SIMULATION OF THE AMPLITUDE-PHASE SPECTRA OF THE FABRY—PEROT INTERFERENCE BY THE ENVELOPE FUNCTION METHOD IN THE REGION OF RESONANT DISPERSION OF THE RESONATOR OPTICAL FUNCTION

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The theoretical simulation of the amplitude-phase Fabry–Perot spectroscopy of the light reflected and transmitted by three-layer plane structures in the region of the resonant dispersion of their dielectric permittivity has been carried out. It has been shown that there is a spectral interval of a certain width where the multibeam interference is of no importance. Beyond this interval, the values of the energy factors of reflection $R_{\max,\min}$, transmission $T_{\max,\min}$, and phase $\phi_{\max,\min}$, which are taken at the extrema of the interference bands considered as the envelopes, describe the amplitude-phase spectra correctly.

Introduction

The influence of absorption on the regularities in forming the light reflection and transmission spectra of non-uniform plane-parallel structures has been investigated for a long time [1]. In work [2], the problems which accompany the determination of spectral parameters were summarized. It turned out that, in this case, different approximations are mainly used [3]. Therefore, this task does remain actual today.

It was substantiated earlier [4, 5] that the instrument-induced characteristics of Fabry–Perot-interference bands are expedient to be analyzed in terms of the half-sums of the reflection, R, and transmission, T, factors, i.e. $\frac{1}{2}(R+T)$, which are evaluated at the extrema (max, min) of the interference bands, because the analysis of the envelope functions $R_{\max,\min}$ and $T_{\max,\min}$ of the amplitude spectra affords an opportunity to express, for certain experimental setups, the phases of the waves reflected and transmitted by an interferometer through experimental values of reflection factors and structural parameters. Later on, a general approach was justified [6] which made it possible to analyze the amplitude-phase spectra of light reflected from or transmitted through transparent three-layer structures which are measured with the help of Fabry–Perot interferometers, by using the envelope method and taking into account the relations between the refraction indices of media.

This work is a sequel to works [4—6]. Here, the envelope method is not only generalized for the analysis of Fabry—Perot amplitude spectra in the region of resonant dispersion of the resonator dielectric function, but is also justified for the first time for the analysis of reflection phase spectra. The envelope method was discussed earlier [7—12] when solving the partial problems of spectroscopy dealing with light reflection and transmission by planar absorbing structures.

Results and Discussion

The three-layer plane structures are considered as being composed of a Fabry—Perot resonator (subscript 2)

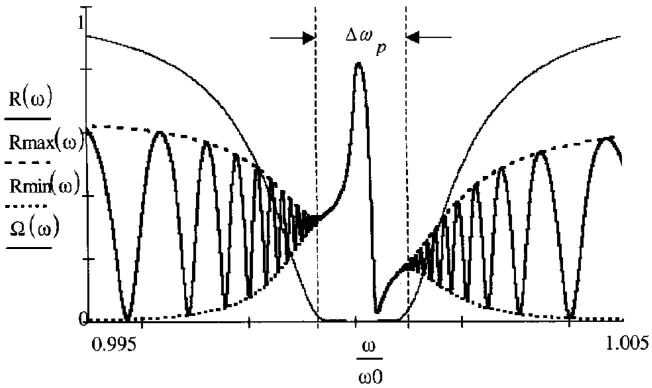

Fig. 1. Reflection spectrum $R(\omega)$, the envelopes $R_{\max}(\omega)$ and $R_{\min}(\omega)$, and the function $\Omega = \exp(-\mathrm{Im}\tilde{\delta})$

with thickness **d** and a complex refraction index n which is surrounded from both sides by transparent semi-bounded dielectric media (subscripts 1 and 3) with refraction indices $n_{1,2}$. The account of the light absorption by the interferometer's body results in a complexity of the resonator phase thickness $\tilde{\delta} = \frac{4\pi d}{\lambda}\tilde{n} = \mathrm{Re}\,\tilde{\delta} + i\,\mathrm{Im}\,\tilde{\delta}$.

Resonant optical characteristics of the resonator are simulated in the single-oscillator approximation by the formula [13]

$$\tilde{\varepsilon}(\omega) = \varepsilon_0 + \frac{4\pi\alpha\omega_0^2}{\omega_0^2 - \omega^2 + -i\omega\gamma} = (n - i\chi)^2, \tag{1}$$

where ε_0 is the background value of dielectric permittivity, ω_0 is the resonant frequency of a transition, $4\pi\alpha$ is the oscillator strength, and γ is the damping parameter.

