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## ON A POSSIBLE MANIFESTATION OF THE FEEDBACK COUPLING BETWEEN GEOMETRY AND MATTER IN THE PHENOMENON OF AN ACCELERATING EXPANSION OF THE UNIVERSE

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UDC 530.12;531.51  
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It is shown that the accelerating expansion of the present-day Universe extracted from the observed luminosity of the type Ia supernovae can be explained by quantum theory which takes into account the feedback coupling between geometry and matter (like that in the Mach's principle). At the same time, the accelerating expansion of the Universe is explained by the influence of a small negative cosmological constant. The comparison with the model with positive cosmological constant (dark energy) which has also obtained its theoretical grounds in the structure of a developed formalism is made. Parameters of the Universe in the states with large quantum numbers are calculated.

### 1. Introduction

The analysis of possible reasons for the observed weak luminosities of the type Ia supernovae (SNe Ia) at the cosmological redshift  $z \approx 0.5$  [1, 2] demonstrates that this phenomenon cannot be put down to the nonstandard evolution of their luminosity, absorption effects of the interstellar dust, gravitational lensing, and other physical processes which are not connected with the overall expansion of the Universe as a whole (see the discussion in [3–5]). In accordance with the principles of general relativity, the observed dimming of the SNe Ia can be interpreted as an evidence of the accelerating expansion of the present-day Universe. Phenomenological models which are used herewith suppose the existence of a nonzero cosmological constant in the Universe treated as the vacuum energy density or hypothetical cosmological liquid with negative pressure (the so-called dark energy [6, 7]). The models of such a type allow to describe the available dataset on the

distance moduli for the SNe Ia depending on redshift by fixing the free parameters (e.g., from a  $\chi^2$  statistic).

Providing a formal agreement with modern astrophysical observational data (SCP [2], HST [4] and WMAP [8] projects), phenomenological models come across difficulties in matters of principle when trying to find a theoretical explanation for the values of their own free parameters and their physical motivation. Among the fundamental problems available here, it is possible to pick out the cosmological constant problem, the task to determine the nature of dark energy, and the puzzle concerning the coincidence between the contributions from dark energy  $\Omega_X \approx 0.7$  and dark matter  $\Omega_M \approx 0.3$  to the total energy density nowadays [4, 5, 9, 10]. It is assumed that, in order to solve them, one should exceed the limits of modern cosmology built on the principles of general relativity [10–12].

In the present article, we analyze the problem of the accelerating expansion of the Universe within the cosmological quantum model [13–16]. The main feature of this approach lies in taking a possible feedback coupling between geometry and matter into account. This coupling should be taken into consideration when one studies the processes, in which the Universe appears as a whole (on the scales that exceed significantly the size of the superclusters of galaxies,  $> 200$  Mpc).

The quantum model of the Universe characterized by the nonzero vacuum energy density,  $\rho_{\text{vac}} \neq 0$ , which takes the feedback coupling between geometry and matter into account, allows us to describe numerically the observed dependence of the distances moduli of the SNe Ia on  $z$  in the whole range of redshift measured

values [5] with the same accuracy that is achieved within the limits of the phenomenological model with positive cosmological constant in classical cosmology. The latter model is also received its theoretical grounds in the structure of the developed formalism.

## 2. Quantum Geometrodynamics in the Minisuperspace Model

### 2.1. Motivation

The available current experimental dataset allows us to state that quantum theory describes adequately the properties of various physical systems. The universal validity of quantum theory demands that the Universe as a whole must obey quantum laws as well. The quest for these laws falls into the realm of research of quantum cosmology. Since gravity dominates on cosmological (very large) scales, any consistent formalism of quantum cosmology must contain the quantum theory of gravity. The driving forces that give reasons for the studies on quantum gravity are not restricted to the aspirations to obtain a unified theory of all interactions, search for mathematical consistency, or determination of the origin and nature of space and time (a review of motivations from different points of view on the problem can be found, e.g., in [17]). There exist the problems which remain unsolved in the standard model of the hot Universe and which, as it seems today, cannot be solved without appeal to quantum cosmology.

It is generally accepted that the early stage of the exponential expansion of the Universe within the framework of the inflationary scenario of classical cosmology withdraws the horizon problem, directs the density parameter  $\Omega$  to unity, and explains the absence of registration acts of monopoles, topological defects, etc. by the very low density of these relics. But a number of problems remains outside the limits of the inflationary model. Among them, there are the mystery of the origin of primordial fluctuations of energy density, explanation of the time arrow, and determination of initial conditions of the evolution of the Universe [18]. Besides this, it is established nowadays that certain problems are solved by the inflationary model in unproper way or these problems can be avoided at all or solved differently [16, 19]. For instance, the horizon problem is, in fact, directly connected with the physical processes in the Planck era [20]. Therefore, one should appeal to quantum gravity in order to solve it. The flatness problem can obtain its solution within the quantum description in the Planck time as well [21].

The inflationary scenario does not allow one to tackle the problem of the presence of singularities in quantum cosmology [19]. Inflation cannot be continued infinitely into the past, mainly because the flat de Sitter metric becomes geodesically incomplete then [22]. The main achievement of the inflationary model is, as is widely accepted, the opportunity to obtain the Universe with current parameters (such as size, energy density contrast, age, etc.) starting from the natural Planck values for different quantities (the so-called *small bang*). However, the same result can be achieved in quantum cosmology as well [13, 14], which does not contradict the inflationary paradigm at this point. Let us note that the inflationary model itself needs quantum cosmology for its motivation, namely in order to ensure the long enough duration of the inflation period (determined by a numerical coefficient in the exponent) which would agree with observations [18].

Cosmology considers the Universe as a system with very large scales, whereas quantum phenomena are typical of microscopic systems. Therefore, the combination of words *quantum* and *cosmology* may seem contradictory. It is commonly accepted that the quantum effects of gravity take place on Planck length scales  $L_P = \sqrt{G\hbar/c^3} \sim 10^{-33}$  cm, whereas the space-time structure on large scales will be classical automatically. Such a point of view is motivated only if a consistent quantum theory of gravity exists, within whose framework the small parameter  $L_P/L$ , where  $L$  is some typical length in the Universe, can be introduced and an appropriate perturbation theory can be constructed, and from the latter it follows that quantum effects are negligibly small for  $L_P/L \ll 1$  [17]. But the assumption that the quantum effects of gravity are small neglects completely the possibility of their non-perturbative character, whose account could provide, in particular, quantum field theory with an appropriate cut-off.

