CHARGE-EXCHANGE PROCESSES WITH EXCITATION OF BARYON RESONANCES AT COLLISION OF HIGH-ENERGY PROTONS

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Within the framework of the diffraction model, a formalism for the description of charge-exchange resonance processes is developed. The satisfactory agreement of the theoretical calculations with the corresponding experimental differential cross sections of the process $p(p, n)\Delta^{++}$ is achieved.

1. Introduction

Last years, the experiments on the interaction of incident light nuclei (beginning from nuclei ${}^{1}_{1}H$) with protons at energies of about 1 GeV per nucleon and more [1 - 6] are carried out and planned at various scientific centers. In such processes, the intermediate nonstrange baryon resonances can be excited with a high probability starting with the lightest Δ -resonance (Δ -isobar), $\Delta(1232)$, as the most energy probable, eventually breaking up to nucleons and mainly to pions in time $\tau \sim 10^{-23}$ s (the corresponding width of resonances is $\Gamma = \frac{\hbar}{\tau} \approx 115$ MeV). The relative easiness of the excitation, first of all, of lightest isobars (having isospin $\frac{3}{2}$) together with nucleon resonances N^* (with isospin $\frac{1}{2}$) at such energies is due to their quark structure that is similar to the nucleon one (we have here quarks only of two lightest flavors u and d). In the corresponding processes of collision of incident nuclei and protons with the excitation of resonances and with the lightest meson exchange between nucleons (lightest mesons, pions, consist of the same u- and d-quarks and the relevant antiquarks) and with the radiation of pions, all the known laws of conservation for strong interactions are satisfied.

At the energies of collision not exceeding noticeably 1 GeV per nucleon in systems of strongly interact particles, a not so large number of degrees of freedom (including non-nucleon ones) will be excited, i.e. a rather small number of baryons and mesons (pions) will take part in the above-mentioned processes. This enables one to explore the viewed processes without excessive complication not only experimentally, but also theoretically, by applying various approaches to their description in the region of high energies, in particular, the diffraction approximation we use here.

In the present paper, we consider the elementary charge-exchange process with excitation of an isobar $p(p, n)\Delta^{++}$ and show how it is possible to adapt the diffraction approximation for the theoretical study of such reactions. Having obtained general theoretical results for the reaction $p + p \rightarrow (N\Delta) \rightarrow n + \Delta^{++}$, we then describe the available experiments [4]. With the use of the developed method, it is possible to study more complicated processes of such a type.

The excitation probability of the intermediate Δ isobar at the collision of two high-energy nucleons (in our case, two protons) will be at most possible, if the total energy E in the center-of-mass system (c.m.s.), i.e. in the rest system of the composite system (N Δ), will be close to some resonant energy E_r defining a quasi-discrete level of this physical system with width $\Gamma = \frac{\hbar}{\tau}$, where τ is the lifetime of a Δ -isobar. On the Riemann surface of complex energies E, the poles of the E-dependent scatte-

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ring matrix Ω , which correspond to such quasistationary states, lie, as is known [7], on the second unphysical sheet (the relevant poles of the scattering matrix in the complex plane of wave numbers lie in the lower half-plane, but not on the imaginary axis).

Note that the lifetime of the composite (intermediate) system $(N\Delta)$ and the lifetime τ of a free Δ -isobar are approximately the same, and therefore we can speak also about a resonance of the $(N\Delta)$ system with practically the same width Γ as that of a Δ -isobar. Here, the situation is similar to the known resonance scattering in nonrelativistic quantum theory [7] generalized to the relativistic case with the allowance for different particles (including unstable ones) in initial and final states [8]. So, we hope that the resonance approximation is also correct for the processes considered in the present work.

In Fig. 1, the Feynman diagram for the considered process $p(p, n)\Delta^{++}$ with the exchange between protons by one virtual charged pion is represented for obviousness. In the diagram, the most probable decay of a Δ^{++} -isobar (figured as a narrow rectangle) on a proton and a positively charged pion is shown as well. But we will now not interest in the destiny of excited Δ -isobars, since we shall find (measured also in experiment) the energy (and angular) distributions of neutrons generated during a recharging. That is, the results obtained by us will be, as a matter of fact, integrated over the characteristics of these isobars. Here, the process of collision of two protons up to the decay of an excited Δ -isobar at a moment of time marked in Fig. 1 as the horizontal line ab will be considered formally theoretically. Such an approach considerably simplifies the study of the process and relevant mathematical calculations.

