FLUCTUATIONS IN DUSTY PLASMAS: KINETIC DESCRIPTION AND NUMERICAL SIMULATION

A.G. ZAGORODNY

Bogolyubov Institute for Theoretical Physics, Nat. Acad. Sci. of Ukraine
(14b, Metrolohichna Str., Kyiv 03143, Ukraine; e-mail: azagorodny@bitp.kiev.ua)

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The kinetic theory of electromagnetic fluctuations in dusty plasmas is developed on the basis of the microscopic description of the grain charging dynamics. The main difference of such a theory from that formulated with the use of a phenomenological assumption is that the effective charging cross-sections are replaced by the k-dependent quantities describing the electron and ion absorption by grains with regard for the influence of a plasma inhomogeneity on the fluxes of absorbing particles. The correlation function of grain charge fluctuations is calculated and compared with the result of numerical simulations.

Introduction

The description of grain fluctuations in dusty plasmas still remains one of the important issues of dusty plasma theory. Recently, various approaches have been used in order to study the correlation properties of fluctuations associated with grain charging [1—10]. In the present contribution, we propose the kinetic theory of electromagnetic fluctuations in dusty plasma developed on the basis of the microscopic description [11]. The main attention is paid to the effects produced by plasma particle discreteness. The correlation function of electric fluctuations is calculated, and the mean-square value of grain charge variance is found. The results of the Brownian dynamics simulations of grain charge fluctuations are discussed and compared with the theoretical predictions.

1. Microscopic Description and Kinetic Equations for Dusty Plasma

In the case of dusty plasma consisting of electrons, ions, and monodispersed grains under the assumption that each grain absorbs all encountering electrons and ions, the microscopic phase densities of plasma particles ($\sigma = e, i$) satisfy the equation (see [11])

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{e_\sigma}{m_\sigma}\mathbf{E}(\mathbf{r},t)\right\} N_\sigma(X,t) =$$

$$= -\int d\mathfrak{X} |\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')| \times$$

$$\times \delta\left(|\mathbf{r}-\mathbf{r}'| - a\right) N_g(\mathfrak{X}',t) N_\sigma(X,t),\ X \equiv \mathbf{r},\mathbf{v}, \tag{1}$$

where $\mathbf{e}_\mathbf{r} = \mathbf{r}/r$, $N_g(\mathfrak{X},t)$ is the microscopic phase density of grains, $\mathfrak{X}$ is the extended phase variable $\mathfrak{X} = (\mathbf{r},\mathbf{v},q)$ which includes the grain charge q as an additional component, the rest of notations is conventional. In order to simplify the problem as much as possible, we neglect grain-grain contact collisions in what follows. In such an approximation, an equation for $N_g(\mathfrak{X},t)$ has the form

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{q}{m_g}\mathbf{E}(\mathbf{r},t)\frac{\partial}{\partial \mathbf{v}}\right\} N_g(\mathfrak{X},t) =$$

$$= -\sum_{\sigma=e,i}\int d\mathfrak{X}' \delta(|\mathbf{r}-\mathbf{r}'| - a) \times$$

$$\times \left\{|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')| N_g(\mathfrak{X},t) - |\mathbf{e}_{\mathbf{r}-\mathbf{r}'}\left(\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_g\right) \times\right.$$

$$\left.\times N_g\left(\mathbf{r},\mathbf{v}-\delta v_\sigma, q-e_\sigma\right)\right\} N_\sigma(X,t), \tag{2}$$

where

$$\delta v_\sigma = -\frac{m_\sigma}{m_g}(\mathbf{v}-\mathbf{v}').$$

Averaging Eqs. (1), (2) and their combinations over the physically infinitesimal time leads to the appropriate BBGKY-hierarchy for dusty plasma [11]. In the approximation of the dominant influence of charging collisions, such a hierarchy generates the following kinetic equations:

