THE BULK PLASMA POTENTIAL AS A TOOL FOR THE DESCRIPTION OF THE INTERACTION OF DUST GRAINS

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The mutual electrostatic influence of isolated electrodes placed in a thermal plasma is investigated. It is shown that the measured value of a floating potential depends on the bulk plasma potential. The spatial distribution of the bulk plasma potential is used to dsecribe the interaction of dust grains. As the ion drag force which is determined by the bulk plasma potential gradient and the force of electric interaction have different directions, the formation of the equilibrium spatial distribution of dust grains in thermal plasmas is possible.

Introduction

The spatial distribution of the electric potential in a neighborhood of a grain in dusty plasmas is frequently defined by the model of a single grain or the Wigner—Seitz model. Zero values of the potential and the electric field at infinity in the first case and on the cell boundary in the second case are used as boundary conditions. The calculated potential distribution in a neighborhood of one dust grain cannot be connected with potential distributions at other dust grains, and it is impossible to solve the problem of interaction of grains with one another under such boundary conditions. Therefore, it was offered to use the concept of bulk plasma potential [1—3] with the purpose to connect different solutions with one another. The bulk plasma potential was defined as a trivial solution of the Poisson—Boltzmann equation in [1].

The applicability of the Poisson—Boltzmann theory to a plasma is defined by the opportunity to use the Boltzmann distribution law. In the present paper, the thermal plasma being at atmospheric pressure with the admixture of alkaline metal atoms is investigated. In this case, the charge carriers are formed by the collision ionization in the plasma volume and the ionization intensity is about $10^{20} - 10^{21}\text{cm}^{-3}\text{s}^{-1}$ that is much more than the diffusion rate of charge carriers. Therefore, the transport of ions and electrons through any microvolume of plasma has no effect (or has a little effect) on the distribution functions.

Processes running on the "dust grain — plasma" boundary may affect the equilibrium as well. However, the frequency of collisions of electrons or ions with dust grains is by two orders less than that with gas particles, there is a thermalization of electronic and ionic gases, and the equilibrium distribution functions are maintained.

Therefore, the Poisson—Boltzmann theory is applicable to thermal plasmas being at atmospheric pressure or to combustion plasmas. The concept of bulk plasma potential as the trivial solution of the Poisson equation is applicable as well and will be used below for the description of the interaction of dust grains with one another.

1. Measurement of the Floating Potential

In experiments, we used the propyl hydride — air flame at a temperature of 1200 K. In the air stream, a 40% aqueous solution of potash was injected. This provided the potassium admixture density $N_A = 10^{10} \div 10^{11}\text{cm}^{-3}$. Under these conditions, the electron and ion equilibrium density was $n_0 \sim 10^6\text{cm}^{-3}$. Along the flame stream, we inlet a planar copper electrode (1 cm × 1 cm) supplied with a thermoelectric couple. The measured values of the electrode floating potential relative to the ground are presented in Fig. 1.

We can calculate the floating potential of the electrode ϕ_s with the boundary condition $\phi(\infty) = 0$. Let us take into account the absence of electric current through the electrode surface, i.e.

$$j_e^T + j_e^{\text{abs}} + j_i^{\text{rec}} + j_a^{\text{ion}} = 0, \tag{1}$$

where

$$j_e^T = -\left(\frac{4\pi e m_e k^2 T^2}{(2\pi\hbar)^3}\right)\exp\left(\frac{-W}{kT}\right) \tag{2}$$

is the thermionic emission current density (the Richardson – Dushman equation), W is the work function;