But it should be noticed that quantum effects are not *a priori* restricted to certain scales. Rather the processes of decoherence (when the coherent superposition of states turns into an incoherent one, and interference effects are absent) through the environment can explain why quantum effects are negligible or important for an object under consideration [23–25].

Since nowadays a consistent quantum theory of gravity has not been formulated, the research here is conducted in a few directions. The method of canonical quantization of constraint systems proposed by Dirac [26] provides a basis for the theory developed in this article.

The structure of constraints which describe the evolution of the intrinsic geometry and extrinsic curvature of a spacelike hypersurface in space-time is such that the true dynamical degrees of freedom cannot be distinguished explicitly from quantities which determine the hypersurface. This leads to famous problems in the interpretation of quantum geometrodynamics constructed on the basis of the Wheeler–de Witt equation [27]. The main reason for these difficulties is that there is no predetermined way to identify spacetime events in generally covariant theories (i.e. one cannot measure the metric, but only the geometry).

In order to solve the problem mentioned above, an approach related to the notion of a medium which determines the reference frame<sup>1</sup> (the so-called *reference fluid*) seems promising [30–33]. The problem lies here in finding an appropriate medium (an additional source in the Einstein equations) which, when quantizing in Dirac’s formalism, would lead to a functional Schrödinger-type equation. The variables which describe a medium (the reference frame is considered as a dynamical system) mark spacetime events. They play the role of the canonical coordinates which determine the *embedding* in the surrounding spacetime, while the new constraints turn out to be linear with respect to the momenta canonically conjugate with the medium variables. Such an additional source is introduced in the action and determines, in particular, the time variable. The invariance of the action remains unbroken here.

The replacement of the Wheeler–de Witt equation by the functional Schrödinger-type equation allows one to introduce the positive definite conserved inner product and to advance essentially in constructing a consistent quantum theory of gravity. But the discovery of the corresponding (physical) medium which defines the reference frame is a nontrivial task by itself. In [13–16], this problem was solved in terms of the minisuperspace model. We consider this case below in more details.

## 2.2. Main equations

Just the same as in the ordinary nonrelativistic and relativistic theories, it is possible to assume that the problem of evolution and research into the properties of the Universe as a whole can be reduced to the solution of a partial differential equation which determines

eigenvalues and eigenfunctions of some Hamiltonian-like operator (in the space of generalized variables, whose roles are played by metric tensor components and matter fields). For simplicity, we restrict our study to the case of a minimal coupling between geometry and matter. Taking into account that scalar fields play fundamental roles both in quantum field theory (see, e.g., [34]) and in cosmology of the early Universe [35–37], we assume that the Universe is filled *ab initio* by a primordial matter in the form of a scalar field  $\phi$  with some potential energy density  $V(\phi)$ .

We suppose that the Universe as a whole is homogeneous, isotropic, and spatially flat, and the scalar field  $\phi$  is uniform. The geometry of such a Universe is determined by the famous Robertson–Walker metric [38]. From the principle of least action, it follows the constraint equation [13, 16, 39],  $\delta S/\delta N = 0$ , where  $S$  is the corresponding action functional, and  $N$  is a lapse function that specifies the time reference scale and plays the role of a Lagrange multiplier in the ADM formalism [40], which is the Einstein–Friedmann equation for the  $\binom{0}{0}$  component.

The structure of the constraint is such that the true dynamical degrees of freedom cannot be singled out explicitly. In the model considered, this difficulty is reflected in that the choice of a time variable is ambiguous (the so-called *problem of time*). In order to solve this problem in general relativity, it will be enough to supplement the field equations with a coordinate condition which does not change the Einstein equations themselves, but only specifies the spacetime platform, from which one observes the gravitational field (the enlarged system of constraints is no longer first class, and it is possible to eliminate *non-dynamical* variables). But this method does not allow one to solve the problem of time for a quantum description [30].

Therefore, we will use another approach, in which a coordinate condition is imposed prior to varying the action functional and is included in it with the aid of a Lagrange multiplier. The parametrization of the action functional (see, e.g., [26, 30, 41]) restores its coordinate invariance, expressing it in arbitrary coordinates. At the same time, the *privileged* time coordinate introduced by means of the coordinate condition is adjoined to the field variables and takes the role of the medium variable which determines the reference frame.

We will choose a coordinate condition in the form

$$T' = N, \tag{1}$$

<sup>1</sup>Application of *material reference frames* has a long history. They were used already by Einstein [28] and Hilbert [29], but in a somewhat idealized form which did not take into account a back action of material reference frames on geometry.

where  $T$  is a new field variable (the privileged time coordinate), while the differentiation with respect to an arbitrary variable (conformal time or *arc parameter*)  $\eta$  is denoted by a prime, the parameter  $\eta$  is related to the synchronous proper time  $t$  by the differential equation  $dt = N a d\eta$ , and  $a$  is a cosmological scale factor.

We include the coordinate condition (1) in the action functional with the aid of a Lagrange multiplier  $P$  and obtain the modified action of the minisuperspace model in the conventional form

$$S_{\text{mod}} = \int d\eta [\pi_a a' + \pi_\phi \phi' + P T' - N H], \quad (2)$$

where  $\pi_a$  and  $\pi_\phi$  are the momenta canonically conjugate with the variables  $a$  and  $\phi$ , and

$$H = \frac{1}{2} \left( -\pi_a^2 + \frac{2}{a^2} \pi_\phi^2 - a^2 + a^4 V(\phi) \right) + P \quad (3)$$

is the Hamiltonian to within the multiplier  $N$ . Here and below, we give all relations between dimensionless quantities. The length is taken in units of the modified Planck length  $l_P = \sqrt{2G\hbar/(3\pi c^3)} = 0.744 \times 10^{-33}$  cm, the energy density is measured in units of  $\rho_P = 3c^4/(8\pi G l_P^2) = 1.627 \times 10^{117}$  GeV cm<sup>-3</sup>, and so on.

The variation of action (2) with respect to  $N$  leads to the constraint equation

$$\delta S_{\text{mod}}/\delta N = 0 \quad \Rightarrow \quad H = 0. \quad (4)$$

The parameter  $T$  can be used as an independent variable for the description of the evolution of the Universe both in classical and quantum cosmology [16].