Due to multiple reflections of the light beam from the resonator surfaces, the ultimate Fresnel reflection amplitude r is calculated as follows [1, 3]:

$$\tilde{r} = \frac{\tilde{r}_{12} + \tilde{r}_{23}\exp\left(-i\,\tilde{\delta}\right)}{1 + \tilde{r}_{12}\,\tilde{r}_{23}\exp\left(-i\,\tilde{\delta}\right)},$$

which leads to the energy reflection factor

$$R = \frac{\sigma_{12}^2 + \sigma_{23}^2\Omega^2 + 2\sigma_{12}\sigma_{23}\Omega\cos\left(\phi_{12} - \phi_{23} + \mathrm{Re}\tilde{\delta}\right)}{1 + \sigma_{12}^2\sigma_{23}^2\Omega^2 + 2\sigma_{12}\sigma_{23}\Omega\cos\left(\phi_{12} + \phi_{23} - \mathrm{Re}\tilde{\delta}\right)}, \tag{2}$$

where $\Omega = \exp\left(-\mathrm{Im}\tilde{\delta}\right)$, $\tilde{r}_{12,23} = \sigma_{12,23}\exp\left(i\,\phi_{12,23}\right)$ are the Fresnel amplitudes for the interfaces with subscripts 12 and 23.

The arguments $F = \phi_{12} \pm (\phi_{23} - \mathrm{Re}\tilde{\delta})$ are different in the numerator and the denominator, but they oscillate identically. Therefore, it is possible to apply the order-reducing formulae to (2). After the corresponding transformations, we obtain

$$R = \frac{R_{\max} + b^2\cos^2\frac{F}{2}}{1 + b^2\cos^2\frac{F}{2}} = \frac{R_{\min} - a^2\sin^2\frac{F}{2}}{1 - a^2\sin^2\frac{F}{2}}, \tag{3}$$

where $a = \frac{2\sqrt{\sigma_{12}\sigma_{23}\Omega}}{1+\sigma_{12}\sigma_{23}\Omega}$, $b = \frac{2\sqrt{\sigma_{12}\sigma_{23}\Omega}}{1-\sigma_{12}\sigma_{23}\Omega}$, $R_{\max} = \left[\frac{\sigma_{12}+\sigma_{23}\Omega}{1+\sigma_{12}\sigma_{23}\Omega}\right]^2$, $R_{\min} = \left[\frac{\sigma_{12}-\sigma_{23}\Omega}{1-\sigma_{12}\sigma_{23}\Omega}\right]^2$.

As an example, the spectra for a symmetric interferometer, i.e. when $n_1 = n_3$, are shown in Fig. 1. We see that the method of envelopes, as the functions of the reflection factors $R_{\max,\min}$ taken at the extrema of interference bands, correctly describes the reflection spectra beyond the spectral region of resonant dispersion of the dielectric function. In the resonant section, it is possible to separate a frequency interval with width $\Delta\omega_p$ which is bounded by an interval of significant absorption where $\Omega(\omega) \to 0$. Therefore, $R_{\max} \approx R_{\min}$ in it and the spectra are formed as if the light wave is reflected from a semibounded medium with resonant dispersion.

Let us pass to the analysis of the spectra of the reflected-wave phase ϕ. According to its definition, $\mathrm{tg}\,\phi = \frac{\mathrm{Im}\tilde{r}}{\mathrm{Re}\tilde{r}}$. Calculating the real, $\mathrm{Re}\tilde{r}$, and imaginary, $\mathrm{Im}\tilde{r}$, parts of Eq. (1), we obtain

$$\mathrm{tg}\phi = \Big(\sigma_{12}(1 - \sigma_{23}^2\Omega^2)\sin\phi_{12} + \sigma_{23}(1 - \sigma_{12}^2)\times$$

$$\times\Omega\sin(\phi_{23} - \mathrm{Re}\,\tilde{\delta})\Big)\Big/\Big(\sigma_{12}(1 + \sigma_{23}^2\Omega^2)\cos\phi_{12} +$$

$$+\sigma_{23}(1 + \sigma_{12}^2)\,\Omega\cos(\phi_{23} - \mathrm{Re}\,\tilde{\delta})\Big). \tag{4}$$

The analysis of this expression shows that the functions (Fig. 2)

$$\phi_{\max,\min} = 2\pi \pm \frac{\sigma_{12}\left(1 - \sigma_{23}^2\Omega\right)\sin\phi_{12} + \sigma_{23}(1 - \sigma_{12}^2)\Omega}{\sigma_{12}\left(1 + \sigma_{23}^2\Omega^2\right)\cos\phi_{12}} \tag{5}$$

are the envelope ones for the phase spectrum beyond the resonant interval $\Delta\omega_p$. In the resonant interval of the spectrum, the phase of light reflected by the Fabry–Perot interferometer is formed as if the light wave is reflected from a semibounded medium with resonant dispersion (Fig. 3).

