SUPERFLUIDITY IN A SYSTEM OF NEUTRONS WITH DIRECT RELATIVISTIC INTERACTION

A.V. NAZARENKO

Bogolyubov Institute for Theoretical Physics, Nat. Acad. Sci. of Ukraine
(14b, Metrolohichna Str., Kyiv 03143, Ukraine; e-mail: nazarenko@bitp.kiev.ua),

Institute for Condensed Matter Physics
(1, Svientsitskii Str., L'viv 79011, Ukraine)

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Using the model with 4-fermionic direct relativistic interaction among nucleons, the energy spectrum of superfluid neutron matter is investigated within the Bardeen—Cooper—Schrieffer approach. The dependence of the energy gap at the Fermi level on neutron density is obtained.

1. Introduction

The importance of neutron matter superfluidity investigations has been induced by Migdal's work [1], where this phenomenon is considered in the context of the physics of neutron stars. Although the real neutron stars do not consist of the pure neutron matter, and the superfluid state cannot be presented in all regions of stars, however, the additional experimental perspective for the verification of nuclear models has arose after the Migdal's work publishing.

The theoretical research of superfluidity in the nuclear matter has been started after the elaboration of the Bardeen–Cooper–Schrieffer (BCS) theory of metal superconductivity (the theory of electron gas superfluidity in metals) [2] and the paper by Bohr with coworkers [3], where a mechanism of creation of Copper pairs of nucleons is explored. If the electron gas superfluidity is provided by the electron-phonon interaction, the nuclear matter superfluidity is associated with the competition between attraction and repulsion forces caused by the exchange by scalar and vector mesons, respectively. At the same time, the existence of the superfluid state demands the suppressing of repulsion by attraction.

Thus numerous works devoted to neutron matter superfluidity have been done in the BCS approximation. Using the phenomenological potentials of internucleon interaction, the results obtained are more or less identical and collected, for instance, in [4]. They have already shown that the BCS approach gives too high values for the energy spectrum of the superfluid state and, therefore, the accounting of polarization effects is needed. Similar conclusions came from relativistic models [5, 6] based on the effective field-theoretic σ-model of quantum hadrodynamics [7], by means of which a number of many-particle nuclear processes have been explained and numerically described.

Taking qualitative predictions only within the BCS approach into account, the attempts to make calculations based on realistic potentials more precise have led indeed to the desirable estimations of the energy spectrum. However, it is achieved at the price of loosing the unity among approaches [8–10]. For this reason, it is now hard to say what outcomes should be standard [11]. On the other hand, various modifications of the random phase approximation (RPA), which promise to be strongly exact, are elaborated and verified with the use of simple models [12–16]. It is understandable that the necessary calculations within the modified RPA including the variational principle are still bulk and complicated to be applied to the models with realistic potentials.

These facts stimulate us to begin our investigations of pure neutron matter superfluidity, using the microscopic model proposed in [17], within the BCS approach. Here, we restrict ourselves by this simple approximation, because the deeper studies of superfluidity and other phenomena have no sense without knowledge of qualitative properties of the model.

The model Hamiltonian of our interest is of 4-fermionic form and linear in the coupling constants. It has been obtained by quantization of the relativistic (Poincaré-invariant) classical Hamiltonian derived from the classical field theory by means of the reduction of the meson fields' degrees of freedom. The idea to apply the 4-fermionic Hamiltonian of interaction (in the terms of the annihilation and creation operators of particles) to high-energy physics has been suggested by Nambu [18]. The models of such a kind serve as an alternative to the field—theoretic σ-model. As known [19, 20], the linear models reflect leading effects in nuclear matter (at low density). Remark that, including nonlinear meson self–interactions, a more wide class of physical situations is able to be described correctly. At a low density, we can study the superfluidity of the nuclear matter and the "liquid–gas" phase transition which was already considered in [17]. There, it was also proved that both effects allow us to neglect the spinor structure of a quantized Dirac field corresponding to nucleons and the antiparticle contribution. The latter becomes evident, remembering that the thermal distribution of antinucleons is described by the expression

$$\frac{1}{1+\exp\left[\left(c\sqrt{m^2c^2+\mathbf{p}^2}+\mu\right)/T\right]}$$

vanishing at $T\to 0$.