$$j_e^{\text{abs}} = (1/4) e n_{es} \bar{C}_e \tag{3}$$

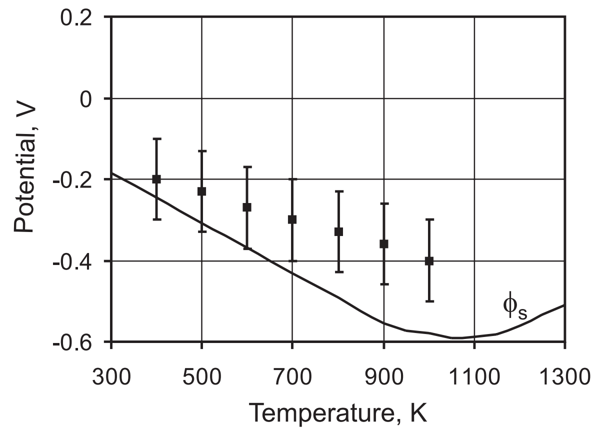

Fig. 1. Floating potential of the copper electrode. The curve is calculated with the condition $\phi(\infty)=0$

is the electron absorption current density, $\bar{C}_e = \sqrt{8kT/\pi m_e}$ is the thermal velocity of electrons, $n_{es} = n_0 \exp(e\phi_s/kT)$ is the surface density of electrons, n_0 is the unperturbed density (in atmospheric-pressure combustion plasmas, the rate of collision ionization is much greater than the diffusion velocity, therefore it possible to use equilibrium distribution functions);

$$j_i^{\rm rec} = -(1/4)\gamma_s e n_{is}\bar{C}_i \tag{4}$$

is the current density upon the surface recombination of ions, $\bar{C}_i$ is the thermal velocity of ions, $n_{is} = n_0\exp(-e\phi_s/kT)$ is the surface density of ions, γ_s is the surface recombination coefficient;

$$j_a^{\rm ion} = (1/4)\beta_s e n_{as}\bar{C}_a \tag{5}$$

is the current density upon the surface ionization of atoms, $\bar{C}_a$ is the thermal velocity of atoms ($\bar{C}_i \cong \bar{C}_a$), $n_{as} = N_A - n_{is}$ is the surface density of atoms, and β_s is the surface ionization coefficient.

The surface ionization coefficient defining the probability of ionization of atoms on the electrode surface [4] is

$$\beta_s = \frac{\exp(e\phi_s/kT)}{1+(g_a/g_i)\exp\left[(I-W)/kT\right]}.$$

Accordingly, the surface recombination coefficient is

$$\gamma_s = \frac{1}{1+(g_i/g_a)\exp\left[(W-I)/kT\right]}.$$

Solving Eq. (1) gives the floating potential ϕ_s shown in Fig. 1.

At the second stage of the experiment, we leave a copper electrode in the flame at a temperature of about 1020 K and inlet a similar aluminum electrode into the flame at a distance of 3 cm from the first one (in this case, the screening length $r_D \sim 0.1$ cm). The surface potential of the aluminum electrode ϕ_{s2} was calculated as the solution of Eq. (1). The results of measurements and calculations are given in Fig. 2.

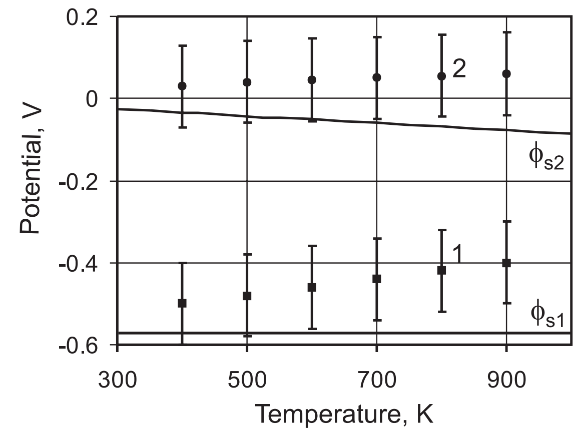

Fig. 2. Floating potential of the copper electrode (*1*) at a stationary temperature of 1020 K and that of the aluminum electrode (*2*). The curve is calculated with the condition $\phi(\infty)=0$

We see that the measured values do not correspond to the calculated ones. Moreover, we observe that the potential of the first copper electrode depends on that of the second aluminum electrode. The surveyed model does not explain this fact as the distance between the electrodes is much greater than the screening length. It is determined by the poor boundary conditions.

