TRANSITION RADIATION OF A MODULATED ELECTRON BEAM IN PLASMA WITH ORIENTED DIPOLE DUST PARTICLES

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Transition radioemission caused by the injection of a modulated electron beam into a dusty plasma with dipole dust particles oriented by the strong magnetic field is theoretically studied. The radiation pattern and the total radiated power are calculated for typical experimental conditions.

Introduction

The development of diagnostic methods for dusty plasma is under discussion. The use of Mie scattering caused by the laser light illumination is the elementary and typical method for the detection of dust particles. In the case of small perturbations of the background plasma, other external influences on dusty structures can be used, in fact, as a base for alternative diagnostics. One of them is beam-excited electromagnetic radiation as a result of the injection of a modulated electron beam in plasma. Recently, the excitation of waves which is caused by a modulated electron beam has been demonstrated in the laboratory experiment [1].

One of the possible mechanisms of radioemission for a modulated electron beam is transition radiation. The presence of dust particles leads to the appearance of background plasma inhomogeneities, and it results in the possibility of transition radiation. In our previous work [2], the non-resonant transition radiation was calculated for a simplified model of dusty plasma inhomogeneities described as a system of equal and randomly distributed spherical dust particles. It was shown that the intensity of transition radiation is directly proportional to the product of the dust particle concentration and the square of the average charge of a dust particle.

Most theoretical studies and experimental researches on dusty plasmas deal with spherical dust particles as a typical model. Nowadays, the complex structures of highly asymmetric dust particles are of special interest [3]. Their longitudinal size is larger than the transverse one. If they are ferromagnetic elongated particles, then dusty structures can be described as a system of dipoles which can be oriented by a magnetic field. It is clear that dipole dust particles can be randomly distributed in the plasma volume. They perturb the plasma concentration causing the appearance of random plasma inhomogeneities. Beam electromagnetic field scattered on the plasma inhomogeneities results in transition radiation. Below, we present the theoretical study of transition radiation of a modulated electron beam acting on dipole dust particles in the magnetic field.

1. Model Description and Initial Equations

We consider a plasma in a constant homogeneous strong magnetic field. Ferromagnetic dipole dust particles are placed in the plasma randomly. To simplify the problem, the dust particles are assumed to be similar. The magnetic field satisfies the condition $H_0 \gg 4\pi n_d d_m$, where n_d is the dust density and d_m is the magnetic moment of a dipole dust particle. This means that all dust particles are oriented along the magnetic field [4]. Such an orientation has been recently observed in the experiment [5]. Dust particles have a constant dipole moment $d = Z_0 e l_0$ ($Z_0 e$ and l_0 are the dipole charge and length, respectively; $l_0 \ll r_d$, r_d is the Debye length). The average distance between dust particles $n_d^{-1/3}$ is larger than the dipole length d, so the magnetic interaction between dipoles can be neglected. The dipole electric field of dust particles perturbs the background plasma density and forms random plasma inhomogeneities. The plasma quasineutrality balance is assumed to be unaffected.

A cylindrical modulated electron beam of radius a is injected along the magnetic field into plasma. The beam is assumed to be charge-compensated. The beam effect on the background plasma is neglected. The alternating beam current density can be predetermined as a given function

$$\vec{j}(r,z,t) = \vec{e}_z j_m(r,z) \exp(i\omega t - i\kappa z). \qquad (1)$$

Here, $\kappa = \omega/v_0$, v_0 and ω are the beam velocity and modulation frequency, respectively. The beam modulation frequency ω is larger than the Langmuir frequency ω_p of the background plasma. Consequently the plasma dielectric permittivity tends to 1. We use the model of cold background plasma.

In our model, the beam appears in the plane $z = 0$, and the alternating current relaxation along the beam is considered to be exponential with relaxation distance L:

$$j_m(r,z) = \begin{cases} j_m \exp\left(-z/L\right), & r < a, \quad z > 0; \\ 0, & r > a, \quad z < 0. \end{cases} \qquad (2)$$

The plasma dielectric permittivity has the matrix form

$$\hat{\varepsilon} = \hat{\varepsilon}^{(0)} + \hat{\varepsilon}^{(1)},$$

$$\hat{\varepsilon}^{(0)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \varepsilon_0 \end{bmatrix},$$

$$\hat{\varepsilon}^{(1)} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_1\left(\varphi, r, z\right) \end{bmatrix}, \quad \left|\varepsilon_1\left(\varphi, r, z\right)\right| \ll \varepsilon_0. \qquad (3)$$

Here, ε_1 is a small random perturbation of the background plasma dielectric permittivity ε_0 ($\varepsilon_0 \to 1$) caused by dipole dust particles.

