PECULIARITIES OF DEPOLARIZATION OF LINEARLY POLARIZED RADIATION BY A LAYER OF THE ANISOTROPIC INHOMOGENEOUS MEDIUM

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The interaction of a linearly polarized radiation with an inhomogeneous layer of the medium possessing a linear phase anisotropy has been studied. A theoretical model of the interaction of a polarized radiation with objects of this class has been developed. Theoretical and experimental researches of the dependences of the intensity and the polarization degree of transmitted radiation on the linear polarization azimuth of incident radiation have been carried out.

1. Introduction

One of the perspective methods of the diagnostics of objects that scatter radiation is the analysis of variations of the polarization state of the electromagnetic radiation which has passed through a researched object. The analysis of polarization allows one to enhance significantly the body of information about the studied object, in particular, the information that cannot be obtained by other methods.

Nowadays, there exists a large bibliography [1–5] devoted to the issue of the scattering of electromagnetic radiation by objects of various nature. In these works, the attention was mainly paid only to the analysis of the scattered radiation intensity or to the scattering of a polarized radiation by inhomogeneous isotropic objects. This has allowed the adequate models describing a number of phenomena both in optics and radiophysics to be developed. A considerably smaller attention was given to the analysis of the polarization structure of scattered radiation and its relations with characteristics of the object under investigation.

When a polarized electromagnetic radiation interacts with objects of certain classes, the transformation of its polarization state as well as the depolarization of incident radiation takes place. Such objects include, e.g., the objects of the bio-medical nature [5–7]. To the latter, a rather high degree of depolarization of transmitted radiation is inherent. In a great many cases, the depolarization degree does not depend on the state of incident radiation [8–11]. This phenomenon was called the isotropic depolarization. For the objects of certain classes, including those of the bio-medical nature, the depolarization degree depends on the polarization state of incident radiation; it is the so-called anisotropic depolarization [9–15]. The dependence of the polarization degree of incident radiation on parameters of the researched object and on the polarization state of a probing radiation reveals additional opportunities for developing the polarization methods for the diagnosics of objects of the bio-medical nature [13–15]. Nevertheless, the anisotropic depolarization, the mechanisms of its emergence, and the conditions needed for this have not been studied enough till now.

This work aims at studying, theoretically and experimentally, the parameters of scattered radiation, in particular, its intensity and degree of polarization, in the case of the interaction of a linearly polarized radiation with an inhomogeneous layer of the medium with a linear phase anisotropy.

The study of objects of this class is also vital, because thin cross-sections of a number of objects of the bio-medical nature, as was shown in [6, 7], are characterized by a linear phase anisotropy. Therefore, the inhomogeneous layer of the medium with a linear phase anisotropy can serve as a model for investigating the processes connected to the increase of the cross-section thickness of those objects.

2. Model and Methods of Research

The geometry of the problem concerned is as follows. The monochromatic radiation $E_0(\rho, z)$ strikes the infinite layer of a substance which is located in the plane $z = 0$ orthogonally to the radiation propagation direction (see Fig. 1). Owing to the interaction of radiation with the medium layer, the spatial modulations of the radiation intensity and polarization will be observed, which can be described making use of the Jones transmission matrix.

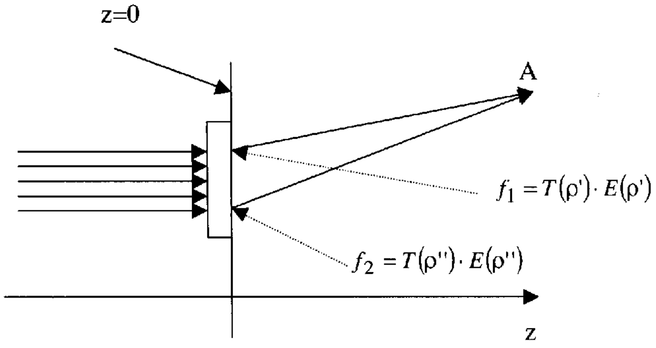

Fig. 1. Geometry of the problem

The Jones matrix describes the linear interaction of a polarized radiation with a deterministic object [16, 17] and satisfies the matrix equation

$$\begin{pmatrix} E_x^{\text{out}} \\ E_y^{\text{out}} \end{pmatrix} = T \begin{pmatrix} E_x^{\text{in}} \\ E_y^{\text{in}} \end{pmatrix}, \tag{1}$$

where E_x and E_y are the Cartesian components of the radiation electric vector, T is the Jones matrix of the researched object, i.e. the medium layer,

$$T(x,y) = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}, \tag{2}$$

and t_{ij} are the complex quantities which are determined by the anisotropy of the researched object.