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{e_\sigma}{m_\sigma}\mathbf{E}(\mathbf{r},t)\frac{\partial}{\partial \mathbf{v}}\right\} f_\sigma(X,t) =$$

$$= -\int d\mathbf{v}' \int dq' \sigma_\sigma\left(q', \mathbf{v}-\mathbf{v}'\right) |\mathbf{v}-\mathbf{v}'| f_\sigma(X,t) \times$$

$$\times f_g\left(\mathbf{r},\mathbf{v}',q',t)\right), \tag{3}$$

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}} + \frac{q}{m_q}\mathbf{E}\frac{\partial}{\partial \mathbf{v}}\right\} f_g(|x,t) =$$

$$= - \sum_{\sigma=e,i} \int d\mathbf{v}' \left[\sigma_\sigma(q, \mathbf{v}-\mathbf{v}')|\mathbf{v}-\mathbf{v}'| f_g(\mathfrak{X},t) - \right.$$

$$-\sigma_\sigma\left(q-e_\sigma, \mathbf{v}-\mathbf{v}'-\delta\mathbf{v}\right)|\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_\sigma|\times$$

$$\left.\times f_g\left(\mathbf{r}, \mathbf{v}-\mathbf{v}-\delta\mathbf{v}_\sigma, q-e_\sigma, t\right)\right] f_\sigma(X,t). \quad (4)$$

Here, the charging cross-section

$$\sigma_\sigma(q,\mathbf{v}) = \pi a^2 \left(1 - \frac{2e_\sigma q}{m_\sigma v^2 c_g}\right)\theta\left(1 - \frac{2e_\sigma q}{m_\sigma v^2 c_g}\right)$$

appears due to the generalization of the Bogolyubov condition for a weakening of initial correlations [11], and c_g is the electric capacity of the grain. In the linear approximation, this quantity can be roughly approximated as $c_g = a(1 + a/\lambda_D)$.

2. Equations for Fluctuation Evolution

Subtracting Eqs. (3), (4) from Eqs. (1), (2) gives the equations for fluctuation evolution. With accuracy up to the terms describing the nonlinear interaction between the electric field and microscopic density fluctuations, these equations can be written as

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right\}\delta N_\sigma(X,t) + \frac{e_\sigma}{m_\sigma}\delta\mathbf{E}(\mathbf{r},t)\frac{\partial f_{0\sigma}(X,t)}{\partial \mathbf{v}} =$$

$$= -\int d\mathfrak{X}' |\mathbf{e}_{\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')\delta(|\mathbf{r}-\mathbf{r}'|-a)\times$$

$$\times\left[\delta N_g(X,t) f_g(\mathfrak{X}',t) + f_\sigma(X,t)\delta N_g(\mathfrak{X}',t)\right], \quad (5)$$

$$\left\{\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right\}\delta N_g(\mathfrak{X},t) + \frac{q}{m_g}\delta\mathbf{E}(\mathbf{r},t)\frac{\partial f_g(\mathfrak{X},t)}{\partial \mathbf{v}} =$$

$$= -\int dX'\delta(|\mathbf{r}-\mathbf{r}'|-a)\left\{\mathbf{e}_{-\mathbf{r}-\mathbf{r}'}(\mathbf{v}-\mathbf{v}')|\times\right.$$

$$\times\left[\delta N_g(\mathfrak{X},t) f_\sigma(X',t) + \delta N_\sigma(X',t) f_g(\mathfrak{X},t)\right] -$$

$$-|\mathbf{e}_{\mathbf{r}-\mathbf{r}'}\left(\mathbf{v}-\mathbf{v}'-\delta\mathbf{v}_\sigma\right)|\times$$

$$\times\left[\delta N_g\left(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_\sigma, q-e_\sigma; t\right) f_\sigma(X',t) + \right.$$

$$\left.\left. + \delta N_\sigma(X',t) f_g\left(\mathbf{r},\mathbf{v}-\delta\mathbf{v}_\sigma; q-e_\sigma; t\right)\right)\right]\right\}. \quad (6)$$

In the limit $m_g \to \infty$ (motionless grains), Eq. (6) yields the following equation for grain charge fluctuations:

$$\frac{\partial \delta q(\mathbf{r},t)}{\partial t} + \nu_{\text{ch}}\delta q(\mathbf{r},t) = \sum_{\sigma=e,i} e_\sigma \times$$

$$\times \int dX'\delta(|\mathbf{r}-\mathbf{r}'|-a)|\mathbf{e}_{\mathbf{r}-r'}(\mathbf{v}-\mathbf{v}')|\delta N_\sigma(X',t). \quad (7)$$

Here,

$$\nu_{\text{ch}} = -\sum_\sigma e_\sigma n_\sigma \int d\mathbf{v} v \sigma'_\sigma(q_0, v) f_{0\sigma}(\mathbf{v}),\ \sigma'_\sigma = \frac{d\sigma_\sigma}{dq}, \quad (8)$$

$f_{0\sigma}(\mathbf{v})$ is the Maxwellian distribution, and n_σ is the average number density of plasma particles far from any grain.

