psi_1}{\partial t^2} - c_0^2\frac{\partial^2\Psi_1}{\partial x^2} - \nu\frac{\partial}{\partial x}A\overline{A} = 0. \quad (2)$$

Here, we denote

$$c_g \equiv \frac{\partial\omega}{\partial k} = \frac{\omega}{k}C_g, \quad C_g = \frac{1}{2} + \frac{1-\sigma^2}{2\sigma}kh,$$

$$c_0 \equiv \sqrt{gh} = \frac{\omega}{k}C_0, \quad C_0 = \sqrt{\frac{kh}{\sigma}},$$

$$\sigma = \tanh kh, \quad p \equiv \frac{1}{2}\frac{\partial^2\omega}{\partial k^2} = \frac{\omega}{k^2}P,$$

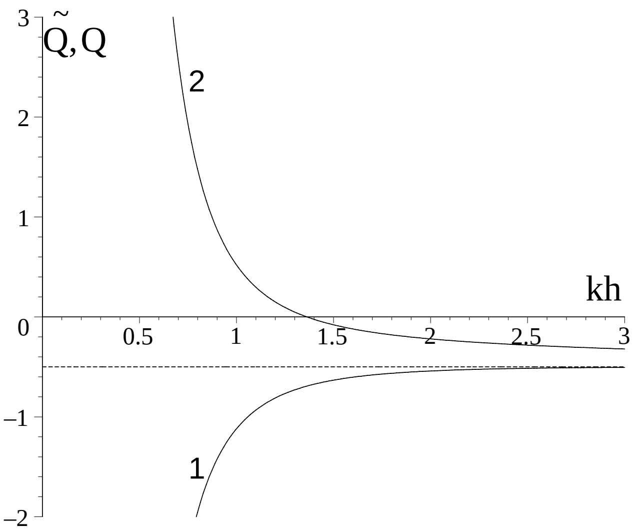


Fig. 1. Coefficients of nonlinearity: *1* — $\widetilde{Q}$, *2* — $Q$. The horizontal asymptote corresponds to the case of the infinite depth

$$P = \frac{1}{8\sigma^2}((\sigma^2-1)(3\sigma^2+1)k^2h^2-$$
$$-2\sigma(\sigma^2-1)kh-\sigma^2),$$
$$\widetilde{q} = \omega k^2\widetilde{Q}, \quad \widetilde{Q} = \frac{1}{16\sigma^4}(2\sigma^6-13\sigma^4+12\sigma^2-9),$$
$$\mu = \frac{k^2}{\omega}M, \quad M = \frac{1}{2}(\sigma^2-1),$$
$$\nu = \frac{\omega^3}{k}N, \quad N = \frac{1}{2\sigma^2}\left[1-MC_g\right],$$

$\omega = \sqrt{gk\sigma}$ is the frequency of a carrier wave, and $g$ is the gravity acceleration. System (1) — (2) as a model of the interaction of long and short waves is used in various physical problems [15, 16], and its simplified version is even solved by the method of inverse scattering problem [17]. Upon the derivation of a nonlinear Schrödinger equation from system (1), (2), it was traditionally supposed, referring to the other authors, that the long-wave component $\Psi_1$ of the velocity potential depends on $x$ and $t$ only in the combination $x - c_g t$, where $c_g = \frac{\partial\omega}{\partial k}$ is the group velocity of the first harmonic $A$. Some authors name it as the transition into the coordinate system which moves with the group velocity of linear waves, and others emphasize a forcing action of the third term in (2) which really evolves with the group velocity. (The relevant literature on the derivation of the NSE for Stokes waves by the method of multiple scales with various generalizations, including NSEs of higher orders, is given in [18]). In any case, the substitution

$$\frac{\partial\Psi_1}{\partial t} = -c_g\frac{\partial\Psi_1}{\partial x} \qquad (3)$$

was made in (1) and (2). Then (2) was integrated, and the derivative $\frac{\partial\Psi_1}{\partial x}$ from (2) was substituted into (1). In this case, (1) became an ordinary NSE

