LONGITUDINAL ELECTRIC CONDUCTIVITY OF LAYERED CRYSTALS IN STRONG MAGNETIC FIELD

P.V. GORSKYI

Yu. Fed'kovich Chernivtsi National University
(2, Kotsyubyns'kogo Str., Chernivtsi 58012, Ukraine; e-mail: goodmike@mail.ru)

UDC 537.611.42

Calculations of longitudinal conductivity (LC) along the c-axis of layered crystals are carried out with regard for the dependence of the chemical potential of an electron gas on a quantizing magnetic field. Two cases are considered: (i) the relaxation time is constant, and (ii) it is proportional to the longitudinal velocity of current carriers. It is shown that, in the quasiclassical approach, a relative contribution of the oscillating conductivity term to the total LC is greater in the former case, than in the latter one. When the field is so strong that the chemical potential depends on magnetic field, the total conductivity approaches zero according to the B^{-2} law in case (i) and the B^{-3} law in case (ii), where B is the magnetic induction.

1. Introduction

As is well known, it is sufficiently great ratio of the Fermi energy to a distance between neighboring Landau levels that makes it difficult to reveal Shubnikov–de Haas oscillations in normal metals. There are striking exceptions from this rule, however. They include semimetals (like bismuth), semiconductors, and organometallic high-T_C superconductors with a superlattice, for which the ultraquantum limit is achievable [1—4]. For these materials, even within the scope of the traditional quasiclassical approach, a relative contribution of the oscillating term to the total conductivity becomes noticeable and reaches a few tenths of a per cent. The effect gets more pronounced upon approaching the ultraquantum limit, where the dependence of the Fermi energy on magnetic field begins to play an essential role [5]. What is more, it turns out that, contrary to the case of transverse conductivity (TC), LC is much more sensitive to a way, in which scattering mechanisms are accounted for. This situation is brought about by the following reasons. For the former case, i.e. for the consideration of TC in a strong magnetic field, the scattering effects on a current carrier motion can be taken into account as perturbations in comparison with the magnetic field effect, because, as the field grows, the electron free path gradually approaches and eventually gets equal to the radius of a cyclotron orbit. By contrast, the motion of electrons in the longitudinal direction can be considered as infinite, and thus, the effect of scattering mechanisms becomes much more essential. The aim of this paper is to consider the combined influence of a set of factors, namely the scattering mechanisms and the dependence of the Fermi energy on magnetic field, on LC of layered crystals, with inclusion of the ultraquantum limit as a particular case.

2. Equation that Determines the Dependence of the Chemical Potential (Fermi Energy) on a Quantizing Magnetic Field, and its Solution

To find the chemical potential of an electron gas in a quantizing magnetic field, let us start from the following expression for an electron energy spectrum:

$$\varepsilon\,(n,x) = \mu^* B(2n+1) + W\,(x)\,. \tag{1}$$

Here, $\mu^* = \mu_B m_0/m^*$, μ_B is the Bohr magneton, B — the magnetic induction, n — the number of a Landau level, $W(x)$ — a dispersion law that describes the interlayer electron motion, $x = ak_z$ — a dimensionless component of the electron quasiimpulse perpendicular to layers, a — the distance between the translationally equivalent layers. We perform calculations in the case where

$$W(x) = \Delta(1-\cos x). \tag{2}$$

Here, Δ is the halfwidth of a miniband in the direction perpendicular to layers. Such an expression for the energy spectrum of a layered crystal was proposed in [6], and it is often used for the description of the energy band spectra of semiconductors with a superlattice, intercalated graphite compounds (fullerenes), organometallic high-T_C superconductors in a normal state (synthetic metals), and semiconductor nanoheterostructures [2—4, 7]. We write an equation that determines the chemical potential, proceeding from the condition of constancy of the number of current carriers per unit volume:

$$n_0 = \sum_\alpha g_\alpha f^0\,(\varepsilon_\alpha)\,, \tag{3}$$

where g_α is the statistical weight of an energy level described with a set of quantum numbers α, and $f^0(\varepsilon_\alpha)$ is the Fermi—Dirac distribution function. To transform the equation (3), represent the Fermi—Dirac distribution function as the sum of terms of an infinitely decreasing geometric progression, namely

$$f^0(\varepsilon_\alpha) = 1 + \sum_{l=1}^{\infty} (-1)^l \exp\left(l\frac{\varepsilon_\alpha - \zeta}{kT}\right) \text{ for } \varepsilon_\alpha \le \zeta, \quad (4)$$

$$f^0(\varepsilon_\alpha) = \sum_{l=1}^{\infty} (-1)^{l-1} \exp\left(l\frac{\zeta - \varepsilon_\alpha}{kT}\right) \text{ for } \varepsilon_\alpha \ge \zeta, \quad (5)$$

where ζ is the chemical potential (the Fermi energy).

To sum over the Landau levels, we use the Poisson formula [1] rewritten as

$$\sum_{n=0}^{\infty} F(2n+1) = 0.5 \int_0^{\infty} F(y)dy +$$

$$+\sum_{s=1}^{\infty}(-1)^s \int_0^{\infty} F(y)\cos(\pi s y)dy. \quad (6)$$

After putting spectrum (1) into Eq. (3), accounting for representations (4)—(6), and taking into account the statistical weight of a Landau level, which, after accounting for both the parity of $W(x)$ and spin degeneracy, equals $2eB/\pi ah$ per unit volume, the integration over y can be performed easily. To do this, we use the linear dependence of the current carrier energy on the numbers of Landau levels and put the proper limits while integrating over y and x. For the double series obtained upon the integration over y, the summation over indices, which retained intact after the integration over x, is carried out with the use of the expression

$$\sum_{m=1}^{\infty} (-1)^{m-1} \frac{m^2}{m^2+\beta^2} = \frac{\pi\beta}{2\sinh(\pi\beta)} \quad (7)$$

which is valid for all values of β [8, 9].

After transformations, Eq. (3) for the determination of ζ takes the form

$$n_0 = \frac{4m^*}{ah^2} \int_{W(x)\le\zeta} [\zeta - W(x)]dx +$$

$$+\frac{8m^*\pi kT}{ah^2} \sum_{l=1}^{\infty} \frac{(-1)^l}{\mathrm{sh}(\pi^2 lkT/\mu^* B)} \times$$

$$\times \int_{W(x)\le\zeta} \sin\left(\pi l \frac{\zeta - W(x)}{\mu^* B}\right) dx + \frac{eB}{\pi ah} \times$$

$$\times \sum_{l=1}^{\infty} \frac{(-1)^{l-1}}{\mathrm{sh}(\mu^* Bl/kT)} \left[\int_{W(x)\le\zeta} \exp\left(l\frac{W(x)-\zeta}{kT}\right) dx + \right.$$

$$\left. + \int_{W(x)\ge\zeta} \exp\left(l\frac{\zeta - W(x)}{kT}\right) dx \right]. \quad (8)$$

This expression allows the limit transitions to the zero magnetic field and to the case of a nondegenerate gas, where the summation over the Landau levels can be performed directly. However, we will concentrate on the cases of a highly degenerated gas and a quantizing magnetic field, where Eq. (8) consists of two first terms only and the non-oscillating (as a function of magnetic field) term can be considered as exponentially small. Then, after the transformation of the integrand in the oscillating term by means of its expansion in terms of the Bessel functions of integer index and real argument [9], Eq. (2) for the $W(x)$ function can be written explicitly as

$$n_0 = \frac{4m^*\Delta}{ah^2}\left[(\gamma-1)\arccos(1-\gamma) + \sqrt{2\gamma-\gamma^2}\right] +$$