In quantum theory, the constraint equation (4) becomes, in accordance with a procedure proposed by Dirac [26], a constraint on the wavefunction  $\Psi$ ,

$$i \partial_T \Psi = \hat{\mathcal{H}} \Psi, \quad (5)$$

with a Hamiltonian-like operator

$$\hat{\mathcal{H}} = \frac{1}{2} \left( \partial_a^2 - \frac{2}{a^2} \partial_\phi^2 - a^2 + a^4 V(\phi) \right). \quad (6)$$

Here, we have introduced the operators  $P = -i \partial_T$ ,  $\pi_a = -i \partial_a$ , and  $\pi_\phi = -i \partial_\phi$  which satisfy the ordinary canonical commutation relations,  $[T, P] = i$ ,  $[a, \pi_a] = i$ ,  $[\phi, \pi_\phi] = i$ , while others vanish.

The wavefunction  $\Psi$  depends on a cosmological scale factor  $a$ , a scalar field  $\phi$ , and time coordinate  $T$ . One can introduce, at least formally, a positive definite scalar product  $\langle \Psi | \Psi \rangle < \infty$  and specify the norm of a state.

<sup>2</sup>Starting from this value of the energy density, the evolution of the Universe in time can be considered in accordance with the classical conceptions [36].

This makes it possible to define a Hilbert space of physical states and to construct quantum mechanics for the model of the Universe being considered.

Equation (5) has a particular solution with separable variables

$$\Psi = e^{\frac{i}{2} E T} \psi_E, \quad (7)$$

where the wavefunction  $\psi_E$  satisfies the time-independent equation

$$\left( -\partial_a^2 + \frac{2}{a^2} \partial_\phi^2 + U - E \right) \psi_E = 0, \quad (8)$$

and

$$U = a^2 - a^4 V(\phi) \quad (9)$$

can be interpreted as an effective potential.

The function  $\psi_E$  is specified in the space of two variables,  $a$  and  $\phi$ . In the classical approximation, the eigenvalue  $E$  determines the components of the energy-momentum tensor

$$\begin{aligned} \tilde{T}_0^0 &= \frac{E}{a^4}, & \tilde{T}_1^1 &= \tilde{T}_2^2 = \tilde{T}_3^3 = -\frac{E}{3a^4}, \\ \tilde{T}_\nu^\mu &= 0 \quad \text{for } \mu \neq \nu, \end{aligned} \quad (10)$$

which describes, in the case  $E > 0$ , an additional source of the gravitational field in the form of a relativistic matter of an arbitrary nature. Equation (8) formally turns into the Wheeler—de Witt equation for the minisuperspace model [27] in the special case  $E \rightarrow 0$ .

### 2.3. Model of a scalar field

The quantum state  $\psi_E$  depends on the form and numerical value of  $V(\phi)$ . We use the model of a scalar field which slowly (in comparison with the rapid motion with respect to the variable  $a$ ) rolls from some initial value  $\phi_{\text{start}}$  with the Planck energy density  $V(\phi_{\text{start}}) \sim 1^2$  to the equilibrium state  $\phi_{\text{vac}}$  with the energy density  $\rho_{\text{vac}} = V(\phi_{\text{vac}}) \ll 1$ . This constant density determines the cosmological constant  $\Lambda = 3\rho_{\text{vac}}$ . At the next stage of the evolution, the scalar field oscillates with a small amplitude near  $\phi_{\text{vac}}$  under the action of quantum fluctuations. In such a model, the motion with respect to  $\phi$  always will be finite.

The analogous model of a scalar field was considered for the first time in connection with the inflationary

scenario (see, e.g., [36, 37] and references therein)<sup>3</sup>. For the inflationary model, the presence of a minimum of the function  $V(\phi)$  is of great importance. The oscillations of the scalar field near a state of equilibrium with subsequent transfer of energy of these oscillations to *real* particles allow the Universe, which has become empty after the exponential expansion, to be filled with hot matter [36, 43].

#### 2.4. Solution of the time-independent equation

For the positive definite function  $V(\phi)$ , the effective potential  $U$  as a function of  $a$  has the form of a barrier. In this case, the Universe described by Eq. (8) can be both in continuum states with  $E > 0$  and quasistationary ones which correspond to complex values  $E = E_n + i\Gamma_n$ , where  $E_n > 0$ ,  $\Gamma_n > 0$  and  $\Gamma_n \ll E_n$  [14, 16]. Quasistationary states are most interesting, since the Universe in such states can be described by a set of standard cosmological parameters (the Hubble constant, deceleration parameter, mean energy density, density contrast, amplitude of fluctuations of the cosmic microwave background radiation temperature, and so on) [16].

Taking into account that the motion with respect to  $a$  in the early Universe is supposed to be rapid in comparison with a slow variation of the state of the scalar field, we find that the wavefunction  $\psi_E$  of a quasistationary state considered as a function of  $a$  at the fixed field  $\phi$  has a sharp peak and is concentrated mainly in the region limited by the barrier (9) [16, 39]. Then, following Fock [44], one can introduce an approximate function which is equal to the exact wavefunction inside the barrier and vanishes outside it. Taking into account the finite motion with respect to  $\phi$ , this function can be normalized and used in the calculations of the expectation values of observed parameters. Such an approximation does not take into account the exponentially small probability of tunneling through the barrier  $U$  in the region of large values of  $a$ , where  $a^2 V > 1$  [14, 16]. It is valid for the calculations of the mean observed parameters of the Universe within its lifetime in a given quasistationary state<sup>4</sup>, when this state

can be considered as a stationary one. Here, we have a close analogy with the corresponding conclusions of ordinary quantum mechanics [45]. In the region of large values of  $a$  outside the barrier, the WKB approximation is valid [13], and the solution of Eq. (8) can be written in an explicit form [13, 14, 16, 39].