$$i(\frac{\partial A}{\partial t}+c_g\frac{\partial A}{\partial x})+p\frac{\partial^2 A}{\partial x^2}+qA^2\overline{A}=0\,, \qquad (4)$$

where

$$q = \widetilde{q} + \frac{k^2}{\omega^3}\frac{2\sigma^2\nu^2}{c_0^2-c_g^2}$$

or, in terms of the dimensionless $Q$,

$$q = \omega k^2 Q, \quad Q = \widetilde{Q} + \frac{2\sigma^2 N^2}{C_0^2 - C_g^2}. \qquad (5)$$

The dependences $\widetilde{Q}(kh)$ and $Q(kh)$ are presented in Fig. 1. It is essential that $Q$ contrary to $\widetilde{Q}$ changes a sign from the positive to negative one at $kh$=1.363. Since the NSE has soliton solutions for zero boundary conditions at $pq > 0$, and $p < 0$ for all $kh$, it cannot have soliton solutions when $kh < 1.363$.

Upon the examination of the modulational (non)stabilities of Stokes waves on the basis of the homogeneous solution of the NSE, namely the Stokes wave, it was found [3] that the imaginary part of the perturbation frequency (the instability increment) is equal to

$$\Im\Omega = \varkappa(2pqA_0^2 - \varkappa^2p^2)^{\frac{1}{2}}, \qquad (6)$$

where $A_0$ is the amplitude of an unperturbed homogeneous wave, and $\varkappa$ is the wave number of a perturbation. Quantity (6) does not exist for any $\varkappa$ if $pq < 0$. Therefore, Stokes waves are modulationally stable at $kh < 1.363$.

A justification of ansatz (3) and the opportunity of its application are problematic [19, 20]. We note that it is necessary to use (3), if we need to deduce an equation of the known form, a NSE [3], from system (1), (2). The study of the modulational instability is carried out by the examination of a specific known solution of a NSE (4) or system (1), (2), namely of a Stokes wave, for its stability. If we consider system (1), (2), the presence of the derivatives with respect to $t$ in (2) increases the order of the dispersion equation for the frequency of a perturbing wave by two. But this allows one to avoid the use of ansatz (3). Therefore, just this approach is applied in this work. System (1), (2) has the trivial solution, namely a Stokes wave

$$A = A_0\, e^{i\alpha\, t}, \quad \Psi_1 = (x - c_0 t)U_0,$$

where

$$\alpha = \widetilde{q}A_0^2 - (k - \mu c_0)U_0,$$

and the amplitudes $A_0$ and $U_0$ do not depend on coordinates and time.

Let's introduce a perturbation ($a$ is a complex number, and $\psi_1$ is a real one)

$$A = (A_0 + \epsilon\, a)\, e^{i\alpha\, t}, \quad \Psi_1 = (x - c_0 t)U_0 + \epsilon\, \psi_1.$$

System (1), (2), being linearized in $\epsilon$, looks as

$$i\frac{\partial a}{\partial t} + c_g\frac{\partial a}{\partial x} + p\frac{\partial^2 a}{\partial x^2} + \widetilde{q}A_0^2(a + \overline{a}) -$$

$$-\left(k\frac{\partial \psi_1}{\partial x} + \mu\frac{\partial \psi_1}{\partial t}\right)A_0 = 0,$$

$$\frac{\partial^2 \psi_1}{\partial t^2} - c_0^2\frac{\partial^2 \psi_1}{\partial x^2} - \nu A_0\left(\frac{\partial a}{\partial x} + \frac{\partial \overline{a}}{\partial x}\right) = 0. \quad (7)$$

