NON-MAXWELLIAN VELOCITY DISTRIBUTION OF GRAINS IN DUSTY PLASMAS WITH ION FLOW

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The concept of generalized friction is used for the calculation of the ion flow influence on the distribution of charged grains. The generalized friction force acting on grains due to ion scattering and atomic friction can be represented via the friction coefficients with atoms β_a and with ions β_i for the respective Fokker—Planck equation for grains. The stationary distribution of grains is found to be non-Maxwellian. A generalization of this method to the case of active particles (including biological objects which are able to self-motion) is discussed.

The concept of generalized friction [1] can be used for the calculation of the ion flow influence on the charged grain distribution. The generalized Fokker—Planck equation for grains has the form:

$$\frac{df_g(\mathbf{P},t)}{dt} = \frac{\partial}{\partial P_i}\left[A_i(\mathbf{P},\mathbf{y})f_g(\mathbf{P}) + \frac{\partial}{\partial P_j}\left(B_{ij}(\mathbf{P},\mathbf{y})f_g(\mathbf{P})\right)\right], \quad (1)$$

where $\mathbf{P} \equiv M\mathbf{V}$, M, and $\mathbf{V}$ are the grain momentum, mass, and velocity; $\mathbf{y} \equiv M\mathbf{u}$, and $\mathbf{u}$ is the ion driven velocity. The coefficients A_i and B_{ij} can be determined if the respective probability transition (PT) functions are known [1]. These coefficients are related to the generalized friction β_i^λ and diffusion $B_{ij}^\lambda = M^2 D_{ij}^\lambda$, where $\lambda = i, a$ for the processes of momentum transfer between grains and ions (i) and atoms (a), respectively. For the function $\mathbf{A}$, we find

$$\mathbf{A}(\mathbf{P}-\mathbf{y}) = \mathbf{A}^i(\mathbf{P}-\mathbf{y}) + \mathbf{A}^a(\mathbf{P}-\mathbf{y}) \approx \approx (\mathbf{P}-\mathbf{y})\,\beta^i(|\mathbf{P}-\mathbf{y}|) + \mathbf{P}\,\beta^a(|\mathbf{P}|). \quad (2)$$

The diffusion coefficients can be written in a similar way. Eq. (2) can be represented as

$$\mathbf{A}(\mathbf{P}-\mathbf{y}) \simeq \left[\frac{(\mathbf{P}\cdot\mathbf{y})}{P^2}\mathbf{P} - \mathbf{y}\right]\beta^i(|\mathbf{P}-\mathbf{y}|) + \mathbf{P}\left[\beta^a(|\mathbf{P}|) + \left(1 - \frac{\mathbf{P}\cdot\mathbf{y}}{P^2}\right)\beta^i(|\mathbf{P}-\mathbf{y}|)\right]. \quad (3)$$

The first term on the right hand-side of Eq. (3) is perpendicular to the vector $\mathbf{P}$. The second term on the right-hand side of Eq. (3) is parallel to $\mathbf{P}$ and describes the effective friction, which can be negative in the case under consideration and for $V \ll u$ due to the acceleration of grains by the ion flow. In general, the manifestation of negative friction is the change of a sign of the friction coefficient as a function of the mesoscopic (e.g., Brownian) particle velocity for some specific parameters and conditions in the system. We suggest that the characteristic velocities $\mathbf{V}, \mathbf{u} \ll v_{Ti}$, that provides the applicability of the Fokker—Planck equation for ion scattering [2]. Therefore we can neglect the velocity dependence of the friction β^a, β^i and diffusion D^a, D^i coefficients, which will be denoted as β_0^a, β_0^i, etc. in this case. Then the effective (velocity-dependent) friction coefficient in the Fokker—Planck equation can be simplified and taken in the form

$$\beta^{\text{eff}}(\mathbf{V}) = \beta_\Sigma - \frac{\mathbf{u}\cdot\mathbf{V}}{V^2}\beta_0^i, \quad (4)$$

where $\beta_\Sigma \equiv \beta_0^a + \beta_0^i$. A stationary solution of the Fokker—Planck equation can be easily found as

$$f_g(\mathbf{V}) = C\exp\left\{-\frac{\beta_\Sigma}{2D_\Sigma}\left(\mathbf{V} - \frac{\beta_0^i}{\beta_\Sigma}\mathbf{u}\right)^2\right\}. \quad (5)$$