Let us consider the solution of Eq. (8) in the approximation of finite motion with respect to the variables  $a$  and  $\phi$ . It is convenient to expand the wavefunction  $\psi_E$  in the basis of the oscillator functions  $\langle a|n\rangle$  satisfying the equation

$$(-\partial_a^2 + a^2 - \epsilon_n^0)|n\rangle = 0. \quad (11)$$

Here,  $a \geq 0$ ,  $\epsilon_n^0 = 4n + 3$ , and  $n = 0, 1, 2, \dots$  is the number of a state. Such an expansion has the form

$$\psi_E = \sum_n |n\rangle f_n. \quad (12)$$

Functions  $f_n(\phi)$  satisfy the set of differential equations

$$\partial_\phi^2 f_n + \frac{1}{2} \sum_{n'} K_{nn'} f_{n'} = 0 \quad (13)$$

with the kernel

$$K_{nn'}(\phi; E) = \langle n|a^2|n'\rangle [\epsilon_n^0 - E] - \langle n|a^6|n'\rangle V(\phi). \quad (14)$$

In classical theory, the gravitational field is determined by the spacetime metric [46]. According to (11), the states  $\langle a|n\rangle$  will describe geometric properties of the Universe as a whole in quantum theory. The motion with respect to  $a$  can be quantized. The corresponding equidistant spectrum of energy has the form  $\mathcal{E}_n = m_P(N + \frac{1}{2})$ , where  $m_P$  is the Planck mass, and  $N = 2n + 1$  gives the number of elementary quantum excitations of the vibrations of oscillator (11). The mass of elementary excitations of the geometry coincides with the mass of known Markov maximons which are particle-like formations with the Planck mass<sup>5</sup>.

<sup>3</sup>We shall note that, in light of the coincidence problem (the contributions from dark energy and dark matter to the total energy density in the Universe have the same order of magnitude), the model of *quintessence* [it is a scalar field  $\varphi$  of the special type with potential energy density  $\mathcal{V}(\varphi)$  modeled by different functions (see, e.g., review [9])] is widely discussed in the literature. There is a fundamental difference between the quintessence  $\varphi$  and the scalar field  $\phi$  of the model being considered in this article. The field  $\phi$  models the primordial matter which is a source of real matter [42], including the quintessence  $\varphi$  (if it really exists). The question concerning the relation between the fields  $\phi$  and  $\varphi$  goes beyond the scope of this article and will not be considered further.

<sup>4</sup>At  $V \lesssim 10^{-122}$ , this time can reach the values close to the age of our Universe [16].

<sup>5</sup>We recall that the notion of massless gravitons as quanta of the gravitational field was introduced within the theory constructed in the weak gravitational field approximation (gravitational waves). It is obvious that this approximation is not valid for cosmologically significant effects.

### 3. Universe in the States with Large Quantum Numbers

In view of a further application of the developed formalism to the interpretation of astrophysical observational data on our Universe, we consider the cosmological equations obtained above in the approximation of large quantum numbers.

The direct calculations [13,14] demonstrate that, in the quantum model of the Universe with the slow-roll potential energy density  $V(\phi)$ , the first quasistationary state emerges when the density reaches the value  $V = 0.08$ . This state is characterized by finite values of the energy density,  $\rho \sim \rho_P$ , and scalar curvature,  $R \sim l_P^{-2}$ , while the singular state with  $\rho \sim \infty$  and  $R \sim \infty$  is excluded from the consideration as non-physical.

When  $V(\phi)$  decreases to the value  $V \ll 0.1$ , the number of available states of the Universe increases up to  $n \gg 1$ . Before the instant when the scalar field reaches its equilibrium state  $\phi_{vac}$ , the Universe may get into the state with the number  $n \gg 1$ . Really, the origin of new quantum levels and the (exponential) reduction of the width of the states that have emerged earlier lead to the competition between the processes of tunneling through the potential barrier  $U$  from a given  $n$ -th state and the allowed transitions between the states,  $n \rightarrow n \pm i$ , where  $i = 1, 2$  [14,16]. The comparison between the probabilities of these processes demonstrates that the process  $n \rightarrow n + 1$  appears to be most probable. Such transitions are realized at the expense of the energy of the scalar field accumulated in the state  $\phi_{start}$ .

Taking into account an explicit form of the matrix elements  $\langle n'|a^2|n \rangle$  and  $\langle n'|a^6|n \rangle$ , we find that, in the limiting case  $n \gg 1$ , the system of equations (13) is reduced to one equation in the approximation  $f_n \approx f_{n \pm j}$ , where  $j = 1, 2, 3$ . This approximation preserves the orthogonality of states with respect to the quantum number ( $s$ ) that characterizes the field  $\phi$ .

An equation for  $f_n$  as a function of the new variable  $x = \sqrt{m/2} (2N)^{3/4} (\phi - \phi_{vac})$ , which describes a deviation of the field  $\phi$  from the equilibrium value  $\phi_{vac}$ , has the form

$$[\partial_x^2 + z - v(x)] f_n(x) = 0, \tag{15}$$

where we denote  $z = (\sqrt{2N}/m) (1 - E/(2N))$ ,  $v(x) = (2N)^{3/2} V(\phi)/m$ , and  $m$  is some parameter. It is convenient to choose  $m^2 = [\partial_\phi^2 V(\phi)]_{\phi_{vac}} > 0$ . We assume that the density  $V(\phi)$  near the point  $\phi_{vac}$  is a rather smooth function. Then, by expanding  $v(x)$  into a

Taylor series near the point  $x = 0$ , we obtain

$$v(x) = v(0) + x^2 + \alpha x^3 + \beta x^4 + \dots, \tag{16}$$

where

$$\alpha = \frac{\sqrt{2}}{3} m^{-5/2} (2N)^{-3/4} [\partial_\phi^3 V(\phi)]_{\phi_{vac}},$$

$$\beta = \frac{1}{6} m^{-3} (2N)^{-3/2} [\partial_\phi^4 V(\phi)]_{\phi_{vac}}.$$

Since  $N \sim n \gg 1$ , we get  $|\alpha| \ll 1$  and  $|\beta| \ll 1$ , and Eq. (15) with potential (16) can be solved using perturbation theory for stationary problems with a discrete spectrum. As the state of the unperturbed problem, we take the state of a harmonic oscillator described by Eq. (15) with potential (16) for  $\alpha = \beta = 0$ . As a result, we obtain

$$z = 2s + 1 + v(0) + \Delta z, \tag{17}$$

where  $s = 0, 1, 2, \dots$  is the number of states of the field  $\phi$ , and  $\Delta z$  takes into account its self-action (an explicit form of  $\Delta z$  is given in [42]). The spectrum of energy states of the field  $\phi$  has the form  $M' = M + \Delta M$ , where

$$M = m \left( s + \frac{1}{2} \right), \tag{18}$$

and  $\Delta M = m\Delta z/2$ . It can be demonstrated that the following estimation is valid:

$$\frac{\Delta M}{M} \ll 1 \quad \text{at} \quad s > m^{-2}. \tag{19}$$

Hence, it appears that, at large enough values of  $s$ , we can neglect the self-action of the field  $\phi$ . It is reasonable to interpret  $M$  (18) as the amount of matter/energy in the Universe represented in the form of a sum of the elementary quantum excitations of vibrations of the field  $\phi$  near the equilibrium state  $\phi_{vac}$  with masses  $m$ , and  $s$  is the number of such excitations. For instance, for  $m \sim 1$  GeV, condition (19) is satisfied at  $s > 10^{38}$ . Assuming  $s \sim 10^{80}$  (the equivalent number of baryons in our Universe), we obtain a lower restriction on the mass of quantum excitations:  $m > 10^{-21}$  GeV.