Moreover, the dependence on finite volume V is restored. However, the form of a relativistic interaction among nucleons should be preserved in order to observe the transition between the constraint and unconstraint states of a nucleonic system. Thus, taken together, these physical simplifications allow us to study the system with relativistic interaction within a non-relativistic (statistical) approach. Although we intend to apply the BCS approximation, some generalizations due to the relativistic form of interaction have to be done. They will consist in the introduction of two-gap functions, whose competition will determine the energy spectrum. As noted above, this competition is represented by the interactions of scalar and vector mesons. Actually, the attraction dominates over repulsion in some region of momentum (or density) values and is suppressed beyond this region. This means that, in these two physically different situations, we need to exploit two different approaches [21]. Therefore, where attraction forces prevail, BCS approximation is available. This natural fact gives us a physical basis for the evaluation of the cut-off momentum which characterizes the superfluid region for relativistic models within the BCS formalism [5].

2. The Formalism

We begin our studies of superfluidity from the Hamiltonian of interacting neutrons from [17]

$$\hat{H}=\hat{H}_0+\hat{W}, \tag{1}$$

where

$$\hat{H}_0=\sum_{\mathbf{p},\sigma}cp^0\hat{a}^+_{\mathbf{p},\sigma}\hat{a}_{\mathbf{p},\sigma},\quad p^0=\sqrt{m^2c^2+\mathbf{p}^2}, \tag{2}$$

is the Hamiltonian of free neutrons. The index $\sigma=\pm$ labels the spin projection on the axis z, namely, $\pm\hbar/2$; $\hat{a}^+_{\mathbf{p},\sigma}$, $\hat{a}_{\mathbf{p},\sigma}$ are regarded as the creation and annihilation operators of free neutrons, respectively; c is the light velocity.

The instantaneous interaction $\hat{W}$ is of the form

$$\hat{W}=\frac{1}{V}\sum_{\mathbf{p}_1,\mathbf{p}_2}W_{\mathbf{p}_1,\mathbf{p}_2}\hat{a}^+_{\mathbf{p}_2,+}\hat{a}^+_{-\mathbf{p}_2,-}\hat{a}_{-\mathbf{p}_1,-}\hat{a}_{\mathbf{p}_1,+}; \tag{3}$$

$$W_{\mathbf{p}_1,\mathbf{p}_2}=\frac{1}{2}\left[\Gamma_{\mathbf{p}_1}\left(\frac{\mathbf{p}_2-\mathbf{p}_1}{\hbar}\right)+\Gamma_{\mathbf{p}_2}\left(\frac{\mathbf{p}_2-\mathbf{p}_1}{\hbar}\right)\right], \tag{4}$$

where only terms corresponding to Cooper pairs are preserved. The summing over momenta is carried out within the interval limited by the cut-off momentum p_c which characterizes the superfluid region and is estimated bellow.

The quantity $\Gamma_{\mathbf{p}}(\mathbf{k})$ in the case of isotropic and homogeneous matter is

$$\Gamma_{\mathbf{p}}(\mathbf{k})=\frac{g_v^2}{\mathbf{k}^2-[(\mathbf{kp})/p^0]^2+\mu_v^2}+\frac{g_\rho^2/4}{\mathbf{k}^2-[(\mathbf{kp})/p^0]^2+\mu_\rho^2}-$$
$$-\frac{g_s^2(mc/p^0)^2}{\mathbf{k}^2-[(\mathbf{kp})/p^0]^2+\mu_s^2}, \tag{5}$$

where g_i^2 $(i=s,v,\rho)$ are the coupling constants; μ_i are the mesons' characteristics associated with the masses m_i as $\mu_i=m_ic/\hbar$.