We represent the solution of the linear system of differential equations (7) as

$$a = a_0\, e^{i\theta} + b_0 e^{-i\theta}, \quad \theta = \varkappa x - \Omega t,$$

$$\psi_1 = \psi_{10}e^{i\,\theta} + \psi_{11}e^{-i\theta}.$$

Substituting it in (7), we obtain the linear system of algebraic equations

$$(\Omega - c_g\varkappa - p\varkappa^2 + \widetilde{q}A_0^2)a_0 + \widetilde{q}A_0^2 b_0 +$$

$$+i(\mu\Omega - k\varkappa)A_0\psi_{10} = 0,$$

$$(\Omega - c_g\varkappa + p\varkappa^2 - \widetilde{q}A_0^2)b_0 - \widetilde{q}A_0^2 a_0 +$$

$$+i(\mu\Omega - k\varkappa)A_0\psi_{11} = 0,$$

$$i\nu\varkappa A_0(a_0 + b_0) + (\Omega^2 - c_0^2\varkappa^2)\psi_{10} = 0,$$

$$i\nu\varkappa A_0(a_0 + b_0) - (\Omega^2 - c_0^2\varkappa^2)\psi_{11} = 0.$$

If we withdraw $\psi_{10}$ and $\psi_{11}$ from the third and fourth equations of the system and substitute them in the first two equations, we obtain

$$[\Omega - \varkappa c_g - p\varkappa^2 + (\widetilde{q} + \delta(\Omega))A_0^2]a_0 +$$

$$+(\widetilde{q} + \delta(\Omega))A_0^2 b_0 = 0,$$

$$[\Omega - \varkappa c_g + p\varkappa^2 - (\widetilde{q} + \delta(\Omega))A_0^2]b_0 -$$

$$-(\widetilde{q} + \delta(\Omega))A_0^2 a_0 = 0,$$

where

$$\delta(\Omega) = \frac{k\varkappa - \mu\Omega}{c_0^2\varkappa^2 - \Omega^2}\nu\varkappa. \quad (8)$$

Equating the determinant to zero gives an equation for the determination of the perturbation frequency $\Omega$:

$$(\Omega - c_g\varkappa)^2 = p^2\varkappa^4 - 2(\delta(\Omega) + \widetilde{q})p\varkappa^2 A_0^2. \quad (9)$$

An equation similar to (9) for the characteristic velocity $\Omega/\varkappa$ was obtained by the method of averaged Lagrangian in [2, Eq. (56)]. To simplify the analysis of solutions of the fourth-order equation of for $\Omega$ in the assumption of small $A_0$, the second term was neglected [2], and the relation $\Omega/\varkappa = c_g$ was imposed on the third term. The quadratic equation obtained in such a way has the solutions whose imaginary part does not exist for any wave vectors of the perturbation at $kh < 1.363$. Thus, the analysis of the behavior of solutions of the characteristic equation carried out in [2] concerns, in fact, the case of infinitesimal $A_0$. An equation of the fourth order for $\Omega$ was obtained also in [21] from the Zakharov equations in the $\varepsilon^3$ approximation. But it was not studied numerically and was simplified up to a second-order equation in the approximation of small $A_0$.

Taking into account (8), we get

$$\Omega^4 - 2c_g\varkappa\Omega^3 - \varkappa^2\left(p^2\varkappa^2 - 2p\widetilde{q}A_0^2 + c_0^2 - c_g^2\right)\Omega^2 +$$

$$+2\varkappa^3\left(c_g c_0^2 + p\mu\nu A_0^2\right)\Omega +$$

$$+\varkappa^4\left(c_0^2(p^2\varkappa^2 - 2p\widetilde{q}A_0^2) - c_0^2 c_g^2 - 2pk\nu A_0^2\right) = 0. \quad (10)$$

Let us renormalize the quantities $\Omega$, $\varkappa$, and $A_0$ to the dimensionless ones $\widehat{\Omega}$, $\widehat{\varkappa}$, and $\widehat{A}_0$:

$$\widehat{\Omega} = \frac{\Omega}{\frac{1}{2}\omega k^2 A_0^2}, \quad \widehat{\varkappa} = \frac{\varkappa}{2k^2 A_0}, \quad \widehat{A}_0 = kA_0.$$