Taking into account the relation between  $z$  and  $E$  and relation (17) we obtain the following expression for the eigenvalue:

$$E = 2N - (2N)^2 \rho_{vac} - 2\sqrt{2N} M'. \tag{20}$$

The wavefunction of the Universe in the state with large quantum numbers,  $n \gg 1$ ,  $s \gg 1$ , has the form

$$\psi_E(a, \phi) = \varphi_n(a) f_{ns}(\phi), \tag{21}$$

where

$$\varphi_n(a) = \left( \frac{4}{2N+1} \right)^{1/4} \cos \left( \sqrt{2N+1} a - \frac{\pi N}{2} \right), \quad (22)$$

$$f_{ns}(\phi) = \left( \frac{m(2N)^{3/2}}{2(2s+1)} \right)^{1/4} \times \\ \times \cos \left( \sqrt{(2s+1) \frac{m}{2} (2N)^{3/2} (\phi - \phi_{\text{vac}}) - \frac{\pi s}{2}} \right). \quad (23)$$

These functions are normalized to unity in the intervals  $0 \leq a \leq a_c$  and  $\phi_- \leq \phi \leq \phi_+$  limited by the classical turning points

$$a_c = \sqrt{2N+1}, \quad \phi_{\pm} = \phi_{\text{vac}} \pm \left( \frac{2(2s+1)}{m(2N)^{3/2}} \right)^{1/2}$$

for the corresponding oscillator potentials. Beyond these intervals, the exact wavefunction decreases exponentially. Here, the exact analogy with the normalization of quasiclassical functions in quantum mechanics is traced (see, e.g., [47]).

Taking into account that the mean value of the scale factor  $\langle a \rangle$  in state (21) is equal to

$$\langle a \rangle = \frac{1}{2} \sqrt{2N+1}, \quad (24)$$

we come to the conclusion that  $v(x)$  in Eq. (15) is the potential energy of the scalar field contained in the Universe of the volume  $\sim \langle a \rangle^3$ , and the variable  $x^2$  characterizes a squared deviation of the field  $\phi$  from an equilibrium state in such a volume. Thus, Eq. (15) describes the stationary states which characterize the scalar field  $\phi$  in the Universe as a whole. The quantities  $v(x)$ ,  $x^2$  and  $M$  are its overall characteristics.

Taking (24) into account, condition (20) can be rewritten in the form of a feedback coupling relation between the geometric and energetic characteristics of the Universe

$$\langle a \rangle = M + \frac{E}{4\langle a \rangle} + 4\langle a \rangle^3 \rho_{\text{vac}}, \quad (25)$$

where we discard a small addition  $\Delta M$  and take into consideration that  $N \gg 1$ . Here, the second term on the right-hand side describes the energy of a relativistic matter, while the third term gives the contribution from the vacuum of the scalar field.

Equation (25) can be interpreted as one of the possible implementations of the famous Mach's principle [48]. Indeed, passing to dimensional quantities, we obtain

$$\frac{G \mathcal{M}}{c^2 \mathcal{R}} \sim 1,$$

where  $\mathcal{M}$  and  $\mathcal{R}$  are the measures of mass (without taking the gravitational interaction between bodies into account) and radius of the Universe's observed part. This relation follows from the Lense–Thirring effect in general relativity as well. In this connection, the Universe appears like a huge system which tracks and adjusts its parameters according to the feedback coupling condition (25) (see also [48]).

#### 4. Cosmological Models

Using the relation for mean values of a product of operators [49]

$$\left\langle -\frac{1}{a^4} \partial_a^2 \right\rangle = \left\langle \left( \frac{1}{a} \frac{da}{dt} \right)^2 \right\rangle,$$

where  $t$  is the synchronous proper time, while the averaging is performed over the state  $\psi_E$  normalized in the way indicated above, we can pass from Eq. (8) to the relation between expectation values. Assuming that the mean  $\langle a \rangle$  in such a state determines the scale factor in the classical description in general relativity, we obtain the Einstein–Friedmann equation in terms of mean values

$$\left( \frac{1}{\langle a \rangle} \frac{d\langle a \rangle}{dt} \right)^2 = \langle \rho \rangle - \frac{1}{\langle a \rangle^2}, \quad (26)$$

where

$$\langle \rho \rangle = \frac{2}{\langle a \rangle^6} \langle -\partial_\phi^2 \rangle + \langle V \rangle + \frac{E}{\langle a \rangle^4} \quad (27)$$

is the mean total energy density. In this equation, the dispersion  $\sigma^2 = \langle a^2 \rangle - \langle a \rangle^2$  and the higher order moments with respect to a deviation of  $a$  from its mean value  $\langle a \rangle$  are not taken into account. For the problems considered in the present article, they can be neglected.

The mean total energy density in state (21) equals to

$$\langle \rho \rangle = \gamma \frac{M}{\langle a \rangle^3} + \rho_{\text{vac}} + \frac{E}{\langle a \rangle^4}, \quad (28)$$

where  $\gamma = 193/12$  is a numerical coefficient which appears in the calculation of expectation values of the operators of the kinetic and potential parts of the energy density of the scalar field in expression (27). The mean density (28) is the sum of the energy density connected with matter (in the form of the elementary quantum excitations of vibrations of the scalar field near the

equilibrium state  $\phi_{vac}$ , the vacuum energy density, and the energy density of the relativistic matter.