Within the superfluid region, we ignore the dependence of matrix elements on the squared transmission momentum $\mathbf{k}^2$ $(= [(\mathbf{p}_1 - \mathbf{p}_2)/\hbar]^2)$. Then we obtain

$$W_{\mathbf{p}_1,\mathbf{p}_2} = C_v^2 + \frac{1}{4}C_\rho^2 - \frac{1}{2}C_s^2\frac{m^2c^2}{(p_1^0)^2} - \frac{1}{2}C_s^2\frac{m^2c^2}{(p_2^0)^2}, \qquad (6)$$

where $p^0_{1,2} = \sqrt{m^2c^2 + \mathbf{p}^2_{1,2}}$; $C_i^2 \equiv g_i^2/\mu_i^2$ $(i = s, v, \rho)$ are the parameters of carriers of the interaction between nucleons. For a given model, C_i^2 were found in [17]. At this stage, we only note that they obey the following inequalities: $C_s^2 > C_v^2 > C_\rho^2$, $C_s^2 > C_v^2 + C_\rho^2/4$.

This approximation simplifies calculations considerably, however, it neglects the relativistic effect of finiteness of the interaction transmission velocity under the exchange of mesons between nucleons. Because of the short Compton wavelengths of heavy mesons as compare to the internucleon distance in the ordinary nuclear matter, the adopted approximation occurs to be reasonable.

For further investigations, let us transform the Hamiltonian with the matrix element (6) as follows:

$$\hat{H}' = \mathrm{e}^{-\hat{S}}\hat{H}\mathrm{e}^{\hat{S}}; \qquad (7)$$

$$\hat{S} = \frac{1}{2V}\times$$

$$\times\sum_{\mathbf{p}_1,\mathbf{p}_2} C_s^2\frac{m^2c}{(p_1^0p_2^0)^2}(p_2^0 - p_1^0)\hat{a}^+_{\mathbf{p}_2,+}\hat{a}^+_{-\mathbf{p}_2,-}\hat{a}_{-\mathbf{p}_1,-}\hat{a}_{\mathbf{p}_1,+}. \qquad (8)$$

This transformation is canonical because the operator $\hat{S}$ is anti-Hermitian, i.e. $\hat{S}^\dagger = -\hat{S}$.

In the linear approximation by which we limit ourselves, we get

$$\hat{H}' = \hat{H}_0 + \hat{W} + [\hat{H}_0, \hat{S}] \equiv \hat{H}_0 + \hat{W}'. \qquad (9)$$

Due to the commutation relation for Fermi operators

$$[a^+_{\mathbf{p},\pm}a_{\mathbf{p},\pm}, \hat{a}^+_{\mathbf{p}_2,+}\hat{a}^+_{-\mathbf{p}_2,-}\hat{a}_{-\mathbf{p}_1,-}\hat{a}_{\mathbf{p}_1,+}] =$$

$$= (\delta_{\mathbf{p},\pm\mathbf{p}_2} - \delta_{\mathbf{p},\pm\mathbf{p}_1})\hat{a}^+_{\mathbf{p}_2,+}\hat{a}^+_{-\mathbf{p}_2,-}\hat{a}_{-\mathbf{p}_1,-}\hat{a}_{\mathbf{p}_1,+}, \qquad (10)$$

the interaction becomes

$$\hat{W}' = \frac{1}{V}\sum_{\mathbf{p}_1,\mathbf{p}_2} W'_{\mathbf{p}_1,\mathbf{p}_2}\hat{a}^+_{\mathbf{p}_2,+}\hat{a}^+_{-\mathbf{p}_2,-}\hat{a}_{-\mathbf{p}_1,-}\hat{a}_{\mathbf{p}_1,+}; \qquad (11)$$

$$W'_{\mathbf{p}_1,\mathbf{p}_2} = C_v^2 + \frac{1}{4}C_\rho^2 - C_s^2\frac{m^2c^2}{p_1^0p_2^0}. \qquad (12)$$

Thus, now one can see that the performed transformation factorizes the scalar relativistic interaction. Such a form is close to the interaction term within the BCS model of a superfluid electron gas, which allows us to apply directly this formalism.