Deriving Eqs. (7) and (8), we took into account that, in the vicinity of a grain, the plasma particle density can be described by the Boltzmann distribution and excluded bound ionic states. We also used the explicit form of the grain charge distribution

$$f_g(q) = \frac{n_g}{\sqrt{2\pi a T^*}} e^{-\frac{(q-q_0)^2}{2aT^*}} \quad (9)$$

obtained as a stationary solution of Eq. (4) [12]. Here,

$$T^* = \frac{(1+Z_i)}{2}\frac{t+z}{1+t+z}T_e; \quad t = \frac{T_i}{Z_i t_e}; \quad z = \frac{e_e q}{T_e c_g}.$$

In the approximation under consideration, Eqs. (5), (7) together with the Poisson equation form the basic system of equations for electric fluctuations in dusty plasmas. These equations are different from those introduced phenomenologically in [1, 2]. The difference concerns the integral terms proportional to $\delta N_\sigma(X,t)$. In the appropriate phenomenological relations, the charging cross-section is present instead of a singular "microscopic" kernels. In the **k**-representation, Eqs. (5), (7) lead to the phenomenological results [1,2] with accuracy up to the replacement of $\sigma_\sigma(q,\mathbf{v})$ by the quantity

$$\sigma_{\sigma\mathbf{k}}(q,\mathbf{v}) = \sigma_\sigma(q,\mathbf{v})\frac{\sin ka}{ka}. \quad (10)$$

This k-dependent cross-section is qualitatively similar to that introduced in [7, 8]:

$$\sigma_{\sigma k}(q,v) = \frac{1}{2k_\perp}\sigma_\sigma(q,v) J_1\left(k_\perp a\sqrt{1 - \frac{2e_\sigma q}{m_\sigma v^2 a}}\right).$$

3. Correlation Functions of Electric Fluctuations

The final result of calculations of electric potential fluctuations can be written as

$$\langle\delta\Phi^2\rangle_{\mathbf{k}\omega} = \frac{\sum_{\sigma=e,i,g}\langle\delta\rho_\sigma^{(0)2}\rangle_{\mathbf{k}\omega}}{|\varepsilon(k,\omega)|^2}, \tag{11}$$

where

$$\varepsilon(\mathbf{k},\omega) = 1 + \sum_{\sigma=e,i,g}\chi_\sigma(\mathbf{k}\omega)$$

$$\chi_\sigma(\mathbf{k}\omega) = -i\sum_{\sigma'=e,i}\frac{4\pi e_\sigma e_{\sigma'} n_\sigma}{k^2 m_{\sigma'}}\times$$

$$\times\int d\mathbf{v}\int d\mathbf{v}' W_{\sigma\sigma'\mathbf{k}\omega}(\mathbf{v},\mathbf{v}')\mathbf{k}\frac{\partial f_{0\sigma'}}{\partial\mathbf{v}'},$$

$$\sigma = e, i,$$

$$\chi_g(\mathbf{k}\omega) = \frac{n_g}{\omega + i\nu_{\text{ch}}}\sum_{\sigma=e,i}\sum_{\sigma'=e,i}\frac{4\pi e_\sigma e_{\sigma'} n_\sigma}{k^2 m_{\sigma'}}\times$$

$$\times\int d\mathbf{v}\int d\mathbf{v}' W_{\sigma\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') v\sigma_{\sigma\mathbf{k}}(q_0 v)\mathbf{k}\frac{\partial f_{0\sigma'}}{\partial\mathbf{v}'}$$

$$W_{\sigma\sigma'\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') = \frac{i\delta_{\sigma\sigma'}\delta(\mathbf{v}-v')}{\omega - \mathbf{kv} + i\nu_\sigma} + \frac{f_{0\sigma}(\mathbf{v})v}{\omega - \mathbf{kv} + i\nu_\sigma}\times$$