Taking (28) into account, Eq. (26) can be rewritten in the form of a relation for the Hubble constant  $H = (1/\langle a \rangle) (d\langle a \rangle/dt)$  as a function of the cosmological redshift  $z = a_0/\langle a \rangle - 1$ ,

$$H^2(z)/H_0^2 = \Omega_M(1+z)^3 + \Omega_{vac} + \Omega_R(1+z)^4 + (1-\Omega_0)(1+z)^2, \quad (29)$$

where

$$\Omega_M = \frac{\gamma M}{a_0^3 H_0^2}, \quad \Omega_{vac} = \frac{\rho_{vac}}{H_0^2}, \quad \Omega_R = \frac{E}{a_0^4 H_0^2}$$

are the components of the total energy density  $\Omega_0 = \Omega_M + \Omega_{vac} + \Omega_R$  at  $z = 0$ ,  $a_0 \equiv \langle a \rangle_{z=0}$ , and  $H_0 \equiv H(0)$ . If the quantity  $M$  is assumed to be constant, then expression (29) will describe the evolution of the Universe in the model with a cosmological constant (MCC) represented in terms of mean values. If one establishes a direct correspondence between the classical values and the corresponding mean values, then such a model will be equivalent to the model with a cosmological constant of classical cosmology [9]. In this case, the feedback coupling between geometry and matter given by relation (25) is not taken into consideration.

Account for (25) in (28) leads to the mean energy density <sup>6</sup>

$$\langle \rho \rangle = \frac{\gamma}{\langle a \rangle^2} + \tilde{\rho}_{vac} + \tilde{\rho}_{rad}, \quad (30)$$

where we denote

$$\tilde{\rho}_{vac} = (1-4\gamma)\rho_{vac}, \quad \tilde{\rho}_{rad} = \left(1 - \frac{\gamma}{4}\right) \frac{E}{\langle a \rangle^4}.$$

The dependence of the Hubble constant on  $z$  in the model with feedback coupling (MFC) which has no analog in classical cosmology takes the form

$$H^2(z)/H_0^2 = \tilde{\Omega}_M(1+z)^2 + \tilde{\Omega}_{vac} + \tilde{\Omega}_R(1+z)^4, \quad (31)$$

where the components with tildes

$$\tilde{\Omega}_M = \frac{\gamma-1}{a_0^2 H_0^2}, \quad \tilde{\Omega}_{vac} = (1-4\gamma)\Omega_{vac},$$

$$\tilde{\Omega}_R = \left(1 - \frac{\gamma}{4}\right) \Omega_R$$

<sup>6</sup>Since relation (25) connects the overall characteristics of the Universe, the energy density in the form (30) describes only its properties as a homogeneous system on very large scales. Density (30), for instance, cannot be used in the calculation of fluctuations of the density near the mean value  $\langle \rho \rangle$ , which lead to the formation of visible structures in the Universe. It is necessary to use representation (28) in order to study such processes. (See also [10].)

satisfy the obvious equality

$$\tilde{\Omega}_M + \tilde{\Omega}_{vac} + \tilde{\Omega}_R = 1. \quad (32)$$

The total energy density at  $z = 0$  equals

$$\Omega_0 = 1 + \frac{\tilde{\Omega}_M}{\gamma-1}. \quad (33)$$

Equation (26) with density (30) can be integrated in an explicit form. Neglecting the contribution from the relativistic matter, we find

$$\langle a \rangle = \frac{a_{in}}{2} \left(1 + \sqrt{1 + \zeta^2}\right) \left\{ e^{\sqrt{\tilde{\rho}_{vac}} \Delta t} - \left(\frac{\zeta}{1 + \sqrt{1 + \zeta^2}}\right)^2 e^{-\sqrt{\tilde{\rho}_{vac}} \Delta t} \right\}, \quad (34)$$

where  $\Delta t = t - t_{in}$  is the time interval counted from some initial value  $t_{in}$ , when the scale factor is equal to  $a_{in} \equiv \langle a \rangle_{t=t_{in}}$ ;  $\zeta^2 = (\gamma-1)/(\tilde{\rho}_{vac} a_{in}^2)$ . This yields that

$$\frac{\ddot{\langle a \rangle}}{\langle a \rangle} = \tilde{\rho}_{vac},$$

where the dots denote the second derivative with respect to time  $t$ .

According to (34) in the epoch, when  $\sqrt{\tilde{\rho}_{vac}} \Delta t \ll 1$ , the law of evolution of the Universe must be close to a linear one,  $\langle a \rangle \approx \sqrt{\gamma-1} \Delta t$ . If  $\sqrt{\tilde{\rho}_{vac}} \Delta t \sim 1$  for some redshift range, then the Universe tends, on the average, to an exponential regime during the expansion during this time interval, namely the expansion is realized with acceleration.

Taking into account the available current astrophysical data, it is interesting to apply the theory developed above to the calculation of parameters of our Universe. Below, we consider the matter-dominant era, when the contribution from  $\Omega_R \sim 10^{-4}$  can be neglected.



## 5. Parameters of the Universe

### 5.1. Distance modulus of a source

If one knows  $H(z)$ , it is possible to calculate the luminosity distance  $d_L$  to a source with redshift  $z$ ,

$$d_L = \frac{c}{H_0} \frac{1+z}{\sqrt{\Omega_0 - 1}} \sin \left( \sqrt{\Omega_0 - 1} H_0 \int_0^z \frac{dz}{H(z)} \right) \quad (35)$$

at  $\Omega_0 > 1$ . In the limiting case  $\Omega_0 \rightarrow 1$ , relation (35) describes the luminosity distance in the spatially flat Universe. The distance modulus  $\mu = m - M$  (here,  $m$  and  $M$  are the apparent and absolute magnitudes, respectively) can be calculated with the help of the equation [38]

$$\mu = 5 \lg d_L + 25, \quad (36)$$

where  $d_L$  is taken in megaparsecs.

In Fig. 1, we show the results of the fitting (from the  $\chi^2$  statistics) of the theoretical models (MCC and MFC) for the observed distance modulus  $\mu$  as a function of  $z$  for 156 type Ia supernovae [5] with “high-confidence” spectroscopic and photometric record for the individual source (*gold* SNe Ia in the terminology of [5]). For MCC (29) (dotted curve), the best agreement between the theory and the observed data at  $0.0104 \leq z \leq 1.755$  is achieved for  $\Omega_{\text{vac}} = 0.71$  with  $h = 0.65$ ,  $\chi^2 = 178$ ,  $\chi_{\text{dof}}^2 = 1.16$ . MCC has two free parameters ( $\Omega_0 = 1$  and  $\Omega_{\text{vac}}$ ). In this part, our calculation agrees with the results represented in [5]. The value  $h = 0.65$  coincides with the Hubble constant which was obtained in [1, 3, 4] according to the dataset on supernovae at  $z \lesssim 0.1^7$ .