Within the statistical approach to superfluidity, where the number of particles is not fixed, we use the extended Hamiltonian

$$\hat{\mathcal{H}} = \sum_{\mathbf{p},\sigma}\varepsilon_p\hat{a}^+_{\mathbf{p},\sigma}\hat{a}_{\mathbf{p},\sigma} + \hat{W}', \quad \varepsilon_p = cp^0 - \mu, \qquad (13)$$

where μ is the chemical potential of neutrons.

Hereafter we apply the formalism developed by Bogolyubov in the theory of superconductivity [22]. An alternative approach of investigations of the phenomena of superfluidity and superconductivity was elaborated by Gorkov on the basis of correlation functions (see [21]).

The study of superfluidity with the use of Hamiltonian (13) is complicated. Then we need to replace (13) by the model Hamiltonian

$$\hat{\mathcal{H}}_\mathrm{m} = \sum_{\mathbf{p},\sigma}\varepsilon_p\hat{a}^+_{\mathbf{p},\sigma}\hat{a}_{\mathbf{p},\sigma} +$$

$$+\frac{1}{V}\sum_{\mathbf{p}_1,\mathbf{p}_2} W'_{\mathbf{p}_1,\mathbf{p}_2}\Big(\lambda^*_{\mathbf{p}_2}\hat{a}_{-\mathbf{p}_1,-}\hat{a}_{\mathbf{p}_1,+} +$$

$$+\lambda_{\mathbf{p}_1}\hat{a}^+_{\mathbf{p}_2,+}\hat{a}^+_{-\mathbf{p}_2,-} - \lambda^*_{\mathbf{p}_2}\lambda_{\mathbf{p}_1}\Big), \qquad (14)$$

where the complex quantity $\lambda_\mathbf{p}$ is determined by the equation

$$\lambda_\mathbf{p} = \langle\hat{a}_{-\mathbf{p},-}\hat{a}_{\mathbf{p},+}\rangle = \frac{\mathrm{Tr}[\hat{a}_{-\mathbf{p},-}\hat{a}_{\mathbf{p},+}\exp(-\hat{\mathcal{H}}_\mathrm{m}/T)]}{\mathrm{Tr}\exp(-\hat{\mathcal{H}}_\mathrm{m}/T)}. \qquad (15)$$

Here, T is the temperature of the system (in energy units).

The validity of the use of the modeling Hamiltonian instead of $\mathcal{H}$ is proved by equivalence of the results obtained on their basis in the thermodynamic limit (see [22]). In practice, such a replacement means that we neglect the contribution of the interaction among Cooper pairs and remain with the one-particle spectrum.

Let us introduce the quantities

$$\Delta_\mathrm{V} \equiv \frac{1}{V}\sum_\mathbf{p}\lambda_\mathbf{p}, \quad \Delta_\mathrm{S} \equiv \frac{1}{V}\sum_\mathbf{p}\frac{mc}{p^0}\lambda_\mathbf{p}; \qquad (16)$$

$$\Delta_p \equiv C_s^2\Delta_\mathrm{S}\frac{mc}{p^0} - \left(C_v^2 + \frac{1}{4}C_\rho^2\right)\Delta_\mathrm{V}. \qquad (17)$$

The quantity Δ_p, as we shall see, plays the role of the energy gap in the spectrum of the superfluid matter and corresponds to the binding energy of a Cooper pair of neutrons. The energy gap which we will look for depends on momenta and is determined by the difference between the scalar and vector interactions.

Now operator (14) is rewritten as

$$\hat{\mathcal{H}}_{\rm m} = \Lambda + \sum_{\mathbf{p},\sigma} \varepsilon_p \hat{a}^+_{\mathbf{p},\sigma} \hat{a}_{\mathbf{p},\sigma} -$$

$$- \sum_{\mathbf{p}} (\Delta_p \hat{a}^+_{\mathbf{p},+} \hat{a}^+_{-\mathbf{p},-} + \Delta^*_p \hat{a}_{-\mathbf{p},-} \hat{a}_{\mathbf{p},+}), \qquad (18)$$

where the asterisk denotes complex conjugation, and the constant Λ is

$$\Lambda \equiv -\frac{1}{V} \sum_{\mathbf{p}_1,\mathbf{p}_2} W'_{\mathbf{p}_1,\mathbf{p}_2} \lambda^*_{\mathbf{p}_2} \lambda_{\mathbf{p}_1}$$

$$= V \left[C_s^2 |\Delta_{\rm S}|^2 - \left(C_v^2 + \frac{1}{4} C_\rho^2 \right) |\Delta_{\rm V}|^2 \right]. \qquad (19)$$