The renormalized equation (10) for $\widehat{\Omega}$ has the form

$$\widehat{A}_0^4\widehat{\Omega}^4 - 8\widehat{\varkappa}C_g\widehat{A}_0^3\widehat{\Omega}^3 - 16\widehat{\varkappa}^2\left(\Lambda\widehat{A}_0^2 + C_0^2 - C_g^2\right)\widehat{A}_0^2\widehat{\Omega}^2 +$$

$$+128\widehat{\varkappa}^3\left(C_g C_0^2 + PMN\widehat{A}_0^2\right)\widehat{A}_0\widehat{\Omega} +$$

$$+256\widehat{\varkappa}^4\left(C_0^2(\Lambda\widehat{A}_0^2 - C_g^2) - 2PN\widehat{A}_0^2\right) = 0, \quad (11)$$

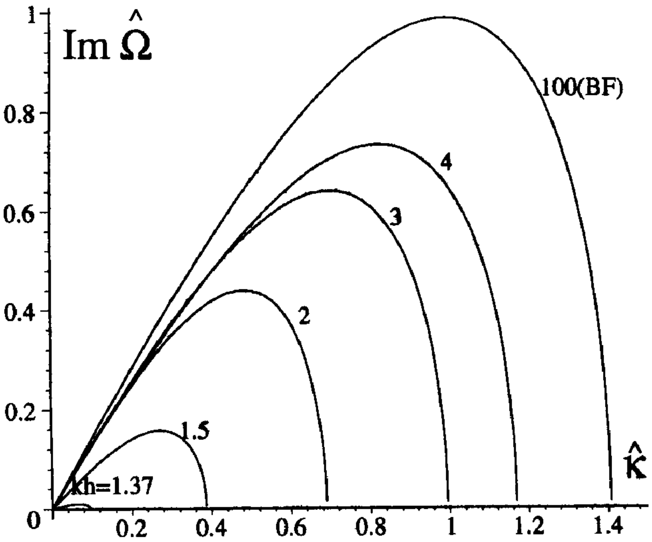


Fig. 2. Dependence $\Im\widehat{\Omega}$ on $\widehat{\varkappa}$ for various $kh$, whose values are indicated on curves, for $\hat{A}_0 = 0$