In the case of MFC (31) (the solid line in Fig. 1), the best agreement between the theory and the SNe Ia observational data is achieved with the value  $\Omega_{\text{vac}} = 0.48$  which corresponds to the density parameter

$$\Omega_{\text{vac}} = -0.0075, \quad (37)$$

with  $h = 0.65$ ,  $\chi^2 = 181$ ,  $\chi_{\text{dof}}^2 = 1.17$ . MFC has only one parameter ( $\tilde{\Omega}_{\text{vac}}$ ). Taking into account the reliability of spectroscopic and photometric measurements of distant sources and their possible adjustment in the future<sup>8</sup>, one can conclude that both models describe the distance modulus of SN Ia considered as a function of redshift  $z$  practically with the same accuracy. The susceptibility

<sup>7</sup>Attachment of additional data from WMAP experiment [8] and HST project [4] gives  $h = 0.71^{+0.04}_{-0.03}$ . Since, in this article, we consider the problem of the accelerating expansion of the Universe in view of the information on SNe Ia only, the value of the Hubble constant which corresponds to these data should be used in self-consistent calculations.

<sup>8</sup>Some of the data for 172 SNe Ia represented in [4] on May, 2003 are not included in the new catalog [5] due to their low confidence with respect to one of the recorded parameters (details see in [5]).

<sup>9</sup>We note that the idea of occupied levels with negative energy [51] leads to a negative energy density as well [52].

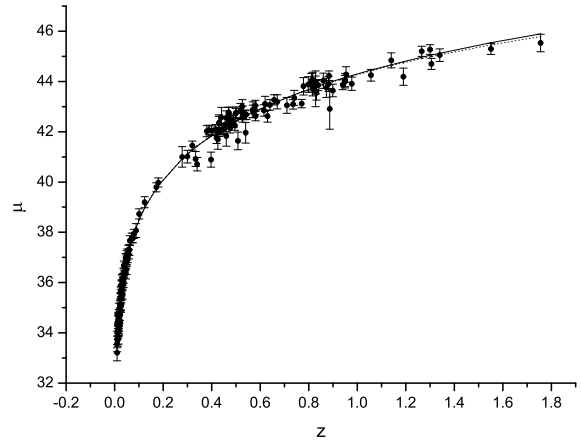


Fig. 1. Dependence of the distance modulus  $\mu$  (36) on the redshift  $z$ . The result of the best fitting (according to the  $\chi^2$  fit statistics) of the quantum model (31) (with the parameter  $\Omega_{\text{vac}} = -0.0075$ ) using SNe Ia data (dots) [5] is shown as the solid line. The model with a cosmological constant (29) is represented as the dotted line (for a flat Universe with  $\Omega_{\text{vac}} = 0.71$ ).

level of the  $\chi^2$  fit statistics can be judged from the following example. For  $h = 0.664$  [50], we obtain  $\Omega_{\text{vac}} = 0.76$  at  $\chi^2 = 184$ ,  $\chi_{\text{dof}}^2 = 1.20$  for MCC and  $\tilde{\Omega}_{\text{vac}} = 0.56$  ( $\Omega_{\text{vac}} = -0.0088$ ) at  $\chi^2 = 189$ ,  $\chi_{\text{dof}}^2 = 1.22$  for MFC. These numbers are close to those mentioned above.

### 5.2. Energetic and geometric scales

In the case of MCC, the density parameter  $\Omega_{\text{vac}} < 0$  and  $|\Omega_{\text{vac}}| \ll 1^9$ . This component of the energy density forms the negative cosmological constant. At (37), it is equal to  $\Lambda = -1.1 \times 10^{-58} \text{ cm}^{-2}$ . This value is in good agreement with the available experimental data,  $|\Lambda| < 10^{-56} \text{ cm}^{-2}$  [9].

The total energy density (33) equals  $\Omega_0 = 1.03$ . This means that, in the redshift range under consideration, the Universe must look like a spatially flat one (to within  $< 4\%$ ). The theoretical value of the density gets within the uncertainty limits for this value,  $\Omega_0 = 1.02 \pm 0.02$  [8], obtained from the combined data of the available astronomical observations.

The scale factor in the current epoch for the obtained value of the parameter  $\tilde{\Omega}_M = 0.52$  turns out to be equal to  $a_0 = 24721$  Mpc. The same value can be obtained directly from solution (34). This number is considerably larger than the corresponding Hubble distance,  $c/H_0 = 4612$  Mpc. Such a correlation between them gives the physical reason why the density  $\Omega_0$  is close to unity.

The feedback coupling relation allows us to estimate the total amount of matter (the sum of bodies' masses in the Universe, being taken separately without regard for their gravitational interaction, according to Eq. (18)) in the present-day Universe. In dimensionless units, this parameter and the scale factor are quantities of the same order of magnitude,

$$M_0 = 1.86 a_0.$$

From here, we obtain  $M_0 = 9 \times 10^{57}$  g in CGS units.

### 5.3. Time scale

The time interval  $\Delta t$  counted from some instant of the expansion  $z = z_{in}$ , taken as a reference point, to another instant fixed by observations with  $z < z_{in}$  can be determined using the famous expression which follows from the definition of the Hubble constant,

$$\Delta t(z) = \int_z^{z_{in}} \frac{dz}{(1+z)H(z)}.$$

Assuming  $z = 0$  and  $z_{in} = 1.755$ , which corresponds to the most distant source SN 1997ff among the known SNe Ia, we obtain  $H_0 \Delta t(0) = 0.74$  for MFC with parameter (37) and  $H_0 \Delta t(0) = 0.71$  for MCC with the parameters  $\Omega_{vac} = 0.71$  and  $\Omega_0 = 1$ . This leads to practically the same time intervals,  $\Delta t(0) = 11.1 \times 10^9$  years for MFC and  $\Delta t(0) = 10.6 \times 10^9$  years for MCC. Supposing that expressions (29) and (31) remain valid up to the singular initial state with  $z = \infty$ , we get  $H_0 t_0 = 1.23$  for MFC and  $H_0 t_0 = 0.97$  for MCC, where  $t_0 = \Delta t(0)|_{z_{in}=\infty}$  is the age of the Universe, or  $t_0 = 18.5 \times 10^9$  years for MFC and  $t_0 = 14.6 \times 10^9$  years for MCC. Since the parameters of both models were fitted in a finite interval of  $z$ , these values can be used for illustrative purposes only. Assume, for example, that, in the interval  $1.755 < z < \infty$ , the Universe is described by MFC with  $\Omega_{vac} = 0$  [39, 49]. Then in this case, we have the numerical values of the dimensionless time parameter and age of the Universe equal to  $H_0 t_0 = 1.10$  and  $t_0 = 16.5 \times 10^9$  years, respectively. These numbers get within the experimental uncertainty range of the corresponding parameters,  $0.72 \lesssim H_0 t_0 \lesssim 1.17$  and

$11 \lesssim t_0 \times 10^{-9} \text{ years}^{-1} \lesssim 17$  [9], obtained in the analysis of old stars under the assumption that stars were formed not earlier than at  $z = 6$ .