2. Resonance Amplitude of the Process $p(p, n)\Delta^{++}$

The resonance amplitude of the NN-collision process accompanied by the excitation of a Δ -isobar in the diffraction (as a matter of fact, quasiclassical) approximation (when the scattering and emission angles of particles in the c.m.s. $\theta' \ll 1$ and the orbital moments of relative motion $l \gg 1$) can be obtained from the common expression for the resonance amplitude in scattering theory [7, 8]

$$f^{r}(\theta') = -\frac{2l_r + 1}{k} \frac{\Gamma/2}{E - E_r + \frac{i}{2}\Gamma} e^{2i\delta_{l_r}} P_{l_r}(\cos\theta') \tag{1}$$

at the substitution of a resonant value of the relative orbital momentum $l = l_r$ for $k\rho_r$, where k is a value of the relative moment, i.e. the moment of an incident nucleon in the c.m.s., and $\rho = \rho_r$ is the relevant impact parameter, the Legendre polynomial $P_{l_r}(\cos\theta)$ for the Bessel function $J_0(q_{\perp}\rho_r)$, where q_{\perp} is the perpendicular component of the momentum change in the c.m.s. ($\hbar = c = 1$), and the factor $\exp(2i\delta_{l_r})$ with resonant phase for the relevant value of the scattering matrix $\Omega(\rho_r) =$ $1 - \omega(\rho_r)$, where $\omega(\rho_r)$ is the profile function [9 - 11]. As a result, the resonance amplitude in the diffraction approximation takes the following form:

$$f^{r}(q_{\perp}) = -\frac{\Gamma\rho_{r}}{E - E_{r} + \frac{i}{2}\Gamma}\Omega(\rho_{r})J_{0}(q_{\perp}\rho_{r}).$$
(2)

The impact parameter of collision of two nucleons ρ_r in (2) should be close quantitatively by sense to the radius of the nuclear *NN*-interaction $R = 2r_0$, where $r_0 = 1.2$ Fm. The energies *E* and E_r in (1) and (2) for *pp*-collisions will be, respectively, equal to [12]

$$E \equiv \sqrt{s} = \left(2M_p^2 + 2M_p\sqrt{P_p^2 + M_p^2}\right)^{1/2} = = \left(4M_p^2 + 2M_pT_p\right)^{1/2},$$
(3)

$$E_r = M_n + M_\Delta,\tag{4}$$

where M_p , M_n and M_Δ are the masses of a proton, a neutron, and a Δ isobar, P_p is the value of the incident proton momentum in the laboratory system (l.s.), and $T_p = \sqrt{P_p^2 + M_p^2} - M_p$ is the relativistic kinetic energy of the proton. The momentum k in the c.m.s. is related to the momentum P_p in the l.s. as follows:

$$k = \frac{M_p P_p}{\sqrt{s}}.$$
(5)

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Except for the most light isobaric Δ -resonance, Δ (1232), with the mass $M_{\Delta} = 1.232$ GeV, the heavier isobaric Δ -resonance with the mass $M_{\Delta} = 1.62$ GeV can be excited as well as two lightest resonances, if the proton energy T_p will be sufficient. If the energy of an incident proton not heavily exceeds the threshold energy, which is necessary for the birth of the isobar Δ (1232), the excitation of one Δ isobar with minimum possible mass will be most probable. Such a process will be considered in what follows.

The isobars can be excited with equal probability both in the proton-target and the incident proton (for the c.m.s., it is obvious), though there are cases for composite nuclei, when Δ -isobars are excited in a target nucleus [2, 4] with noticeably greater probability, which has not yet received, however, a rigorous theoretical explanation.

Note that, within the composite system $(N\Delta)$, the numerical values of M_{Δ} and Γ can essentially differ from the relevant values of free isobars [2, 6]. Thus, the effective mass of an isobar M_{Δ} decreases due to the interaction of the Δ isobar with a nucleon (or with several nucleons in the case of composite nuclei), as well as the effective mass of a nucleon in a composite nucleus, and the width of a "bound" isobar Γ increases (this fact can be qualitatively explained by the increased "looseness" of the isobar in a composite system).

Since, in the process $p(p, n)\Delta^{++}$ considered here, the recharge of baryons takes place, and the profile function [11] looks as

$$\omega(\rho) = \omega^0(\rho) + (\vec{\tau}_1 \vec{\tau}) \omega^1(\rho), \tag{6}$$

where $\frac{1}{2}\vec{\tau_1}$ and $\frac{1}{2}\vec{\tau}$ are the isospin operators of the incident proton and the proton-target, respectively. The function $\omega^1(\rho)$ is related to the part V^1 of the NN-potential $V = V^0 + (\vec{\tau_1}\vec{\tau})V^1$, and V^1 is less than V^0 [13]. Taking into account that the isospin part of the potential V^1 is poorly investigated so far, we assume, as in [11], that

$$\omega^1(\rho) = \xi \omega^0(\rho), \quad \xi < 1, \tag{7}$$

where the quantity ξ does not depend on the impact parameter ρ , but can depend on the incident proton energy.