$$\times\frac{n_g\sigma'_\sigma(q_0 v)}{\omega + i\nu_{\text{ch}}}\frac{ie_{\sigma'}n_{\sigma'}\sigma_{\sigma'\mathbf{k}}v'}{g(k,\omega)(\omega - \mathbf{kv}' + i\nu_{\sigma'}}$$

$$g(\mathbf{k},\omega) = 1 - \frac{n_g}{\omega + i\nu_{\text{ch}}}\sum_{\sigma=e,i} e_\sigma n_\sigma\times$$

$$\times\int d\mathbf{v}\frac{\sigma_{\sigma\mathbf{k}}(q_0 v)}{\omega - \mathbf{kv} + i\nu_\sigma}\sigma'_\sigma(q_0 v) f_{0\sigma'}(v)v^2,$$

$$\nu_\sigma = n_g v\sigma_{\sigma\mathbf{k}}(q_0 v), \tag{12}$$

and the correlation functions of the Langevin sources are given by

$$\langle\delta\rho_\sigma^{(0)2}\rangle_{\mathbf{k}\omega} = \sum_{\sigma'} e_\sigma e_{\sigma'} n_\sigma\times$$

$$\times\int d\mathbf{v}\int d\mathbf{v}' W_{\sigma\sigma'\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') f_{0\sigma'}(\mathbf{v}') + \text{c.c.},\quad \sigma = e, i$$

$$\langle\delta\rho_g^{(0)2}\rangle_{\mathbf{k}\omega} = \frac{n_g^2}{|\omega + i\nu_{\text{ch}}|^2}\sum_{\sigma\sigma'=e,i} e_\sigma e_{\sigma'} n_{\sigma'}\int d\mathbf{v}\int d\mathbf{v}' vv'\times$$

$$\times\sigma_{\sigma\mathbf{k}}(q_0, v)\sigma_{\sigma'\mathbf{k}}(q_0, v') W_{\sigma\sigma'\mathbf{k}\omega}(\mathbf{v},\mathbf{v}') f_{\sigma'}(\mathbf{v}') + \text{c.c.} \tag{13}$$

In the equilibrium case, Eq. (11) is considerably simplified:

$$\langle\delta\Phi^2\rangle_{\mathbf{k}\omega} = \frac{T}{2\pi\omega k^2}\text{Im}\left[-\frac{1}{\varepsilon(k,\omega)}\right]. \tag{14}$$

Analogously,

$$\langle\delta\rho_\sigma^2\rangle_{\mathbf{k}\omega} = \frac{Tk^2}{2\pi\omega}\text{Im}\frac{\chi_\sigma(k,\omega)[\varepsilon(k,\omega) - \chi_\sigma(k,\omega)]}{\varepsilon(k,\omega)}. \tag{15}$$

The appropriate static correlations are described by

$$\langle\delta\Phi^2\rangle_{\mathbf{k}} = \frac{T}{4\pi k^2}\left[1 - \frac{1}{\varepsilon(\mathbf{k},0)}\right], \tag{16}$$

$$\langle\delta\rho_\sigma^2\rangle_k = \frac{Tk^2}{4\pi}\chi_\sigma(\mathbf{k},0)\left[1 - \frac{\chi_\sigma(\mathbf{k},0)}{\varepsilon(\mathbf{k},0)}\right]. \tag{17}$$

The last relation can be used to calculate the static correlation function of grain charge fluctuations. For example, in the case of a rarefied grain subsystem, we get

$$\langle\delta\rho_g^2\rangle_k \simeq \frac{Tk^2}{4\pi}\chi_g(\mathbf{k},0) \simeq 2n_g T^* c_g\frac{\sin ka}{ka} \tag{18}$$

or

$$\langle\delta\rho_g^2\rangle_{\mathbf{r}-\mathbf{r}'} = 2n_g T^* c_g\frac{\delta(|\mathbf{r}-\mathbf{r}'| - a)}{4\pi a^2}. \tag{19}$$

The presence of δ-function in Eq. (19) shows that, in the case under consideration, the correlations are related to the same grain, and thus

$$\langle\delta q^2\rangle = \frac{1}{2}\frac{1}{n_g V}\int d\mathbf{r}\int d\mathbf{r}'\langle\delta\rho_g^2\rangle_{\mathbf{r}-\mathbf{r}'} = T^* c_g. \tag{20}$$

This result corresponds to the single-grain approximation. In the case of a dense grain subsystem, it is necessary to use the more general equation (17).