$$+\frac{8m^*\pi kT}{ah^2} \sum_{l=1}^{\infty} \frac{(-1)^l}{\mathrm{sh}(\pi^2 lkT/\mu^* B)} \left\{ \sin\left(\pi l\frac{\zeta-\Delta}{\mu^* B}\right) \times \right.$$

$$\times \left[\arccos(1-\gamma) J_0\left(\frac{\pi l\Delta}{\mu^* B}\right) + \right.$$

$$\left. + \sum_{r=1}^{\infty} (-1)^r \frac{\sin(2r\arccos(1-\gamma))}{r} J_{2r}\left(\frac{\pi l\Delta}{\mu^* B}\right) \right] +$$

$$+\cos\left(\pi l\frac{\zeta-\Delta}{\mu^* B}\right) \times$$

$$\left. \times 2\sum_{r=0}^{\infty}(-1)^r \frac{\sin[(2r+1)\arccos(1-\gamma)]}{2r+1} J_{2r+1}\left(\frac{\pi l\Delta}{\mu^* B}\right) \right\},$$

$$\text{for} \quad 0 \le \gamma \le 2, \quad (9)$$

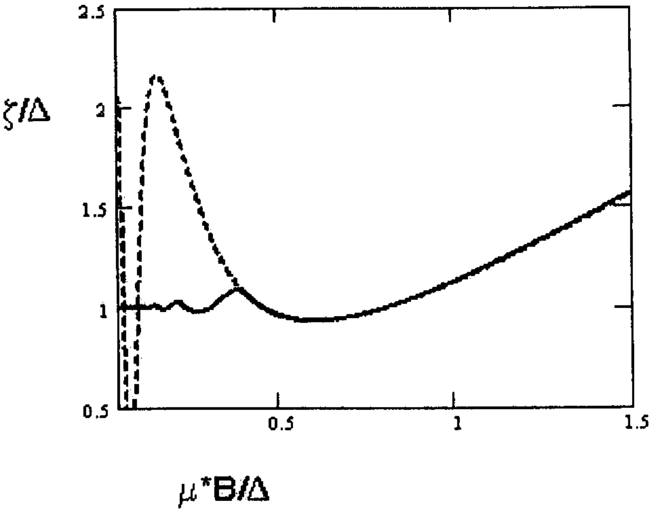

Fig. 1. Field dependence of the chemical potential of a gas of current carriers at $\gamma = 1$ (the dashed line corresponds to the asymptotic solution)

or

$$n_0 = \frac{4m^*\pi\Delta}{ah^2}(\gamma - 1) + \frac{8m^*\pi^2 kT}{ah^2}\sum_{l=1}^{\infty}\frac{(-1)^l}{\mathrm{sh}\left(\pi^2 lkT/\mu^* B\right)}\times$$

$$\times \sin\left(\pi l\frac{\zeta - \Delta}{\mu^* B}\right) J_0\left(\frac{\pi l\Delta}{\mu^* B}\right), \quad \text{for} \quad \gamma \geq 2. \qquad (10)$$

where

$$\gamma = \zeta/\Delta. \qquad (11)$$

In the case where the temperature equals the absolute zero, and a magnetic field is not applied, the concentration of current carriers can be expressed through the Fermi energy ζ_0 of the electron gas as

$$n_0 = \frac{4m^*\Delta}{ah^2}\left[(\gamma_0 - 1)\arccos(1 - \gamma_0) + \sqrt{2\gamma_0 - \gamma_0^2}\right],$$

$$\text{for} \quad \gamma_0 \leq 2 \quad , \qquad (12)$$

or

$$n_0 = \frac{4\pi m^*\Delta}{ah^2}(\gamma_0 - 1), \ \text{for } \gamma_0 \geq 2, \qquad (13)$$