For studying the interaction of radiation with the medium layer, we consider the model of an anisotropic phase screen in the single-scattering approximation. The anisotropic phase screen induces different random phase shifts between the orthogonal components of the electric vector of radiation at every point of the $x0y$ plane. In this case, the quantities t_{ij}, depend generally speaking, upon the position of a point on the phase screen.

The Jones vector of radiation at the point $(x, y, 0)$ just behind the output surface of the medium layer looks as

$$E^{\text{out}}(x,y,0) = T(x,y)E^{\text{in}}(x,y,-d). \tag{3}$$

To describe the radiation polarization, which can be, in general, partially polarized, at the point of observation, we use the coherence matrix G [16, 17]. The coherence matrix contains all available information concerning the polarization characteristics of a partially polarized radiation.

Then, if a certain distribution of the Jones vectors of radiation $E_{\text{out}}(x,y,0)$ is given across the plane $z = 0$, the elements of the coherence matrix at the point of observation located on the plane z (Fig. 1) can be found from the relation [18]

$$G_{ij}(x,y,z) = \int\limits_{-\infty}^{\infty} dx_1 \int\limits_{-\infty}^{\infty} dy_1 \int\limits_{-\infty}^{\infty} dx_2 \int\limits_{-\infty}^{\infty} dy_2 \times$$
$$\times E_{\text{out}}(x_1,y_1,0)\, E_{\text{out}}(x_2,y_2,0)\, \Gamma_{ij}(x_1,y_1,x_2,y_2,z), \tag{4}$$

where $\Gamma(x_1,y_1,x_2,y_2,z)$ is the correlation function of radiation between two points (x_1,y_1) and (x_2,y_2) on the screen and $G_{ij}(x,y,z)$ is the element of the radiation coherence matrix at the point of observation.

An explicit form of the correlation function $\Gamma(x_1,y_1,x_2,y_2,z)$ is determined by the statistical parameters of the screen surface relief and the parameters of the screen anisotropy. The calculation of the correlation function was carried out according to a method described in [13]. Thus, having found the values of $G_{ij}(x,y,z)$ from Eq. (4), we obtain the complete information concerning the polarization structure of scattered radiation at the point of observation. In particular, the degree of polarization p and the intensity of radiation I are determined as [16]

$$P = \left[1 - \frac{4\det(G)}{(\text{Sp}(G))^2}\right]^{1/2} \tag{5}$$

and

$$I = \text{Sp}\,(G)\,. \tag{6}$$

The experiment was carried out by measuring, with the help of a polarimeter described in [19], the Müller matrix of the specimen. The latter was a plane-parallel plate cut out of the tiff crystal, $CaCO_3$, in parallel to its optical axis and possessing a linear phase anisotropy. The presence of inhomogeneities was simulated by treating one of the layer sides by abrasive powders with different average dimensions of particles. In total, four samples, Nos. 1—4, were studied. The average particle dimension in the abrasive powder applied to each of the specimens and, accordingly, the average size of inhomogeneities in them were 10, 20, 30, and 50 μm.

The measurements were carried out at the wavelength $\lambda = 0.63$ μm. The refraction indices of the birefringent crystal at this wavelength are $n_o = 1.65505$ and $n_e = 1.4890$. The results of the measurements of the Müller matrices for all the specimens are presented in the table. The matrices are the result of averaging over 500 measurements. The experimental measurements of the intensity and the polarization degree of transmitted radiation were carried out for a number of the fixed values of the azimuth θ_n of the linear polarization of incident radiation. In addition, the values of the polarization degree and the radiation intensity were calculated making use of relations (4)—(6).