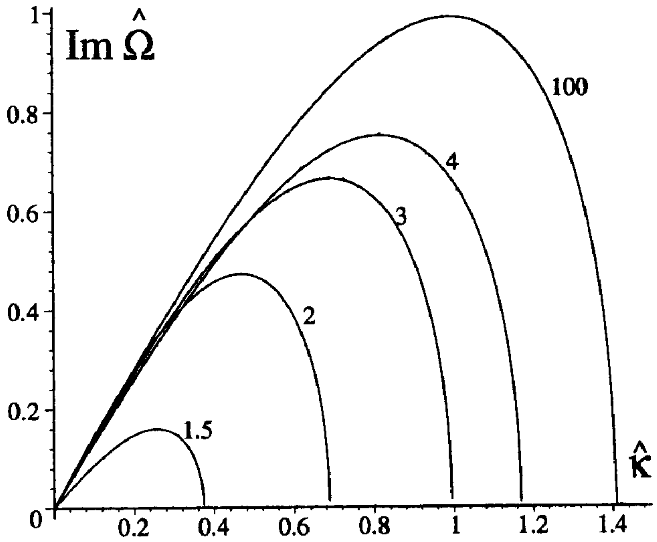


Fig. 3. The same as in Fig. 2 for $\widehat{A}_0 = 0.1$

where we denote

$$\Lambda = 2(2P^2\widehat{\varkappa}^2 - P\widetilde{Q}).$$

Let us find $kh$ and $\widehat{\varkappa}$, at which the solutions

$$\widehat{\Omega} = \Re\widehat{\Omega} + i\Im\widehat{\Omega} \tag{12}$$

of Eq. (11) can have the nonzero imaginary part $\Im\widehat{\Omega}$ and, hence, a homogeneous Stokes wave is unstable to a harmonic perturbation with wave vectors $\widehat{\varkappa}$. To this end, we will deduce the equation for $\Im\widehat{\Omega}$. Such an equation is a resultant of the system of two nonlinear equations for $\Re\widehat{\Omega}$ and $\Im\widehat{\Omega}$ in the variable $\Im\widehat{\Omega}$, which originate after the substitution of (12) in (11). Executing some algebraic transformations in the general case of the fourth-order equation for the variable $\widehat{\Omega}$, we obtain an equation for the imaginary part (the above-mentioned resultant). The last equation has the sixth order as for the quantity $(\Im\widehat{\Omega})^2$:

$$\sum_{j=1}^{6}\sum_{l=2(j-1)}^{10} b_{jl}\widehat{A}_0^l(\Im\widehat{\Omega})^{2j} + \sum_{l=0}^{8} b_{0l}\widehat{A}_0^l = 0. \tag{13}$$

The coefficients $b_{00}$, $b_{01}$, $b_{02}$, $b_{10}$, $b_{11}$, $b_{12}$, and $b_{22}$ are given in Appendix.

Let's start from (13) at small $\widehat{A}_0$ which are naturally under consideration, because our theory is weakly nonlinear from the very beginning:

1) At $\widehat{A}_0 \to 0$, we have $(\Im\widehat{\Omega})^2 = -\frac{b_{00}}{b_{10}}$, i.e.

$$(\Im\widehat{\Omega})^2 = -32\,\widehat{\varkappa}^2\left(2P^2\widehat{\varkappa}^2 - P\left(\widetilde{Q} + \frac{2\sigma^2N^2}{C_0^2 - C_g^2}\right)\right), \tag{14}$$

or

$$(\Im\Omega)^2 = -\varkappa^2\left(\varkappa^2p^2 - 2p\left(\widetilde{q} + \frac{k^2}{\omega^3}\frac{2\sigma^2\nu^2}{c_0^2 - c_g^2}\right)A_0^2\right)$$

in the unnormed form (like in [3]) for the comparison with (6).

Take into account that $P < 0$ for all $kh$. So, waves are modulationally stable (at $\widehat{A}_0 \to 0$) for those $kh$, for which

$$\widetilde{Q} + \frac{2\sigma^2N^2}{C_0^2 - C_g^2} > 0. \tag{15}$$

$Q = \widetilde{Q} + \frac{2\sigma^2N^2}{C_0^2-C_g^2}$ becomes positive at $kh < 1.3628$ (Fig. 1.) Just this expression was obtained by other methods in [1] and [2]. It is also the coefficient of a NSE obtained for the first time in [3] (see (5)).

In Fig. 2, we show the imaginary part of the frequency of a harmonic perturbation $\Im\widehat{\Omega}$ (14) vs the wave number of a perturbation $\widehat{\varkappa}$ according to formula (14). In the case of great $kh$ (the upper curve), the result coincides with the well-known one for the infinite depth. The lowest curve ($kh = 1.37$) demonstrates the disappearance of the modulational instability at approaching $kh = 1.363$.

2) To analyze the imaginary part $\Im\,\widehat{\Omega}$ at arbitrary $\widehat{A} \neq 0$ (small, but not infinitesimal), we used the

analytical solution (13) with account of terms $O(\widehat{A}^2)$ in it,

$$b_{22}\widehat{A}_0{}^2((\Im\widehat{\Omega})^2)^2+[b_{12}\widehat{A}_0{}^2+b_{11}\widehat{A}_0+b_{10}](\Im\widehat{\Omega})^2+$$