The electromagnetic field of the modulated electron beam is scattered by random plasma inhomogeneities. It results in the appearance of transition radioemission.

A wave equation for the electromagnetic field excited by current (1) in plasma with the random spatial dielectric permittivity (3) has the form

$$\mathrm{rotrot}\vec{E} + \frac{\hat{\varepsilon}}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = -\frac{4\pi}{c^2}\frac{\partial \vec{j}}{\partial t}. \qquad (4)$$

It can be solved by using the method of successive approximations in the small parameter ε_1.

The zero-order equation

$$\mathrm{rotrot}\vec{E}^{(0)} + \frac{\hat{\varepsilon}^{(0)}}{c^2}\frac{\partial^2 \vec{E}^{(0)}}{\partial t^2} = -\frac{4\pi}{c^2}\frac{\partial \vec{j}}{\partial t} \qquad (5)$$

allows us to find out the electromagnetic field of the modulated electron beam caused by current (1) in a homogeneous plasma. The non-resonant transition radiation can be calculated from the equation

$$\mathrm{rotrot}\vec{E}^{(1)} + \frac{\hat{\varepsilon}^{(0)}}{c^2}\frac{\partial^2 \vec{E}^{(1)}}{\partial t^2} = -\frac{\hat{\varepsilon}^{(1)}}{c^2}\frac{\partial^2 \vec{E}^{(0)}}{\partial t^2} \qquad (6)$$

in the first-order approximation as a result of the scattering of the beam electromagnetic field by random plasma inhomogeneities.

The Fourier transformation with respect to z and Fourier—Bessel transformation with respect to r are used to build up the solution of (5) and (6). Additionally, the Fourier series with respect to φ has to be applied to (6).

2. The Spectral Intensity of Plasma Inhomogeneities

Random spatial perturbations of the background dielectric permittivity are considered to be a stationary random process in all variables. Its spatial spectral intensity $G(k_r, k_z)$ can be introduced by the following relation [6]:

$$\left\langle \varepsilon_n(k_r, k_z)\varepsilon^*_{n'}(k_r^{'}, k_z^{'})\right\rangle = \frac{G(k_r, k_z)}{k_r}\times$$

$$\times\delta_{nn'}\delta(k_r - k_r^{'})\delta(k_z - k_z^{'}) \quad (n = 0, \pm 1, \pm 2, \ldots), \qquad (7)$$

where the broken brackets define the ensemble averaging over realizations.

A single dipole dust particle is described as the system of two equal point charges with opposite signs. It is supposed that the contribution of each point charge to the spatial perturbation of the plasma dielectric permittivity falls off exponentially beyond the characteristic Debye length r_d. The spatial spectrum of plasma permittivity perturbations caused by a single dipole dust particle can be written as

$$\Phi(k_r, k_z) = 8i\frac{Z_0}{m_e}\left(\frac{e}{\omega}\right)^2 \frac{\sin\left(k_z l_0/2\right)}{1 + \left(k_r^2 + k_z^2\right)^2 r_d^2}. \qquad (8)$$

It is assumed that every dipole dust particle is located randomly and independently from other particles as well as the spatial location and the total quantity of dipole dust particles are statistically independent. After generalizing the model of a random sequence of pulses, one can obtain the spatial spectral intensity for the stationary distribution of dielectric permittivity fluctuations as

$$G(k_r, k_z) = 2\pi n_d \left|\Phi(k_r, k_z)\right|^2 =$$

$$= 128\pi n_d \left(\frac{Z_0}{m_e}\right)^2 \left(\frac{e}{\omega}\right)^4 \frac{\sin^2\left(k_z l_0/2\right)}{\left[1 + \left(k_r^2 + k_z^2\right)^2 r_d^2\right]^2}. \qquad (9)$$