Accordingly, the resonant charge-exchange amplitude (2) of the process $p(p, n) \Delta^{++}$ in the diffraction approximation $(k\rho_r \gg 1)$ can be presented in the simple form

$$f^r_{\rm pp \to n\Delta^{++}}(q_{\perp}) = \frac{-\sqrt{3/4}\Gamma\rho_r\omega^0(\rho_r)(1+\xi)}{E - E_r + \frac{i}{2}\Gamma} \times$$

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$$\times J_0(q_\perp \rho_r), \quad q_\perp \approx k \theta'_n \tag{8}$$

where θ'_n is the neutron emission angle in the c.m.s.

3. Formulas for Differential Cross Sections

The angular distribution of the neutrons arising in the reaction $p(p, n)\Delta^{++}$ will be determined by

$$\frac{d\sigma}{d\Omega'_n} = \left| f^r_{\rm pp \to n\Delta + +}(q_r) \right|^2, \quad d\Omega'_n = \sin \theta'_n d\theta'_n d\varphi'_n, \qquad (9)$$

in the c.m.s. and by

$$\frac{d\sigma}{d\Omega_n} = \frac{d\sigma}{d\Omega'_n} B_n(\theta_n), \quad B_n(\theta_n) = \frac{d\Omega'_n}{d\Omega_n} = \frac{\sin\theta'_n d\theta'_n}{\sin\theta_n d\theta_n} \quad (10)$$

in the l.s., where θ_n is the neutron emission angle in the l.s. (θ'_n) is the function of the angle θ_n , and $\theta_n \leq \theta'_n$. The function $B_n(\theta_n)$ in an explicit form is given in [14]. The quantity $B_n(\theta_n \to 0)$ will be given below.

According to the diffraction approximation, the effective values of the neutron emission angles are $\theta'_n \leq \frac{1}{k\rho_r}$. For $k\rho_r \gg 1$, $\theta'_n \ll 1$ and $\theta_n \ll 1$. For this reason, the smooth function $B_n(\theta_n)$ in (10) can be approximately substituted by the quantity \overline{B}_n , which is the value of $B_n(\theta_n)$ as $\theta_n \to 0$ (when $\theta'_n \to 0$ as well) and which looks as [14]

$$\bar{B}_n = \gamma^2 (1 + \frac{V}{V'_n})^2, \quad \gamma = (1 - V^2)^{-\frac{1}{2}},$$

$$V'_n = \frac{P'_n}{\sqrt{M_n^2 + (P'_n)^2}},$$
(11)

where V is the c.m.s. velocity (in terms of the speed of light), and V'_n and P'_n are the velocity and momentum of the emitted neutron in the c.m.s. Thus,

$$V = \frac{P_p}{M_p + \sqrt{M_p^2 + P_p^2}},$$

$$(P'_n)^2 = \frac{\left[4\left(k^2 + M_p^2\right) + M_\Delta^2 - M_n^2\right]^2}{16\left(k^2 + M_p^2\right)} - M_\Delta^2.$$
 (12)

In the nonrelativistic limit and in the approximation $M_n \to M_p$, we have: $\gamma \to 1, V'_n \to V, \bar{B}_n \to 4$.

Following [10] and using (9) and (10), the double (over the energy and neutron emission angles) differential section of the process $p(p, n)\Delta^{++}$ in the l.s. can be written as

$$\frac{d^2\sigma}{dT_n d\Omega_n} = \left| f_{pp \to n\Delta^{++}}^r (q_{\perp}) \right|^2 B_n(\theta_n) \times 215$$

$$\times \frac{\delta}{2\pi \left[\left(T_n - \bar{T}_n \right)^2 + \frac{1}{4} \delta^2 \right]}, \quad \delta \sim \Gamma,$$
(13)

where $T_n = \sqrt{P_n^2 + M_n^2} - M_n$ is the kinetic energy of the escaping neutron, δ is the parameter of "spreading" of the energy delta function $\delta(T_n - \bar{T}_n)$ related to experimental errors and uncertainties in the energy smearing by the large width of the isobar Γ .