MFC predicts the distance to SN 1997ff equal to  $r_0 = c \Delta t(0) = 3396$  Mpc. This value lies between the distances  $r = 3317$  Mpc and  $r = 5245$  Mpc for sources with  $z = 1$  and  $z = 2$ , respectively, calculated in [53] for astrophysical data in the standard model with  $\Omega_0 = 1$  and the normalization  $a_0 = c/H_0$ . Using the known relation  $r(t) = \chi a(t)$ , where  $r(t)$  is the distance to a source at the instant of time  $t$  [46], we obtain the value of the coordinate (angular distance)  $\chi$  for the source SN 1997ff,  $\chi = 0.137$ . That is, it is more than 20 times smaller than the maximum possible value  $\chi_{max} = \pi$ .

### 5.4. Deceleration parameter

Assume that, near  $z = 0$ , the deceleration parameter  $q(z) = -\ddot{a}/(aH^2(z))$  can be approximated by the simple expression

$$q(z) = q(0) + z \left( \frac{dq}{dz} \right)_{z=0}. \quad (38)$$

By determining the free parameters  $q(0)$  and  $(dq/dz)_{z=0}$  from the  $\chi^2$  statistics for SNe Ia, the conclusion can be drawn [5] that the transition between the current epoch of accelerating expansion and the previous phase with cosmic deceleration may take place at  $z_t = 0.46 \pm 0.13$ , where  $q(z_t) = 0$ . At the same time,  $q(0)$  restored by the *gold* sample of SNe Ia lies in the interval from  $-1$  to  $-0.5$  (at the 68 % confidence level) or from  $-1.4$  to  $-0.2$  (at the 99 % confidence level). MCC with the parameters  $\Omega_{vac} = 0.71$  and  $\Omega_0 = 1$  in approximation (38) leads to the values  $q(0) = -0.57$  and  $z_t = 0.46$ .

For MFC (31) in the approximation (38) linear in  $z$ , we obtain  $q(0) = -0.48$  and  $z_t = 0.95$ . In other words, both models predict the accelerating expansion of the Universe in the current epoch and a possible deceleration at  $z > 1$ .

From solution (34), it follows that the inflationary expansion of the Universe may be realized, theoretically, both in the early Universe (with a large enough value of  $\tilde{\rho}_{vac}$ ) and in a later epoch. This conclusion agrees with the point of view which is widespread nowadays that the present-day Universe goes through the period of inflationary expansion again [10, 19]. (In MFC, we have  $\sqrt{\tilde{\rho}_{vac}} \Delta t(0) = 0.5$  in the interval  $0.0104 \leq z \leq 1.755$ , that corresponds to the observed SNe Ia.)

We note that the linear approximation (38) may come to an agreement with the SNe Ia data at  $w < -0.5$  in the equation of state  $p = w\rho$ , where  $p$  is the pressure

(for MCC,  $w = -1$ ). The more refined models which take into account the possible dependence of  $w$  on  $z$  lead to a nonlinear dependence of the deceleration parameter on the redshift when processing the observational data for supernovae [5].

## 6. Conclusion

In the present article, we have demonstrated that the accelerating expansion of the Universe observed nowadays for the SNe Ia data [1, 2, 5] may give the evidence in favour of the presence of a small negative cosmological constant,  $\Lambda = -1.1 \times 10^{-58} \text{ cm}^2$ , and be a direct confirmation of the existence of the feedback coupling between geometry and matter on the scales that exceed significantly the size of the superclusters of galaxies anticipated by the Mach's principle [48]. In the quantum model of the Universe, this principle is not introduced from the outside as an additional condition. It is contained in the theory by itself in the form of the condition on eigenvalues  $E$  (20). The parameters calculated in accordance with the quantum mechanical principles are in good agreement with the observational data. In particular, the quantum model does not contradict the idea of the decelerating expansion of the Universe in the epoch  $z > 1$ . The largest possible distance between sources  $r_{\text{max}} = \pi a_0 = 77663 \text{ Mpc}$  in the Universe described by quantum theory can be compared with the effective particle horizon  $14283 \text{ Mpc}$  calculated in [53] for a spatially flat Universe.

Exceeding the bounds of the aim of the present paper, we note that the quantum cosmological model allows us to solve the dark matter problem and give a natural explanation for the presence of one more additional component in the energy density in the Universe. Here, the matter component of the energy density is formed as a result of the dynamic process, in which the elementary quantum excitations of vibrations of the primordial matter (being considered as the uniform scalar field in this article) decay into the real (visible and invisible) matter mainly under the action of gravity [42, 54]. These excitations themselves are uniformly distributed in space. They practically do not interact between themselves and do not make clusters with the real matter, i.e. they have properties ascribed to the invisible (dark) energy [4, 6, 7], with the exception of a negative pressure, perhaps. Properties of the elementary quantum excitations of vibrations of the primordial matter allow us to identify them with the invisible energy for a better reason than with the invisible matter. The percentage of the *matter* and *energy* components in the total energy

density can be made consistent with observations on the reasonable assumptions about the baryon contribution and the energy released in the decay of quantum excitations [42].

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Received 20.01.05

ПРО МОЖЛИВИЙ ПРОЯВ ЗВОРОТНОГО ЗВ'ЯЗКУ  
ГЕОМЕТРІЇ З МАТЕРІЄЮ У ЯВИЩІ  
ПРИСКОРЕНОГО РОЗШИРЕННЯ  
ВСЕСВІТУ

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Р е з ю м е

Показано, що прискорене розширення сучасного Всесвіту, на користь якого свідчать дані про спостережувану світність наднових типу Ia, можна описати квантовою теорією, яка враховує зворотний зв'язок геометрії з матерією (типу принципу Маха). Прискорене розширення Всесвіту при цьому пояснюється впливом малої від'ємної космологічної сталої. Зроблено порівняння з моделлю з додатною космологічною сталою (темною енергією), яка також отримала своє теоретичне обґрунтування в структурі розвинутого формалізму. Розраховано параметри Всесвіту у станах з великими квантовими числами.