In order to estimate the experimental errors δf, the following method was used. When probing an arbitrary Müller matrix

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \qquad (7)$$

by radiation which was described by the Stokes vector

$$S = \begin{bmatrix} 1 \\ \cos 2\theta \\ \sin 2\theta \\ 0 \end{bmatrix}, \qquad (8)$$

the intensity and the polarization degree of transmitted radiation can be represented as

$$I = m_{11} + m_{12} \cos 2\theta + m_{13} \sin 2\theta, \qquad (9)$$

and

$$p = \frac{\sqrt{\sum\limits_{j=2}^{4} \left(m_{j1} + m_{j2} \cos 2\theta + m_{j3} \sin 2\theta\right)^2}}{m_{11} + m_{12} \cos 2\theta + m_{13} \sin 2\theta} = \frac{\text{Upper}}{I}, \qquad (10)$$

respectively. Neglecting the error of determination of the azimuth of the linear polarization of incident radiation and supposing the Gaussian distribution, we obtain

$$\delta f(M) = \sqrt{\sum_{i,j=1}^{4} \left(\frac{df\,(m_{ij})}{dm_{ij}} \Delta m_{ij} \right)^2}, \qquad (11)$$

where Δm_{ij} are the measurement errors of the Müller matrix elements. In the present experiment, the measurements of the Müller matrix elements were assumed to be of equal accuracy. The value of the quantity $\Delta m \approx 2\%$ was determined according to a method described in [20].

Experimental Müller matrices of the researched samples

Sample 1	$\begin{bmatrix} 1.00 & 0.48 & 0.05 & 0.01 \\ 0.46 & 0.97 & 0.08 & -0.02 \\ 0.04 & 0.09 & -0.03 & -0.00 \\ 0.00 & 0.03 & 0.00 & -0.04 \end{bmatrix}$
Sample 2	$\begin{bmatrix} 1.00 & 0.41 & -0.01 & 0.00 \\ 0.39 & 0.96 & 0.01 & -0.00 \\ 0.01 & 0.03 & -0.00 & -0.00 \\ 0.01 & 0.00 & -0.00 & 0.00 \end{bmatrix}$
Sample 3	$\begin{bmatrix} 1.00 & 0.37 & 0.02 & -0.00 \\ 0.35 & 0.95 & 0.03 & 0.00 \\ 0.02 & 0.05 & 0.01 & 0.01 \\ 0.00 & 0.00 & -0.02 & 0.01 \end{bmatrix}$
Sample 4	$\begin{bmatrix} 1.00 & 0.35 & -0.01 & 0.01 \\ 0.33 & 0.89 & -0.04 & 0.03 \\ -0.01 & -0.03 & -0.02 & 0.00 \\ -0.00 & -0.02 & -0.01 & -0.01 \end{bmatrix}$

Then, the errors of measurements for the intensity ΔI and the polarization degree Δp are

$$\Delta I = \sqrt{2} \Delta m, \qquad (12)$$

and

$$\Delta p = \frac{\sqrt{2\left(1 + I^2\right) \sum\limits_{j=2}^{4} \left(m_{j1} + m_{j2} \cos 2\theta + m_{j3} \sin 2\theta\right)^2}}{I^2 \text{Upper}} \Delta m, \qquad (13)$$

respectively.

Expressions (12) and (13) yield that, under the conditions mentioned above, the error of evaluating the transmitted radiation intensity ΔI is constant and equals approximately 3%, whereas the error Δp depends on the transmitted radiation intensity, being within the interval of 4—11%.