2. Potential Barrier and Floating Potential

The spatial distribution of the potential $\varphi(r)$ in a thermal plasma can be found as a solution of the Poisson—Boltzmann equation,

$$\nabla^2\varphi = 4\pi e[n_{e0}\exp\left(e\varphi/kT\right) - n_{i0}\exp\left(-e\varphi/kT\right)], \tag{6}$$

where n_{e0} and n_{i0} are the electron and ion densities at a point with the zero potential.

It is easy to see that Eq. (6) has the trivial solution relevant to the case where the potential is equal to some value $\varphi = \varphi_0$, at which $\nabla^2\varphi_0 = 0$,

$$\varphi_0 = (kT/2e)\ln\left(n_{i0}/n_{e0}\right) \tag{7}$$

and any of two replacements $\varphi(r) = \varphi_0 \pm \phi(r)$ reduces Eq. (6) to the form

$$\nabla^2\phi = 8\pi e\sqrt{n_{e0}n_{i0}}\sinh\left(e\phi/kT\right), \tag{8}$$

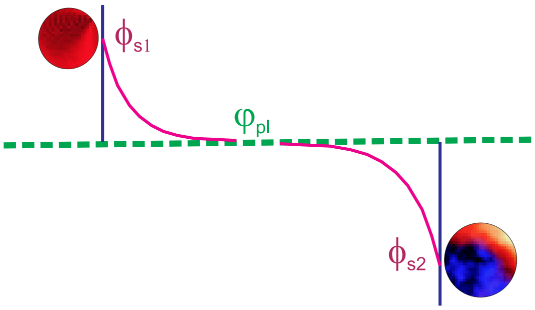

Fig. 3. Bulk plasma potential

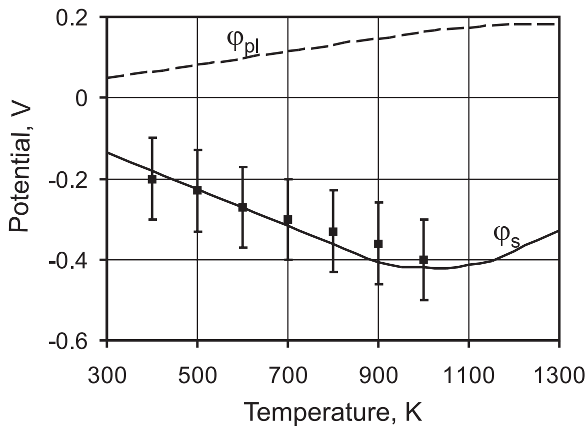

Fig. 4. Floating potential of the copper electrode. The curve is calculated with regard for the bulk plasma potential

where $\sqrt{n_{e0}n_{i0}} = n_q$ is the quasi-unperturbed density which is equal to n_0 if $\varphi_0 = 0$.

This means that all solutions of Eq. (6) are symmetric concerning (7), and each solution which is distinct from the trivial solution may not touch it by virtue of the theorem of existence and uniqueness. This means that the point R, where $\phi'(R) = \phi(R) = 0$, does not exist in a restricted area. The constant value $\varphi_0 \equiv \varphi_{\rm pl}$ is named as the bulk potential of a plasma, and it is necessary in order to interconnect the different solutions of the Poisson—Boltzmann equation within the Wigner—Seitz model for separate dust grains (see Fig.3).

The bulk plasma potential characterizes a value of the work which should be made in order that plasma has gained some volumetric charge $Q_{\rm pl}$. The bulk plasma potential and the volumetric charge determine the electrostatic energy of a plasma volume

$$U = (1/2)Q_{\rm pl}\varphi_{\rm pl}.$$

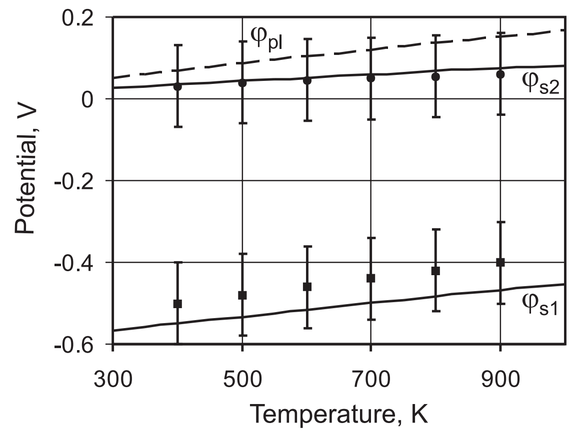

Fig. 5. Floating potential of the copper electrode (*1*) at a stationary temperature of 1020 K and that of the aluminum electrode (*2*). The curves are calculated with regard for the bulk plasma potential