The quantity \overline{T}_n in (13) is a root of the equation following from the laws of conservation (generally, there will be two roots):

$$\varphi(\bar{T}_n) = 0, \tag{14}$$

$$\begin{split} \varphi(T_n) &= T_n - T_p + (M_n - M_p) - M_p + \\ &+ \left[T_n^2 + 2M_n T_n + T_p^2 + 2M_p T_p + M_\Delta^2 - \right. \\ &- 2\cos\theta_n \cdot \sqrt{(T_n^2 + 2M_n T_n)(T_p^2 + 2M_p T_p)} \right]^{1/2}, \end{split}$$

where all kinematic quantities are defined in the l.s. It is clear that the root \overline{T}_n will be a function of the kinetic energy of the incident proton T_p and the neutron emission angles θ_n in the l.s.

Note that, owing to the almost 30% difference between the masses of a nucleon and an isobar, some parallel component of the momentum change $q_{\parallel} = \sqrt{q^2 - q_{\perp}^2}$ can also arise even at small angles θ'_n . Although we did not single out it in the formulas, it is taken into account by us according to the laws of conservation in all the kinematic relations. Upon the passage from (1) to (2), we introduce the perpendicular component of the momentum change $q_{\perp} = k\theta'$ which appears in the diffraction approximation when we going over from the exact expression $P_{l_r}(\cos \theta')$ to the approximate one $J_0(\theta' k \rho_r)$. This is mathematically rigorous at $\theta' \ll 1$ and $l_r \gg 1$, so that $q_{\perp} = k\theta'$ in the argument of $J_0(q_\perp \rho_r)$ can be still considered here as a designation. In our case, it is not obligatory to write the exponent $\exp(iq_{\parallel}z)$ (see [15, 16]) at the single profile function $\omega(\rho)$ in (6) and further, since this exponent in the cross sections is under modulus as a factor and therefore does not affect the result.

4. Results of Calculations and Description of Observed Differential Cross Sections

The inclusive experiments [1, 4] analyzed here correspond to the registration of only one most fast particle (in our case, a neutron) resulting in the reaction. Therefore, there is a hope for some adequate description of the observed dependences with the help of cross sections (13), (8) obtained namely for this case in the diffraction approximation. In the indicated experiments, the kinetic energy of incident protons is $T_p \geq 650$ MeV, when the isobars are excited with a high probability. At $T_p \geq 800$ MeV, the processes with the excitation of intermediate resonances should generally dominate and, first of all, with the excitation of the lightest $\Delta(1232)$ isobar [4]. It appears that the process $p(p, n)\Delta^{++}$ noticeably is more probable here, than the less investigated process $p(p, p)\Delta^{+}$.

The following qualitative picture of the excitation of resonances in the processes of pp-collisions is still observed. The birth threshold for the lightest $\Delta(1232)$ isobar at pp-collisions is approximately near $T_p = 0.647$ GeV (or $P_p = 1.27 \text{ GeV}/c$), and the great width of the isobar Γ spreads it. In the energy interval of 0.8 $\leq T_p \leq 2.8 \text{ GeV}$ for protons, the processes with the birth of one pion predominate. In this energy interval T_p , the maximum of the pp-collision cross section with the excitation of the isobar $\Delta(1232)$ is also observed. At energies $T_p > 2 \text{ GeV}$, two and more pions can arise with a high probability, when intermediate resonances and ones heavier than the $\Delta(1232)$ can be born.

In Figs. 2, a, 2, b, and 2, c, the differential sections of the process $p(p,n)\Delta^{++}$ calculated by formulas (13) and (8), namely $\frac{d^2\sigma}{dP_n d\Omega_n} = \frac{P_n}{T_n + M_n} \cdot \frac{d^2\sigma}{dT_n d\Omega_n}$ at $\theta_n = 0$ depending on the escaping neutron momentum P_n , are presented for the energies T_p of incident protons in the l.s. of 0.805, 0.771, and 0.647 GeV, respectively, and the corresponding experimental points are marked [1, 4]. For the indicated energies T_p , the following numerical parameters in (8) and (13) are taken: $\omega^0(\rho_r) = 0.38$; 0.34; 0.233 and $\xi = 0.06$; 0.06; 0.06 (in the considered interval of energies, there is some tendency to a decrease in the parameter $\omega^0(\rho_r)$ with a decrease in the energy); $\delta = \Gamma = 0.115 \text{ GeV}$ [6]; $M_{\Delta} = 1.213 \text{ GeV}$ for Figs. 2, *a* and 2, *b*, $M_{\Delta} = 1.195 \text{ GeV}$ for Fig. 2, *c*.