To diagonalize our Hamiltonian, following Bogolyubov [22], we go over from the operators of neutrons to the operators of quasiparticles:

$$\hat{a}_{\mathbf{p},+} = u_p \hat{b}_{\mathbf{p},+} + v_p \hat{b}^+_{-\mathbf{p},-}, \qquad (20)$$

$$\hat{a}_{-\mathbf{p},-} = u_p \hat{b}_{-\mathbf{p},-} - v_p \hat{b}^+_{\mathbf{p},+}. \qquad (21)$$

Such a transformation is canonical if $|u_p|^2 + |v_p|^2 = 1$.

Inserting these operators into the Hamiltonian and rewriting the complex quantities as $u_p = |u_p| {\rm e}^{i\phi}$, $v_p = |v_p| {\rm e}^{i\psi}$, $\Delta_p = |\Delta_p| {\rm e}^{i\xi}$, we derive the relation $\phi + \psi - \xi = 0$. A given condition connects the phases of the complex quantities but does not determine them. Here we eliminate this freedom by fixing the additional (gauge) conditions: $\xi = 0$, $\phi = -\psi$. Taking it into account, the absolute values of complex numbers are related, at the same time, by the equation

$$2\varepsilon_p |u_p||v_p| = \Delta_p (|u_p|^2 - |v_p|^2). \qquad (22)$$

Finding solutions to this equation, we arrive immediately at

$$|u_p|^2 = \frac{1}{2} \left(1 + \frac{\varepsilon_p}{\sqrt{\varepsilon_p^2 + \Delta_p^2}} \right),$$

$$|v_p|^2 = \frac{1}{2} \left(1 - \frac{\varepsilon_p}{\sqrt{\varepsilon_p^2 + \Delta_p^2}} \right). \qquad (23)$$

Substituting these quantities into the model Hamiltonian, we come to the expression

$$\hat{\mathcal{H}}_{\rm m} = K + \sum_{\mathbf{p}} \sqrt{\varepsilon_p^2 + \Delta_p^2} \left(\hat{b}^+_{\mathbf{p},+} \hat{b}_{\mathbf{p},+} + \hat{b}^+_{-\mathbf{p},-} \hat{b}_{-\mathbf{p},-} \right); \qquad (24)$$

$$K = \Lambda - \sum_{\mathbf{p}} \left[\sqrt{\varepsilon_p^2 + \Delta_p^2} - \varepsilon_p \right]. \qquad (25)$$

Now we need to obtain the equations for the parameters $\Delta_{\rm V}$, $\Delta_{\rm S}$ which are expressed through $\lambda_{\mathbf{p}}$. The average $\lambda_{\mathbf{p}}$ is easily found as

$$\lambda_{\mathbf{p}} \equiv \langle \hat{a}_{-\mathbf{p},-} \hat{a}_{\mathbf{p},+} \rangle = \frac{1}{2} \frac{\Delta_p}{\sqrt{\varepsilon_p^2 + \Delta_p^2}} \tanh \frac{\sqrt{\varepsilon_p^2 + \Delta_p^2}}{2T}, \qquad (26)$$

where we use the thermal distribution of quasiparticles in the form

$$\left\langle \hat{b}^+_{\mathbf{p},\sigma} \hat{b}_{\mathbf{p},\sigma} \right\rangle = \frac{1}{\exp\left(\sqrt{\varepsilon_p^2 + \Delta_p^2}/T \right) + 1}. \qquad (27)$$