On the other hand, the energy of the arisen electric field E is determined by the equation

$$U = (1/8\pi)\int\limits_V E^2 dV.$$

In the plane case, we have [5]

$$\varphi_{\rm pl} = -2\frac{kT}{e}\tanh\left(\frac{e\phi_s}{4kT}\right). \qquad (9)$$

Then the full floating potential of the electrode is $\varphi_s = \varphi_{\rm pl} + \phi_s$, where ϕ_s is the potential barrier on the "electrode—plasma" boundary calculated by Eq. (1). In Fig. 4, we see that these values correspond to the measured values.

If we have two parallel planes in a plasma with surface potential barriers with respect to the bulk plasma potential ϕ_{s1} and ϕ_{s2}, then the bulk plasma potential is

$$\varphi_{\rm pl} = -2\frac{kT}{e}\tanh\left(\frac{e\phi_{s1} + \phi_{s2}}{4kT}\right), \qquad (10)$$

and the total potential of each plane depends on the surface potential of other plane: $\varphi_1 = \varphi_{\rm pl}(\phi_{s1}, \phi_{s2}) + \phi_{s1}$, $\varphi_2 = \varphi_{\rm pl}(\phi_{s1}, \phi_{s2}) + \phi_{s2}$.

In the second experiment, the bulk plasma potential is defined by both potentials ϕ_{s1}(1020 K) and $\phi_{s2}(T)$. We see in Fig. 5 that it gives different signs of the relative and total potentials of the aluminum electrode. We see also that the change of the bulk plasma potential leads to the change of the total potential of the first copper electrode by 0.1 V, i.e. the second isolated electrode inleted into the flame influences the potential of the first electrode.

3. Interaction of Dust Grains

It is obvious that the same happens under the interaction of dust grains in plasma. However, we assume that one dust grain does not perturb all plasma volume, because the perturbation induced by a grain diminishes with increase in the distance from it. Then every dust grain should give the own value of the bulk plasma potential around it which changes with increase in the distance.

We note that any solution of the relevant Laplace equation has all properties of the trivial solution. In other words, the law $\varphi_{\rm pl} \sim 1/r$ completely satisfies Eq. (6).

Let us consider the momentum transport to a unit area of the grain. The total momentum is defined by the momenta of electrons, ions, and atoms (further, we neglect the momentum transferred by electrons). We have

$$p_i = -\frac{1}{3}\lambda_i \bar{C}_i m_i \nabla(n_i v_i)\Delta S \Delta t,$$

$$p_a = -\frac{1}{3}\lambda_a \bar{C}_a m_a \nabla(n_a v_a)\Delta S \Delta t. \quad (11)$$

Taking into account that the velocities of gas particles $v_a = \bar{C}_a$, $v_i = \bar{C}_i + v_E$, where v_E is the drift velocity; thermal velocities $\bar{C}_a \cong \bar{C}_i = \sqrt{8kT/\pi m_i}$; masses $m_i \cong m_a$, free lengths $\lambda_i \cong \lambda_a$, and $n_i + n_a = N_A$ = const, we have

$$p = p_i + p_a = -\frac{1}{3}\lambda_i \bar{C}_i m_i \nabla(n_i v_E)\Delta S \Delta t. \quad (12)$$

The ion density in the space charge shell of a grain is defined by the expression [6]

$$n_i = n_q \exp\frac{-e\phi(r)}{kT} = n_0 \exp\frac{-e\varphi_{\rm pl}/2 - e\phi(r)}{kT}.$$