$$+[b_{02}\widehat{A}_0{}^2+b_{01}\widehat{A}_0+b_{00}]=0,$$

and, for the verification, calculated numerically the imaginary part of the solutions of the general equation (11). In Fig. 3, we give $\Im\,\widehat{\Omega}$ vs $\widehat{\varkappa}$ for the same cases as in Fig. 2, but at $\widehat{A}=0.1$ and, in addition, at $kh=1.32$. Our calculations show that the disappearance of the instability namely at $kh=1.363$ is intrinsic only to the special case $\widehat{A}\to 0$ (the infinitesimal amplitude of an unperturbed homogeneous wave). For $\widehat{A}=0.1$, the instability disappears already at $kh=1.37$. The restriction by only infinitesimal $\widehat{A}$ was underlined in [1] and was taken into account in [2] upon the derivation of the instability limit $kh=1.363$. But we have first demonstrated analytically the displacement of this limit to the side of greater $kh$ with increase in $\widehat{A}$. This displacement was derived also in numerical calculations on the basis of the exact equations of motion of the ideal fluid [22, 23].

## 3. Conclusion

Without the use of the additional assumption as for a motion of the mean flow with the group velocity of the first harmonic, the performed analysis has shown that

a) the limit of the modulational instability of Stokes waves, $kh=1.363$, at $A_0\to 0$ coincides with that obtained in the theories which use ansatz (3);

b) the limit of the modulational instability at small (but not infinitesimal) $A_0$ shifts to $kh$ greater than $kh=1.363$ at $A_0\to 0$;

c) the phenomenon of the restabilization of a fundamental wave for small wave numbers of a perturbation, which is known in more nonlinear theories [4, 6, 24—26] and from the numerical calculations of the exact hydrodynamic equations of the ideal fluid, does not appear in the weakly nonlinear theory studied here.

### APPENDIX

Below, we give the expressions for the coefficients

$$b_{00}=512\widehat{\varkappa}^4\,P\,C_0^2(C_0^2-C_g^2)^2\left[2\,\widehat{\varkappa}^2\,P-\widetilde{Q}-\frac{2\sigma^2N^2}{C_0^2-C_g^2}\right],$$

$$b_{01}=0,$$

$$b_{02}=-256P^2\widehat{\varkappa}^4$$

$$\Big\{\frac{N^2}{(C_0^2-C_g^2)^2}[4(8C_0^4+20C_0^2C_g^2-C_g^4)+4(\,C_g^5-29\,C_g^3\,C_0^2-$$

$$-26C_g\,C_0^4\,)M+(35\,C_g^4\,C_0^2+74\,C_g^2\,C_0^4\,-C_0^6)M^2]-$$

$$-4\,\frac{N}{C_0^2-C_g^2}(C_g^4+13\,C_g^3\,C_0^2\,M-25C_0^2\,C_g^2+$$

$$+23\,C_g\,C_0^4\,M-12\,C_0^4)(\widetilde{Q}-2\,\widehat{\varkappa}^2\,P)+$$

$$+16C_0^2\,(C_0^2+C_g^2)\,(\widetilde{Q}-2\,\widehat{\varkappa}^2\,P)^2\Big\},$$

$$b_{10}=16\widehat{\varkappa}^2C_0^2(C_0^2-C_g^2)^2,$$

$$b_{11}=0,$$

$$b_{12}=-32\widehat{\varkappa}^2P[(C_g^3\,M-2\,C_g^2+5\,C_gC_0^2\,M+8C_0^2)\,N+$$

$$+(C_g^4+10C_0^2\,C_g^2+13\,C_0^4)\,(\widetilde{Q}-2\,\widehat{\varkappa}^2\,P)],$$