Here, C is the constant of normalization, and the effective diffusion coefficient is $D_\Sigma = D_0^a + D_0^i$. For the friction and diffusion coefficients, we can use the specific expressions: $\beta_0^a = 8\frac{\sqrt{2\pi}}{3}(m_a/M)a^2 n_a v_{T_a}$ (scattering of a point atom with mass m_a by a hard spherical grain with mass M and radius a), $\beta_0^i \simeq 2A_0\Gamma^2 \ln\Lambda$, where $A_0 \equiv \frac{\sqrt{2\pi}}{3}(m_i/M)a^2 n_i v_{T_i}$ (m_i, n_i are the mass and density of ions), and $\Gamma \equiv e^2 Z_g Z_i/aT_i \gg 1$ is the characteristic ion-grain interaction parameter. The Landau logarithm $\ln\Lambda$ for dusty plasmas can be taken in the form given in [3,4]. In the case of a strong interaction between ions and grains, the logarithmic approximation is not suitable, and a more elaborated consideration of ion scattering is needed [5]. However, the conclusion on the negative friction and the shifted velocity distribution of grains is independent of the specific form of the ion scattering expression.

In the simple case where $\mathbf{u}$ is parallel to $\mathbf{V}$, Eq. (4) yields the total negative effective friction coefficient considered earlier [6]:

$$\beta^t(V) = \beta^a(0)\left[1 - \frac{u m_i n_i v_{T_i}}{2V m_a n_a v_{T_a}}\Gamma^2 \ln\Lambda\right]. \quad (6)$$

It is necessary to mention that, for the condition $(\mathbf{y}\cdot\mathbf{P}) > 0$, negative friction is always exists for enough small values of the particle velocity V.

We now turn to the case of active particles, e.g. some small biological objects (BO) in the ambient medium which are able to self-motion. The phenomenological model of such a motion and the comparison with experiments has been done in [7]. Recently, a microscopic consideration of self-motion in the case of the average self-acceleration in the direction of motion on the basis of the appropriate PT function has been developed in [1].

In general, as was mentioned already in [1], the direction of acceleration for BO is related with some inner vector of BO which can rotate and takes the instantaneous direction to the external gradients of temperature, or a food density, or other BO density in the system. We named this vector as a "driver" $\tilde{\mathbf{D}}$. The appropriate Fokker—Planck equation for such a type of motion has the form

$$\frac{df_g(\mathbf{P},t)}{dt} = \frac{\partial}{\partial P_i}\left[(P_i\beta(\mathbf{P}) - \tilde{D}_i)f_g(\mathbf{P}) + \frac{\partial}{\partial P_j}\left(B_{ij}(\mathbf{P})f_g(\mathbf{P})\right)\right]. \quad (7)$$

In general, $\tilde{\mathbf{D}}$ can be a smooth function of the grain velocity [1]. If this dependence is negligible the term with a "driver" has a typical structure of a force term in the kinetic equation, $\tilde{\mathbf{D}}\frac{\partial f_g(\mathbf{P})}{\partial \mathbf{P}}$.

It is easy to show that there is some analogy in the description of this process and the acceleration of grains by an ion flow. In both cases, we have the acceleration of particles due to the momentum transfer to the ambient medium (with a loss of the inner energy of BO) or to a grain due to the ion flow scattering. In the case of BO, the effective friction coefficient can be represented as

$$\beta_{\mathrm{BO}}^{\mathrm{eff}}(\mathbf{V}) = \beta^M - \frac{\mathbf{D}\cdot\mathbf{V}}{V^2}\beta^{\mathrm{BO}}. \quad (8)$$

Here, β^M is the friction coefficient for the ambient medium, and β^{BO} is some characteristic function for the momentum production by BO.

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НЕМАКСВЕЛЛІВСЬКИЙ РОЗПОДІЛ ЧАСТИНОК ЗА ШВИДКОСТЯМИ У ЗАПОРОШЕНІЙ ПЛАЗМІ З ІОННИМ ПОТОКОМ

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Р е з ю м е

Для визначення впливу іонного потоку на розподіл заряджених порошинок використано концепцію узагальненого тертя. Показано, що узагальнена сила тертя, що діє на порошинку внаслідок розсіяння іонів, а також атомне тертя можуть бути описані за допомогою коефіцієнтів тертя (β_a — для атомів і β_i — для іонів) у відповідному рівнянні Фоккера—Планка для порошинок. Одержано вигляд стаціонарного немаксвеллівського розподілу порошинок. Обговорюється узагальнення даного методу на випадок активних частинок, у тому числі біологічних об'єктів, здатних до самодовільного руху.