Taking into account, further, that $v_E = e(\tau/m_i)E$, where τ is the lifetime and $\bar{C}_i = \lambda_i/\tau$, we obtain

$$p = -\frac{1}{3}\lambda_i^2 n_i e\left[\nabla E_s + \frac{e}{kT}E_s^2 - \frac{e}{2kT}E_s\nabla\varphi_{\rm pl}\right]\Delta S \Delta t. \quad (13)$$

The effective force acting on a grain is defined by summing the momentum flow, Eq.(13,) over the surface. If the distance between grains is much more than r_D, the electric field is raised only by the given grain and is radially symmetric, therefore in expression for force remains

$$\mathbf{F} = \frac{\lambda_i^2 e^2 n_{is} E_s}{6kT}\int\limits_S \nabla\varphi_{\rm pl} dS. \quad (14)$$

The equilibrium on the "dust grain—plasma" boundary demands the existence of some surface value of the bulk plasma potential $\varphi_{{\rm pl}s}$ independent of the presence of other grains. However, the gradient $\nabla\varphi_{\rm pl}$ at the surface of the chosen dust grain depends on the influence of neighbor grains on a degree of plasma ionization, as it is the sum of the bulk plasma potential gradients given by the chosen grain $\nabla\varphi_{{\rm pl}s}$ and raised by grain k at the surface of the chosen grain $\nabla\varphi_{\rm pl} = \nabla\varphi_{{\rm pl}s} + \nabla\varphi^k_{\rm pl}$. Thus, if $\nabla\varphi_{{\rm pl}s}$ has radial symmetry and converts a surface integral to zero, $\nabla\varphi^k_{\rm pl}$ has no radial symmetry when the neighbor grains are located not evenly.

Such a representation is very convenient, as the task remains within the scope of the Poisson—Boltzmann theory. We may calculate the surface potential of each dust grain φ^k_s, the bulk plasma potential near grains $\varphi^k_{{\rm pl}s}$, and the spatial distribution of the bulk plasma potential $\varphi^k_{\rm pl}(r) = \varphi^k_{{\rm pl}s} a_k/r$, where a_k is the grain radius. As a result, we can calculate the ion pressure force (or the ion drag force) at the surface of the chosen dust grain if the distances between grains are known. In view of the discreteness of the arrangement of grains, Eq. (14) becomes

$$\mathbf{F} = \frac{2\pi a^2 \lambda_i^2 e^2 n_{is} E_s}{3kT}\sum_k \frac{\varphi^k_{{\rm pl}s} a_k}{R_k^2}\mathbf{e}_k, \quad (15)$$

where R_k is the distance to grain k, $\mathbf{e}_k$ is the unit vector directed to the given particle from the neighbors (the positive direction is to the chosen grain).

It is possible to describe this force by the Coulomb interaction of dust grains with effective charges $Q_{\rm eff} = (2\pi a^2 \lambda_i^2 e^2 n_{is} E_s)/(3kT)$ for the chosen grain and $q_k = \varphi^k_{{\rm pl}s} a_k$ for neighbors:

$$\mathbf{F} = Q_{\rm eff}\sum_k \frac{q_k}{R_k^2}\mathbf{e}_k.$$

For example, let us consider the atmospheric plasma with the admixture of Cs with the density $N_A = 6 \times 10^{15}{\rm cm}^{-3}$ (I = 3.6 eV), containing a dust grain of aluminum (W = 3.7 eV) with a radius of 1 μm at the temperature T = 2200 K (0.2 eV). These parameters provide the unperturbed density $n_0 \sim 4 \times 10^{13}{\rm cm}^{-3}$ and the screening length $r_D \sim 0.4$ μm. It is admissible that the distance from the surface of the chosen dust grain to the surface of the left-hand grain is 3 μm and to surface of the right-hand grain is 5 μm. The distribution of the bulk plasma potential at the grain surface in this case is given in Fig. 6.