$$b_{22}=17C_0^4+14C_g^2C_0^2+C_g^4.$$


1. *Benjamin T.B.*// Proc. Roy. Soc. (London) — **A299**(1967) 59.
2. *Whitham G.B.*// J. Fluid Mech. **27**(1967) 399.
3. *Hasimoto H., Ono H.*// J. Phys. Soc. Jpn. **33**(1972) 805.
4. *Yuen H.C., Lake B.M.*, Phys. Fluids. **18**(1975) 956; *Yuen H.C., Lake B.M.*// Nonlinear Dynamics of Deep-Water Gravity Waves (New York: Acad. Press, 1982).
5. *Tulin M.P., Waseda T.*// J. Fluid Mech. **378**(1999) 197.
6. *Zakharov V.E.*// Prikl. Mekh. Tekh. Fiz. N 2 (1968) 86 [J. Appl. Mech. Tech. Phys. (Engl. Transl.) **9**, N2 (1968) 190]; Zh. Eksp. Teor. Fiz. **51** (1966) 1107.
7. *Dysthe K.B.*// Proc. Roy. Soc. (London.) **A369** (1979) 105; *Trulsen K., Kliakhandler I., Dysthe K.B., Velarde M.G.*// Phys. Fluids. **12** (2000) 2432.
8. *Ablowitz M.J., Hammack J., Henderson D., Schober C.M.*, Phys. Rev. Lett. **84** (2000) 887; Physica D **152-153** (2001) 416.
9. *Shemer L., Kit E., Haiyuing Jiao, Eitan O.*// J. Waterway, Port, Coastal, and Ocean Eng. **124** (1998) 320.
10. *Shemer L., Haiyuing Jiao, Kit E., Agnon Y.*// J. Fluid Mech.**427** (2001) 107.
11. *Kharif C., Pelinovsky E.*// Europ. J. Mech. B: Fluids. **22**(2003) 603.
12. *Tajiri M., Watanabe Y.*// Phys. Rev. E **57**(1998) 3510.
13. *Benney D.J., Roskes G.J.*// Stud. Appl. Math. **48** (1969) 377.
14. *Sedletsky Yu.V.*// Zh. Exp. Teor. Fiz. **97** (2003) 180.
15. *Zakharov V. E.*// Ibid. **35** (1972) 908.
16. *Newell A.C.* Solitons in Mathematics and Physics. — University of Arizona, 1989.
17. *Yajima N., Oikawa M.*// Progr. Theor. Phys. **56** (1976) 1719.
18. *Sedletsky Yu.V.*//Ukr. Fiz. Zhurn. **48** (2003) 110.

19. *Brinch-Nielsen U., Jonsson I.G.*// Wave Motion. **8** (1986) 455.

20. *Zakharov V.E., Kuznetsov E.A* // Uspekhi Fiz. Nauk. **167** (1997) 1137 [Phys.-Usp. **40** (1997) 1097].

21. *Zakharov V.E., Kharitonov V.G.*// Prikl. Mekh. Tekh. Fiz. **N 5** (1970) 45; J. Appl. Mech. Tech. Phys. (Engl. Transl.) **11**, N5 (1970) 741.

22. *McLean J.W.*// J. Fluid Mech. **114** (1982) 2445.

23. *Francius M., Kharif C.*// Phys. Fluids. **15** (2003) 2445.

24. *Longuet-Higgins M.S.*// Proc. Roy. Soc. (London) **A360** (1978) 489.

25. *Janssen P.A.E.M.*// J. Fluid Mech. **126** (1983) 1.

26. *Stiasnie M., Shemer L.*// Ibid. **143** (1984) 47.


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ДО ТЕОРІЇ МОДУЛЯЦІЙНОЇ НЕСТІЙКІСТІ СТОКСОВИХ ХВИЛЬ

*Ю.В. Седлецький*

Р е з ю м е

Модуляційна нестійкість стоксових хвиль вивчається без застосування традиційного припущення про рух середньої течії з груповою швидкістю першої гармоніки. Показано, що і без цього припущення можна отримати відому межу $kh = 1,363$ переходу між станами модуляційно стійкої і нестійкої рідини для випадку слабонелінійних хвиль. Продемонстровано, що із збільшенням нелінійності ця межа зсувається в бік більших значень $kh$. Показано також, що зникнення модуляційної нестійкості (рестабілізація) для малих хвильових векторів збурення, яке раніше вже було передбачено при урахуванні нелінійних доданків порядку $\epsilon^4$ на основі рівнянь Захарова і чисельних розрахунків, не випливає із слабонелінійної теорії, навіть і без згаданого припущення.