Further, we deal with the system at $T = 0$, when $\tanh = 1$. Using expressions (16), (17), we get the desirable set of coupled equations

$$\Delta_{\rm V} =$$

$$= \frac{1}{2V} \sum_{\mathbf{p}} \frac{C_s^2 \Delta_{\rm S} mc/p^0 - (C_v^2 + C_\rho^2/4)\Delta_{\rm V}}{\sqrt{\varepsilon_p^2 + [C_s^2 \Delta_{\rm S} mc/p^0 - (C_v^2 + C_\rho^2/4)\Delta_{\rm V}]^2}}, \qquad (28)$$

$$\Delta_{\rm S} =$$

$$= \frac{1}{2V} \sum_{\mathbf{p}} \frac{mc}{p^0} \frac{C_s^2 \Delta_{\rm S} mc/p^0 - (C_v^2 + C_\rho^2/4)\Delta_{\rm V}}{\sqrt{\varepsilon_p^2 + [C_s^2 \Delta_{\rm S} mc/p^0 - (C_v^2 + C_\rho^2/4)\Delta_{\rm V}]^2}}. \qquad (29)$$

Due to the account of relativistic effects, these equations generalize the equation for the energy gap within usual BCS theory. In the next section, we investigate the energy spectrum of the system in the superfluid state with the help of the derived system of equations.

3. The Results and Discussion

Now let us focus on the binding energy of a Cooper pair of neutrons. In order to estimate the values of the gap, it is necessary to know the values of the model parameters C_i^2 which must reproduce preferably equilibrium properties of nuclear matter. They are found in [17]: $C_s^2/\hbar c = 51.962$ fm^2, $C_v^2/\hbar c = 47.269$ fm^2, $C_\rho^2/\hbar c = 8.614$ fm^2, where $\hbar c = 197.35$ MeV $\cdot$ fm.

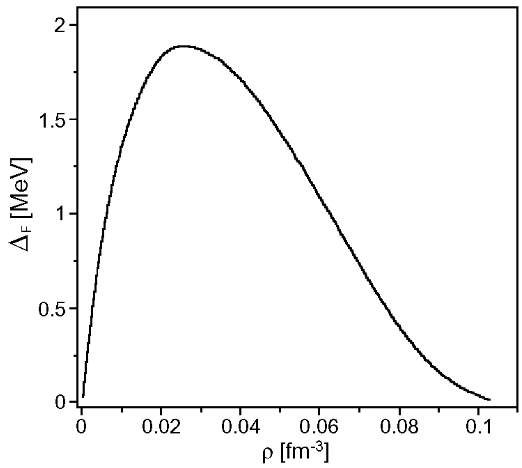

Fig. 1. Dependence of the energy gap at the Fermi level on the density of neutrons

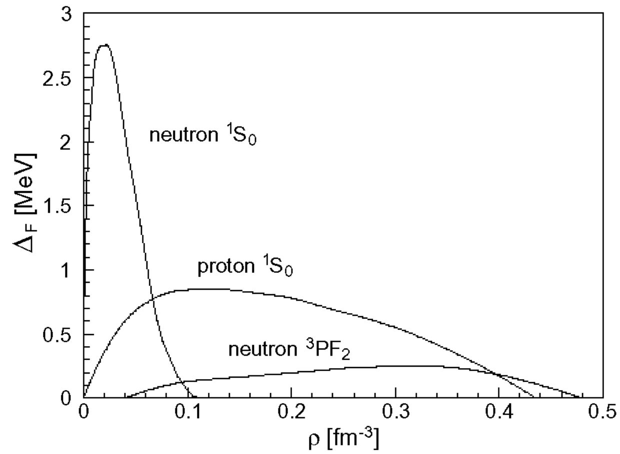

Fig. 2. Typical dependences of the gap on the density of nucleons for different spin configurations