3. Discussion and Conclusions

The theoretical and experimental dependences of the intensity I and the polarization degree p of radiation after its passage through a medium layer on the azimuth θ of the incident linearly polarized radiation are presented in Fig. 2 for all the researched specimens. Four specimens were oriented in such a manner that the slow axis of the linear phase anisotropy of the tiff crystal coincided with the x-axis of the laboratory coordinate system. One can see that there is a good qualitative agreement between experimental dependences. When calculating the intensity and the polarization degree theoretically, the selection of the parameters of the anisotropic phase screen was done, which minimized the norms

$$\left\| I^{\text{exp}}\left(\theta_n\right) - I^{\text{theor}}\left(\theta_n\right) \right\|,$$
$$\left\| p^{\text{exp}}\left(\theta_n\right) - p^{\text{theor}}\left(\theta_n\right) \right\|, \qquad (14)$$

where $\|\ldots\|$ is the Euclidian norm, $I^{\text{exp}}(\theta_n)$, $p^{\text{exp}}(\theta_n)$, $I^{\text{theor}}(\theta_n)$, and $p^{\text{theor}}(\theta_n)$ are the $(n \times 1)$-vectors of the intensity and the polarization degree, obtained

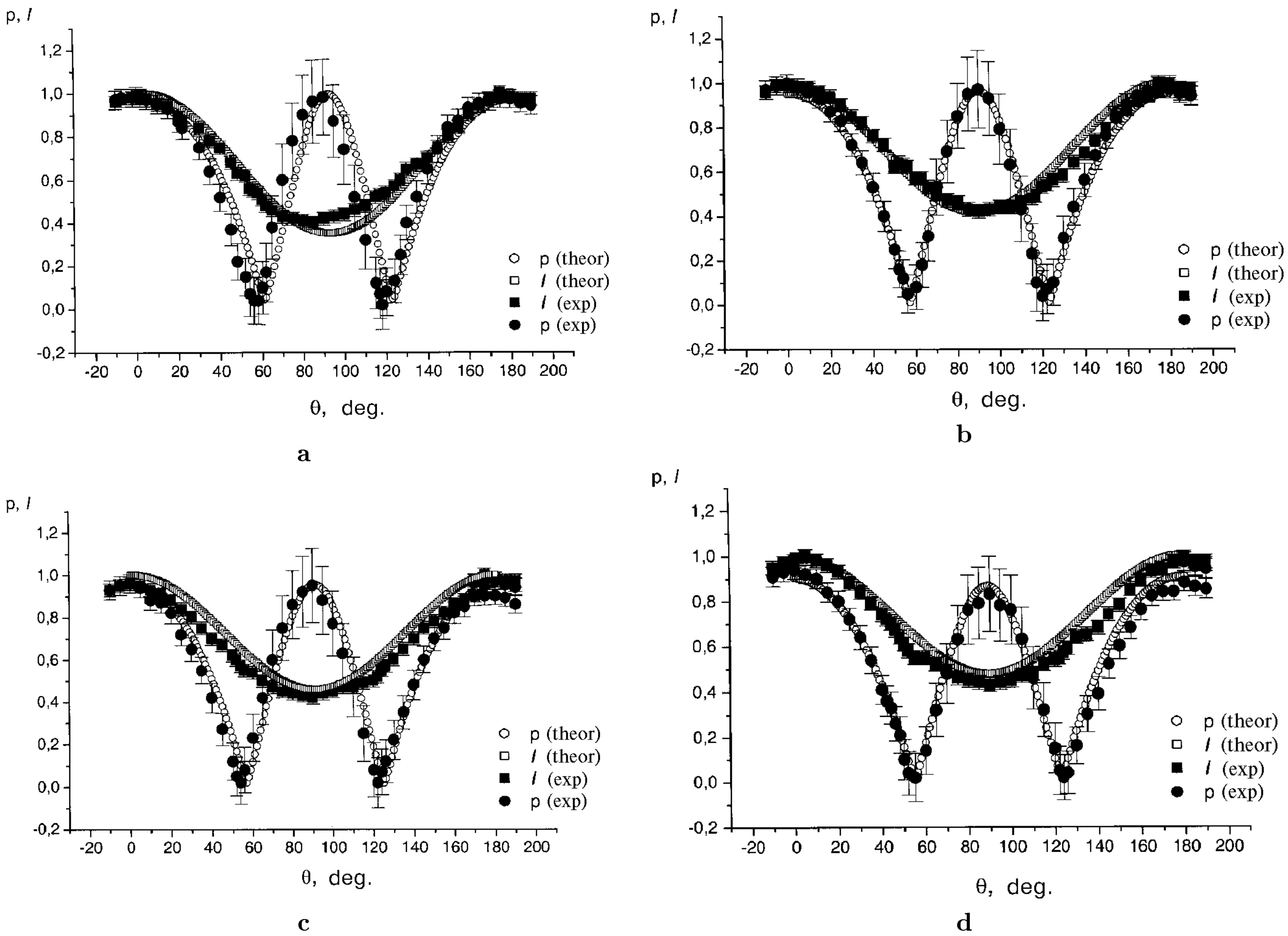

Fig. 2. Theoretical and experimental dependences of the intensity I and the polarization degree p of transmitted radiation on the azimuth θ of incident linearly polarized radiation: specimen 1 (*a*), 2 (*b*), 3 (*c*), and 4 (*d*)

experimentally and theoretically at the corresponding values θ_n of the azimuth of the linear polarization of incident radiation. Nevertheless, there were certain discrepancies between theoretical and experimental dependences. Mainly, it concerns the intensity. These discrepancies can be explained by additional errors made at the determination of the polarization azimuth of incident radiation in experiment.

From Fig. 2, it follows that the polarization degree and the intensity of transmitted radiation essentially depend on the azimuth of the linear polarization of incident radiation. The maximum of the polarization degree was observed at three values of the linear polarization azimuth, namely, 0, 90, and 180°, for all four specimens under investigation. Those values of the azimuth of incident radiation corresponded to orientations of the axes of the linear phase anisotropy in the specimens. At those azimuths, the influence of inhomogeneities turned out minimal, and the transmitted radiation remained completely polarized. The minimal values of the polarization degree (in essence, these were the cases of the total depolarization of incident radiation) were observed at azimuths of about 57 and 123° and did not depend on the dimensions of the specimens' inhomogeneities.