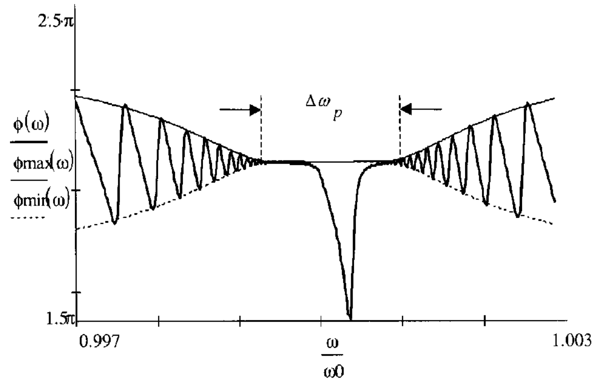

Fig. 2. Phase spectrum $\phi(\omega)$ and envelopes $\phi_{\max}(\omega)$ and $\phi_{\min}(\omega)$ for light reflected from a symmetric three-layer structure with a resonant character of the resonator body dispersion

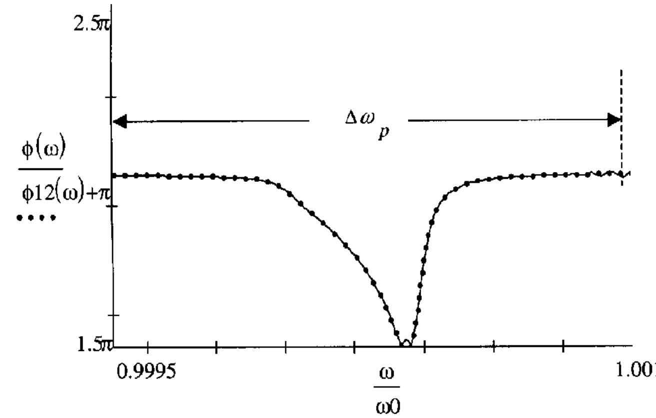

Fig. 3. Phase spectra of light reflected from a symmetric three-layer structure with a resonator with the resonant dispersion $[\phi(\omega)]$ and from a semibounded medium with the resonant dispersion $[\phi_{12}(\omega)]$, in the spectral interval $\Delta\omega_p$

Taking into account that, beyond $\Delta\omega_p$, the shifts of phases are $\phi_{12} \approx \pi$ and $\phi_{23} \approx 2\pi$, formula (5) becomes simpler:

$$\phi_{\max,\min} \approx 2\pi \pm \frac{\sigma_{23}(1-\sigma_{12}^2)\Omega}{\sigma_{12}(1+\sigma_{23}^2\Omega^2)}. \quad (6)$$

Note that

$$\mathrm{Re}\,\tilde{r} \cong \frac{\frac{\sigma_{12}\mp\sigma_{23}\Omega}{1\mp\sigma_{12}\sigma_{23}\Omega} \pm \frac{1+\sigma_{12}^2}{2\sigma_{12}} b^2 \sin^2 \frac{\mathrm{Re}\,\tilde{\delta}}{2}}{1+b^2\sin^2\frac{\mathrm{Re}\,\tilde{\delta}}{2}}$$

and

$$\mathrm{Im}\,\tilde{r} \cong \frac{\mp\frac{1-\sigma_{12}^2}{2\sigma_{12}} b^2 \sin^2 \frac{\mathrm{Re}\,\tilde{\delta}}{2}}{1+b^2\sin^2\frac{\mathrm{Re}\,\tilde{\delta}}{2}} \,\mathrm{ctg}\frac{\mathrm{Re}\,\tilde{\delta}}{2},$$

where the upper sign corresponds to the case where light transmits through the interface from the optically denser medium into the optically less dense one. Then,

$$\mathrm{tg}\,\phi \approx \mathrm{tg}\frac{\mathrm{Re}\,\tilde{\delta}}{2}\left[\mp\frac{1+\sigma_{12}^2}{1-\sigma_{12}^2} - \frac{\sigma_{12}}{1-\sigma_{12}^2}\frac{\sqrt{R_{\min/\max}}}{\sin^2\frac{\mathrm{Re}\,\tilde{\delta}}{2}}\right]. \quad (7)$$