In the thermodynamic limit, we replace the summing by integration,

$$\frac{1}{V}\sum_{\mathbf{p}}(.....) = \int(.....)\frac{\mathrm{d}^3p}{(2\pi\hbar)^3} = \frac{1}{2\pi^2\hbar^3}\int\limits_0^{p_c}(.....)p^2\mathrm{d}p,$$

in Eqs. (28), (29). The effective cut-off momentum p_c is determined for relativistic models from the condition of vanishing the integrand dependent on momentum, as shown in [5]. Since numerical analysis gives us $p_c/\hbar \approx 1.5\ \mathrm{fm}^{-1}$, the expression mc/p^0 weakly differs from 1. Thus, with a good approximation, we take $\Delta = \Delta_\mathrm{V} \approx \approx \Delta_\mathrm{S}$. After that, the coupled equations for Δ_V, Δ_S are reduced to the relation

$$1 = \frac{1}{4\pi^2\hbar^3}\times$$

$$\times\int\limits_0^{p_c}\frac{C_s^2mc/p^0 - C_v^2 - C_\rho^2/4}{\sqrt{\varepsilon_p^2 + \Delta^2[C_s^2mc/p^0 - C_v^2 - C_\rho^2/4]^2}}p^2\mathrm{d}p, \tag{30}$$

and the momentum-dependent expression for the gap becomes

$$\Delta_p = \Delta\left(C_s^2\frac{mc}{p^0} - C_v^2 - \frac{1}{4}C_\rho^2\right). \tag{31}$$

Since $\mu_i > 3\ \mathrm{fm}^{-1}$ $(i = s, v, \rho)$ empirically, taking into account that $\mu_i > p_c/\hbar$, we can argue in favor of the replacement of the matrix element (4) by (6).

Performing numerical calculations, we put the chemical potential μ identical to $c\sqrt{m^2c^2 + p_\mathrm{F}^2}$, where p_F is the Fermi momentum. Since $\varepsilon_{p_\mathrm{F}} = 0$, it is interesting to consider the dependence of the gap at the Fermi level, i.e. $\Delta_\mathrm{F} \equiv \Delta_{p_\mathrm{F}}$, on the density ρ connected with the Fermi momentum by the formula

$$p_\mathrm{F} = \hbar\left(\frac{6\pi^2\rho}{\gamma}\right)^{1/3}.$$

Here, $\gamma = 2$ corresponds to the spin degeneration.

Our numerical calculations result in Fig. 1 where the maximal value of the energy gap for the neutron matter (in singlet $^1S_0$ state) is 1.902 MeV at $\rho = 0.025\ \mathrm{fm}^{-3}$. In order to compare this curve with the known ones, we present the typical dependences of the gap on density in the BCS approximation for different spin configurations in Fig. 2 from [4]. It is seen that the gap maximum for the typical dependence for the pure neutron matter is about 2.8 MeV in the singlet state. The contradiction of values of the gap maximum in Figs. 1 and 2 can be explained by the difference between the microscopic potential exploited here and phenomenological potentials (like the Argonne one) used in the derivation of Fig. 2. Moreover, the parameters of our model have been fitted in accordance with equilibrium properties of the nuclear matter but not with the pure neutron matter data. Such a choice gives us a complete picture of possibilities provided by the model with a single-parameter set which has been successfully applied to the investigation of the nuclear "liquid—gas" phase transition [17]. Of course, another choice of parameters can lead to a better coincidence, and we can then expect a satisfactory agreement with the current estimations. Nevertheless, the other characteristics (the maximum location, the existence interval of the superfluid state) in these figures are the same. From this point of view, our results pretend to be adequate, and our model can be applied to the description of superfluidity. On the other hand, both 2.8 and 1.9 MeV are not realistic values. They are too high, which is connected with the use of the usual BCS approach. Therefore, the forthcoming research should consist in going beyond the BCS approximation, in accounting polarization effects in the medium [4].

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НАДПЛИННІСТЬ В СИСТЕМІ НЕЙТРОНІВ
З ПРЯМОЮ РЕЛЯТИВІСТСЬКОЮ ВЗАЄМОДІЄЮ

А.В. Назаренко

Р е з ю м е

Використовуючи модель з 4-ферміонною прямою релятивістською взаємодією між нуклонами, досліджено енергетичний спектр надплинної нейтронної матерії в рамках підходу Бардіна—Купера—Шріффера. Одержано залежність енергетичної щілини на рівні Фермі від густини нейтронів.