Thus, using the terminology of work [21], it is possible to state that the objects of the investigated class are characterized by a certain type of "polarization memory", in the sense that for definite azimuths of a linear polarization of radiation (in our case, these are 0, 90, and 180°), the interaction of radiation with an object of this class occurs without any changes of the polarization degree, i.e. the object acts as if it "remembers" those polarization states. Some other states with a linear polarization (at azimuths of 57 and 123°) became totally depolarized (they were destroyed completely), i.e. the object "did not remember" them. An important difference from the results of work [21] is the fact that now this effect was observed in a single scattering mode.

The maximal intensity of transmitted radiation was observed at two values of the polarization azimuth of incident radiation, 0 and 180°. In these cases, the incident and transmitted intensities were equal. The minimum of the intensity was observed at the 90°-azimuth of the linear polarization of incident radiation. The values of the azimuth, at which the extreme values of the intensity of transmitted radiation were observed, were identical for each of the researched specimens. Nevertheless, the minimal values of the intensity were different for different specimens and amounted to 0.55 (specimen 1), 0.59 (specimen 2), 0.63 (specimen 3), and 0.67 (specimen 4). We note that the differences between the minimal values of the intensity of transmitted radiation were greater than the maximal value of errors.

Therefore, provided the given direction of observation, the objects of the concerned class affect the intensity of the incident linearly polarized radiation to such an extent that they can be regarded as linear polarizers, whose effectiveness depends on the inhomogeneities' dimensions.

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ОСОБЛИВОСТІ ДЕПОЛЯРИЗАЦІЇ ЛІНІЙНО ПОЛЯРИЗОВАНОГО ВИПРОМІНЮВАННЯ ШАРОМ АНІЗОТРОПНОГО НЕОДНОРІДНОГО СЕРЕДОВИЩА

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Резюме

Досліджено взаємодію лінійно поляризованого випромінювання з об'єктом, який являє собою неоднорідний шар речовини з лінійною фазовою анізотропією. Розроблено теоретичну модель взаємодії поляризованого випромінювання з об'єктами цього класу. Проведено теоретичне й експериментальне дослідження залежності інтенсивності і ступеня поляризації вихідного випромінювання від азимута лінійної поляризації вхідного випромінювання.