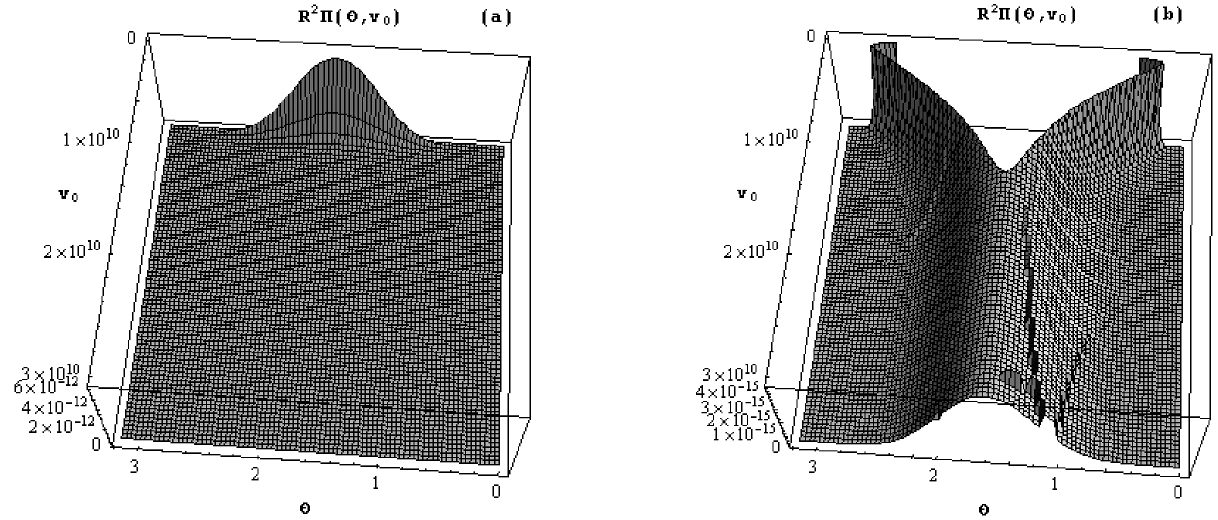

Fig. 1. Radiation pattern $R^2\Pi(R,\Theta)$ $[W]$ as a function of Θ [radian] and v_0 [cm/s] calculated for $\omega_p = 2.5\times10^9$ s^{-1}, $\omega = 7.6\times10^9$ s^{-1}, $n_e = 2\times10^9$ cm^{-3}, $n_d = 2\times10^5$ cm^{-3}, $T_e = 5$ eV, $Z_0 = 10^3$, $r_d = 3.7\times10^{-2}$ cm, $l_0 = 7\times10^{-4}$ cm, $a = 0.5$ cm, $L = 100$ cm, $j_m = 1$ A/cm^2

3. Transition Radiation on Dipole Dust Particles

Transition radiation is p-polarized in the model treated. For the far zone of radiation and a stationary random distribution of dielectric permittivity perturbations of plasma, the radial component of the energy flux of transition radiation has the form

$$\Pi(R,\Theta)=\frac{c\varepsilon_0^{1/2}}{4\pi}\left\langle E_\Theta^{(1)}E_\Theta^{(1)^*}\right\rangle=$$

$$=\frac{c}{4}\frac{k_r^6(\Theta)}{\varepsilon_0^{3/2}k_0^2R^2}\int\limits_0^\pi d\psi\int\limits_{-\infty}^{\infty}dK_z\int\limits_0^\infty K_r dK_r\left|E_z^{(0)}(K_r,K_z)\right|^2\times$$

$$\times G\left(\sqrt{K_r^2+2k_r(\Theta)K_r\cos\psi+k_r^2(\Theta)},k_z(\Theta)-K_z\right),\quad(10)$$

where $\left|E_z^{(0)}(K_r,K_z)\right|^2$ is the spectrum intensity of the electromagnetic field of the electron beam, $k_0=\omega/c$, and

$$k_r(\Theta)=\frac{\varepsilon_0k_0\sin\Theta}{\sqrt{\cos^2\Theta+\varepsilon_0\sin^2\Theta}},$$

$$k_z(\Theta)=-\frac{k_0\cos\Theta}{\sqrt{\cos^2\Theta+\varepsilon_0\sin^2\Theta}}$$

are the components of the electromagnetic wave vector.