The used slightly decreased (in comparison with $M_{\Delta} = 1.232$ GeV) value of the mass M_{Δ} of the Δ isobar, as mentioned above, can be qualitatively explained by the interaction of the Δ isobar with a nucleon N in the composite system (N Δ), and, as the calculations [1] showed, the central parts of the observed wide maxima of the differential cross sections for all three indicated energies of protons T_p just correspond kinematically to the value $M_{\Delta} = 1.2$ GeV. The decrease of M_{Δ} is connected, probably, also with virtuality of the Δ -resonance. More specifically, this shift towards smaller values M_{Δ} is explained in [1] with the use of

a diagram technique, firstly, by the interaction of a generated π^+ meson with the final proton, which is taken into account phenomenologically in the form of a factor that is proportional to the total cross section of the π^+p interaction and, secondly, by the behaviour of other factors in the cross section connected, first of all, with the propagators of the one-pion exchange theory.

From Figs. 2, a and 2, b, for which the energy of proton T_p is close to 0.8 GeV, it is seen that the differential cross sections vs the escaping neutron momentum P_n calculated within the diffraction model describe the experimental cross sections quite well both in the region of maxima and at some distance from them, where the cross sections are already small enough as compared to the maximal values.

However, at $T_p = 0.647$ GeV, the agreement with experiment in the viewed model, as is clear from Fig. 2, c, is only qualitative. The right maximum is well described by diffraction theory [1,4]. But one more (left-hand) maximum is seen to the left of it, where the calculated cross section is noticeably less than the observed one. Note that the similar situation for $T_p = 0.647 \text{ GeV}$ arises also upon the calculations of cross sections and within the framework of other theoretical approaches [4]. This discrepancy with experiment may be explain as follows: the energy $T_p = 0.647$ GeV just corresponds, as mentioned earlier, to the birth threshold of the lightest Δ -isobar. Therefore, here one could expect anomalies in the behaviour of the cross section, which is noticeably less (as one could foresee as well) in the region of both maxima (or one wide maximum) in Fig. 2, c than the maximal values of the cross sections at $T_p \approx 0.8 \text{ GeV}$ in Figs. 2, a and 2, b. It is obvious that, at $T_p = 0.647$ GeV, other processes with the emission of detected neutrons, including processes without excitation of intermediate Δ -isobars, can essentially contribute to the cross section. We do not take these processes into account similarly to most of the used theories. In addition, at the energies $T_p < 0.7~{\rm GeV},$ the situations are kinematically possible, when the relative energy of two final baryons in the c.m.s. will be small enough (< 10 MeV), which leads to the noticeable interaction between the reaction products, which was not taken into account by us and in a series of other works [4], but can strongly distort the dependences of cross sections on P_n . Thus, it is possible in our approach to expect a satisfactory description of the differential cross sections only at $T_p > 0.7$ GeV, as in Figs. 2, a and 2, b, i.e. at energies T_p that significantly exceed the birth threshold energy of the lightest isobar.

$d^2\sigma/dP_n d\Omega_n$, mb/(sr GeV/c)





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d^2\sigma/dP_n d\Omega_n,mb/(sr GeV/c)
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Fig.2

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5. Conclusions

1) The differential cross sections for the process $p(p, n)\Delta^{++}$ are obtained in an explicit form with the use of the diffraction nuclear model, the formalism of which was reconstructed with the purpose of the allowance for resonant charge exchange scattering. The developed approach can be applied also to the investigation of resonant charge exchange processes with involvement of composite colliding nuclei.

2) The reached generally satisfactory agreement of the calculated cross sections with the experimental data testifies to that the physical picture of the considered process is reproduced correctly at least qualitatively with the use of the constructed semiphenomenological theory. The last is based on the diffraction approximation and gives us the hope for the adequate description of more compicated processes with the birth of various resonances, experiments on the investigation of which are planned and already is under way.

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ПЕРЕЗАРЯДНІ ПРОЦЕСИ З УТВОРЕННЯМ БАРІОННИХ РЕЗОНАНСІВ ПРИ ЗІТКНЕННІ ВИСОКОЕНЕРГЕТИЧНИХ ПРОТОНІВ

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Резюме

У рамках дифракційної ядерної моделі розвинений формалізм для опису перезарядних резонансних процесів, з використанням якого отримано задовільне узгодження теоретичних розрахунків з відповідними експериментальними диференціальними перерізами процесу $p(p,n)\Delta^{++}$.