In the case $Z_i = 1$ and $c_g = a$, Eq. (20) recovers the result obtained in [3] and reproduced by another method in [7, 8]. We note that Eq. (20) follows immediately also from solution (9) as well as from the Nyquist formula, if one assumes that the system under consideration can be treated as an electric circuit with the capacity

$$c(\omega)|_{\omega=0} = c_g\frac{t+z}{1+t+z}\frac{(1+Z_i)}{2}.$$

In fact, according to the Nyquist formula, we have

$$\langle\delta q^2\rangle_\omega = \frac{\langle\delta I^2\rangle_\omega}{\omega^2} = \frac{2T}{\omega^2}\text{Re}\frac{1}{Z(\omega)},$$

where $Z(\omega)$ is the impedance of the circuit. After integrating over ω, one obtains

$$\langle\delta q^2\rangle = T\lim_{\omega\to 0}\text{Im}\left[-\frac{1}{\omega Z(\omega)}\right] = Tc(0).$$

As for the dynamical properties of fluctuations in dusty plasmas, some of them are discussed in [5].

4. Numerical Simulations of Grain Charge Fluctuations in Weakly Ionized Plasma Background

In order to study the basic features of grain charge fluctuations from the first principles, the Brownian dynamics simulation of the time autocorrelation function and the static correlator was performed [10]. A single spherical grain embedded in weakly ionized plasma background is considered. The evolution of the grain charge due to the absorption of plasma particles was simulated. It is assumed that the plasma particle motion is governed by the Langevin equation with δ-correlated sources. A similar method has been used earlier to study the grain screening in weakly ionized plasma [13]. The performed simulations confirm the exponential decay of grain charge correlations in time with the damping coefficient of the order of $1/\nu_{\rm ch}$. Another important conclusion is that, for the parameters used in simulations,

$$\langle \delta q^2 \rangle = Ta. \tag{21}$$

It is different from the known result [3]

$$\langle \delta q^2 \rangle = Ta\frac{t+z}{1+t+z}. \tag{22}$$

Notice that, in the case under consideration, the agreement between the theory and simulation can be improved by putting $c_g = a(1 + a/\lambda_{\rm D})$.

Summary and Conclusions

1. On the basis of the microscopic description of dusty plasma, the general theory of fluctuations produced by plasma particle discreteness is developed with regard for the self-consistent interactions of all components. The correlation functions of various quantities (including the grain charge) are calculated.

2. The microscopic approach is a natural basis for theoretical studies of fluctuations in dusty plasmas. It gives the possibility to improve the phenomenological theory, in particular, to introduce consistently the k-dependent charging cross-section.

3. In the appropriate approximation, the theory developed recovers the results for charge fluctuations of a single grain. However, in the case of a dense grain subsystem, general relations of the type of Eq. (17) should be used.

4. Numerical simulations qualitatively confirm the theoretical predictions concerning the exponential decay of grain charge correlations and a static value of the charge variance.

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ФЛУКТУАЦІЇ В ЗАПОРОШЕНІЙ ПЛАЗМІ: КІНЕТИЧНИЙ ОПИС ТА ЧИСЛОВЕ МОДЕЛЮВАННЯ

А.Г. Загородній

Р е з ю м е

На основі мікроскопічного опису процесу заряджання порошинок плазмовими струмами розвинуто кінетичну теорію електромагнітних флуктуацій у запорошеній плазмі. Показано, що такий опис приводить до заміни ефективних перерізів заряджання, що фігурують у феноменологічних теоріях, на їхні модифіковані вирази, залежні від хвильового числа. Тим самим враховано вплив неоднорідності плазми на формування потоків плазмових частинок, що поглинаються порошинками. Розраховано кореляційні функції флуктуацій заряду порошинок і виконано їх порівняння з результатами числового моделювання.