From this formula, the well-known result follows [5] that, for the symmetrical structure,

$$\mathrm{tg}\,\phi = -\frac{1-\sigma_{12}^2}{1+\sigma_{12}^2}\mathrm{tg}\frac{\mathrm{Re}\,\tilde{\delta}}{2}.$$

Consider the transmission spectra. At normal incidence, the energy transmission factor of the three-layer structure is determined as $T = \frac{n_3}{n_1}\tilde{t}\cdot\tilde{t}^*$, where

$$\tilde{t} = \frac{\tilde{t}_{12}\tilde{t}_{23}\exp\left(-i\,\tilde{\delta}/2\right)}{1+\tilde{r}_{12}\tilde{r}_{23}\exp\left(-i\,\tilde{\delta}\right)}$$

is the Fresnel amplitude and $\tilde{t}_{12,23} = 1 + \tilde{\mathbf{r}}_{12,23}$. After relevant transformations, we obtain that the transmission spectrum is expressed through the envelopes $T_{\max,\min}$ as

$$T = \frac{T_{\min}}{1-a^2\sin^2\frac{F}{2}} = \frac{T_{\max}}{1+b^2\cos^2\frac{F}{2}}, \quad (8)$$

where $T_{\max} = \frac{n_3}{n_1}\frac{T_{12}T_{23}}{\left(1-\sigma_{12}\sigma_{23}\Omega\right)^2}\Omega$, $T_{\min} = \frac{n_3}{n_1}\frac{T_{12}T_{23}}{\left(1+\sigma_{12}\sigma_{23}\Omega\right)^2}\Omega$, $T_{12,23} = \tilde{t}_{12,23}\cdot\tilde{t}^*_{12,23}$.

It is problematic to describe the phase spectrum of light transmitted by a symmetric or non-symmetric interferometer using the envelope method. We note only that, at the maxima of transmission bands, the phase of light transmitted by an interferometer is $\Phi = 2\pi$. This means that the period of oscillations of the phase spectrum is twice as large as that of the transmission one.

To summarize, we note the following. If the relation $\frac{T_{\min}}{T_{\max}} = \left(\frac{a}{b}\right)^2$ is taken into account, then, transforming formula (3) to the form

$$\frac{R-R_{\min}}{R_{\max}-R} = \frac{T_{\min}}{T_{\max}}\mathrm{tg}^2\frac{F}{2},$$

we obtain that the reflection factor of the non-symmetric three-layer system is calculated according to the formula

$$R = \frac{R_{\max}T_{\max} + R_{\min}T_{\min}\mathrm{tg}^2\frac{F}{2}}{T_{\max}+T_{\min}\mathrm{tg}^2\frac{F}{2}}. \quad (9)$$

1. Conclusions

1. The method of envelopes which are considered as the values of the energy reflection, $R_{\max,\min}$, and transmission, $T_{\max,\min}$, factors taken at the band extrema of the light interference in the three-layer structures with a Fabry—Perot resonator describes correctly the amplitude spectra in the region of resonant dispersion of the dielectric function.

2. The phase spectra are described correctly by this method only in a reflection geometry.

3. In the resonant region of the spectrum, there is a frequency interval of a certain width, where the influence of the multibeam Fabry—Perot interference on the character of the formation of an amplitude-phase spectrum is not essential. The spectra are formed as if light is reflected from a semibounded medium with the resonant dispersion of the dielectric function.

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МОДЕЛЮВАННЯ МЕТОДОМ ОБВІДНИХ АМПЛІТУДНО-ФАЗОВИХ СПЕКТРІВ ІНТЕРФЕРЕНЦІЇ ФАБРІ—ПЕРО В ОБЛАСТІ РЕЗОНАНСНОЇ ДИСПЕРСІЇ ФУНКЦІЇ ДІЕЛЕКТРИЧНОЇ ПРОНИКНОСТІ

П.С. Кособуцький, А. Моргуліс

Р е з ю м е

Проведено теоретичне моделювання амплітудно-фазової спектроскопії Фабрі—Перо для світла, відбитого і пропущеного тришаровими плоскими структурами в області з резонансною дисперсією діелектричної проникності. Показано, що існує спектральний інтервал деякої ширини, в якому багатопроменева інтерференція не актуальна. За межами цього інтервалу значення енергетичних коефіцієнтів відбиття $R_{\max,\min}$, пропускання $T_{\max,\min}$ та фази $\phi_{\max,\min}$ в екстремумах смуг інтерференції як обвідних коректно описують амплітудно-фазові спектри.