One can see from (10) that the effectiveness of transition radiation depends on the overlap of the spatial spectra of the modulated electron beam $E_z^{(0)}(K_r,K_z)$ and random plasma inhomogeneities $G(K_r,K_z)$. After the substitution of the explicit function $E_z^{(0)}(K_r,K_z)$, the flux of radiation can be presented as

$$\Pi(R,\Theta)=\frac{(j_ma)^2}{\varepsilon_0^{7/2}cR^2}\frac{k_r^6(\Theta)}{k_0^4}\times$$

$$\times\int\limits_0^\pi d\psi\int\limits_{-\infty}^{\infty}dK_z\int\limits_0^\infty dK_r\frac{\left(K_z^2-k_0^2\right)^2}{(K_z+\kappa)^2+1/L^2}\times$$

$$\times\frac{J_1^2(K_ra)}{K_r\left(K_z^2+K_r^2/\varepsilon_0-k_0^2\right)^2}\times$$

$$\times G\left(\sqrt{K_r^2+2k_r(\Theta)K_r\cos\psi+k_r^2(\Theta)},k_z(\Theta)-K_z\right).\quad(11)$$

The total radiated power of transition radiation can be given by

$$P=2\pi R^2\int\limits_0^\pi\Pi(R,\Theta)\sin\Theta d\Theta.\qquad(12)$$

It is clear that the magnitude of transition radiation is determined by the beam current density, and it depends on the parameters of dusty plasma.

Both integrals (11) and (12) can be numerically calculated.

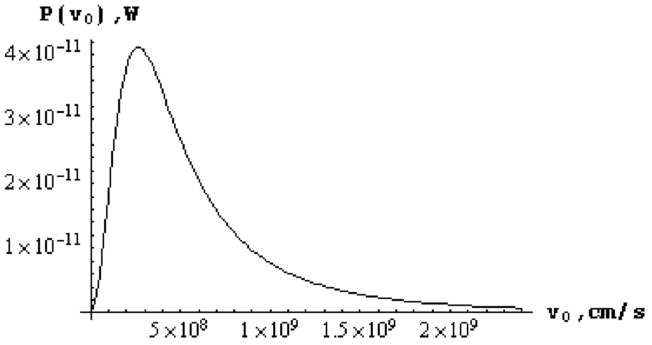

Fig. 2. Total power of transition radiation as a function of the beam velocity

4. Numerical Computations

The numerical computations of transition radiation were made for typical parameters of the modulated electron beam and dusty plasma.

Fig. 1,*a*,*b* shows the transition radiation pattern as a function of the beam velocity v_0 and angle Θ. The angle Θ determines the direction to the observation point. Fig. 1,*a* indicates that there is the maximum of radioemission that is normally directed to the beam motion and lies in the narrow velocity range.

The maximum of transition radiation meets the spatial resonance condition between the propagation vector of the beam electromagnetic field and the wave vector of the spatial spectral intensity of dusty plasma inhomogeneities, when the overlap of the spatial spectra of the beam electromagnetic field and plasma inhomogeneities is more effective. For a non-relativistic beam $(\kappa \gg k_0)$, it can be reached when $\kappa \sim 1/rd$ in the z direction and $a \sim r_d$ in the r direction. Fig. 1,*b* shows the detailed fragment of Fig. 1,*a*, i.e. the radiation pattern for an ultrarelativistic beam that is close to fulfilling the Cherenkov resonance condition.

Discussion

The dependence of the total radiated power on the beam velocity is shown in Fig. 2. It indicates the reduction of radioemission for an ultrarelativistic beam as a result of the spatial resonance condition violation. The effectiveness of transition radiation caused by dipole dust particles is essentially lower than that for spherical dust particles with the same charge due to reducing a perturbation of the plasma concentration.

Dust particles are located randomly and are considered to have the same orientation of dipoles under the model condition. In the case of the random orientation of dipoles, the transition radiation is expected to be even smaller than that obtained in this work.

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ПЕРЕХІДНЕ ВИПРОМІНЮВАННЯ МОДУЛЬОВАНОГО ЕЛЕКТРОННОГО ПУЧКА В ПЛАЗМІ З ОРІЄНТОВАНИМИ ДИПОЛЬНИМИ ПОРОШИНКАМИ

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Резюме

Теоретично розраховано перехідне випромінювання модульованого електронного пучка, який інжектується у плазму з дипольними пиловими частинками. Обчислено діаграму напрямленості та повну потужність випромінювання для типових параметрів експерименту.