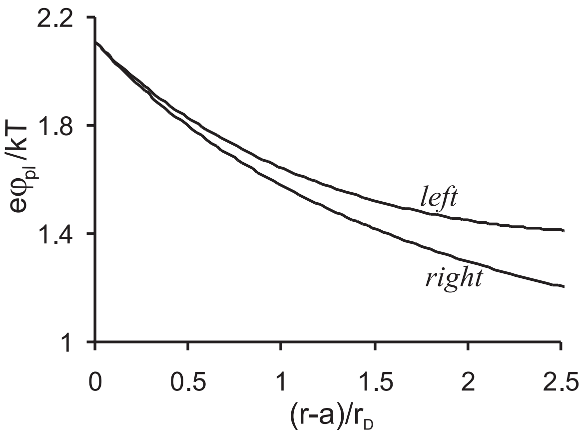

Fig. 6. Distribution of the bulk plasma potential

It is visible that the bulk plasma potential gradient on the left is less, than that on the right. Therefore, there is the force of 5×10^{-15} N which is directed to the left-hand grain and ensures an acceleration of about 30 cm$\cdot$s^{-2}. Hence, if a fluctuation diminishes the distance between two identical dust grains, they proceed to approach each other by the ion drag force, i.e. grains tend to form agglomerates.

Conclusions

Thus, we have shown that the concept of bulk plasma potential allows one to explain the influence of one electrode on the potential of the other one in the thermal plasma. It is not enough to know the potential barrier on the "electrode—plasma" boundary for the complete description, because the bulk plasma potential influences the results of measurements.

The concept of bulk plasma potential allows us to describe the ion drag force as a result of the plasma ionization displacement. In this case, the interaction among dust grains can be presented as the Coulomb one if we use the effective charges of grains which take into account the bulk plasma potential gradient.

It is necessary to note that the sign of the effective charge of the neighboring grains is opposite to the sign of the real charge, sign($\varphi_{\text{pl}s}$) = − sign(φ_s). Similarly to the Coulomb interaction, likely charged grains are attracted and oppositely charged grains are repelled in this case. It occurs because the nature of the long-range interaction of grains is not the electric one, but is determined by the ionization degree anisotropy and the ionic pressure at the grain surface. When grains approach at the distances comparable to the screening length, the electric repulsive force begins to operate, and, as a result, some equilibrium spatial arrangement of grains can appear.

1. *Vishnyakov V.I., Dragan G.S., Margaschuk S.V.* // Chemistry of Plasmas / Ed. by B. Smirnov. — Moscow: Energoatomizdat, 1990. — Issue 16. — P. 98—120 (in Russian).
2. *Barnes M.S., Keller J.H., Forster J.C., O'Neill J.A.* // Phys. Rev. Lett. — 1992. — **68**. — P. 313—316.
3. *Knott M., Ford I. J.* // Phys. Rev. E. — 2001. —**63**. — P. 031403.
4. *Vishnyakov V.I., Dragan G.S.* // Ukr. J. Phys. — 2004. — **49**. — P. 229—235.
5. *V.I. Vishnyakov, G.S. Dragan* // 30th EPS Conf. on Controlled Fusion and Plasma Physics, St.Petersburg, 2003. — ECA 27A. — P-4.119.
6. *Vishnyakov V.I., Dragan G.S.* // Ukr. J. Phys. — 2004. — **49**. — P. 132—137.

ПОТЕНЦIАЛ ПЛАЗМИ ЯК ЗАСIБ ОПИСУВАННЯ ВЗАЄМОДIЇ ПОРОШИНОК У ПЛАЗМI

В.I. Вишняков

Р е з ю м е

Проведено дослiдження електростатичного впливу iзольованих електродiв один на одного у термiчнiй плазмi. Показано, що вимiрюване значення плаваючого потенцiалу залежить вiд потенцiалу плазми. Запропоновано використовувати просторовий розподiл потенцiалу плазми для опису взаємодiї порошинок. Оскiльки захоплювальна сила iонiв, яка описується градiєнтом потенцiалу плазми, i сила електростатичної взаємодiї напрямленi в рiзнi боки, то можливе встановлення рiвноваги у просторовому розподiлi частинок.