where

$$\gamma_0 = \zeta_0/\Delta. \qquad (14)$$

Since the series in the Bessel functions of integer index and real argument, $J_k(...)$, converge rapidly, Eqs. (9)—(10), along with expressions (12)—(13) taken into account, can be solved numerically. The example of such a solution is shown in Fig. 1 by a solid line. As can be seen from the figure, in the case where magnetic fields are so weak that the quasiclassical approach is applicable, the chemical potential weakly depends on magnetic field. However, as the ultraquantum limit is approached, the amplitude of oscillations of the chemical potential grows. Almost the linear character of the dependence of the chemical potential on magnetic field, observed at sufficiently high values of B, results from the fact that, within the field range mentioned, the asymptotic expression

$$\zeta = \mu^* B + \Delta\left(1 - \cos\left(\frac{\varphi(\gamma_0)\Delta}{2\mu^* B}\right)\right) \qquad (15)$$

is valid, where

$$\varphi(\gamma_0) = (\gamma_0 - 1)\arccos(1 - \gamma_0) + \sqrt{2\gamma_0 - \gamma_0^2},$$

$$\text{for} \quad 0 \leq \gamma_0 \leq 2, \qquad (16)$$

and

$$\varphi(\gamma_0) = \pi(\gamma_0 - 1), \ \text{for } \gamma_0 \geq 2. \qquad (17)$$

It is seen from Eq. (15) that, as the field grows in the ultraquantum limit, the distance between the bottom of the only Landau subband, which is filled, and the Fermi level decreases as a result of the increase in the statistical weight of a Landau level. The character of the dependence of the chemical potential (Fermi energy) on magnetic field is also determined by the dispersion law (2). For comparison, the asymptotic solution of Eq. (15) is shown in Fig. 1 by the dashed line. It is clearly seen that both the lines almost coincide after the last oscillation maximum. This means that, in the vicinity of the ultraquantum limit, the degree of filling of the only Landau subband decreases upon the field increase, provided that the concentration of current carriers is constant. This fact should be taken into account in the calculations of LC.

3. Calculation of LC and Discussion of the Results

To calculate LC in the quantizing magnetic field, we use the Kubo formula in the relaxation time approximation [10] which can be rewritten as

$$\sigma_{zz} = e^2\frac{\partial}{\partial\zeta}\sum_{\alpha}\tau_\alpha \nu_{z\alpha}^2 g_\alpha f^0(\varepsilon_\alpha), \qquad (18)$$

where τ_α is the relaxation time of the longitudinal quasiimpulse, and $v_{z\alpha} = 2\pi a W'(x)/h$ is the longitudinal component of the electron velocity. As concerns the rest of notations, they are either explained above, or generally accepted.

The further calculations can completely be carried out in the general form provided that the relaxation time is invariable in the quantum numbers or depends only on the longitudinal quasiimpulse, i.e. $\tau_\alpha \equiv \tau(x)$. Then, after making use of Eq. (8), we obtain the following general expression for LC:

$$\sigma_{zz}\left(B\right) = \sigma_0 + \sigma_{os}\left(B\right) + \sigma_{mr}\left(B\right). \qquad (19)$$

Here,

$$\sigma_0 = \frac{16\pi^2 e^2 m^* a}{h^4} \int\limits_{W(x)\le\zeta} \tau\left(x\right) \left|W'\left(x\right)\right|^2 dx, \qquad (20)$$

$$\sigma_{os}(B) = \frac{32\pi^2 e^2 m^* a}{h^4} \sum_{l=1}^{\infty} (-1)^l f_l^\sigma \times$$

$$\times \int\limits_{W(x)\le\zeta} \tau(x) \left|W'(x)\right|^2 \cos\left(\pi l \frac{\zeta - W(x)}{\mu^* B}\right) dx, \qquad (21)$$

$$\sigma_{mr}(B) = \frac{16\pi^2 e^2 m^* a}{h^4} \sum_{l=1}^{\infty} (-1)^l h_l^\sigma \times$$

$$\times \left[\int\limits_{W(x)\le\zeta} \tau(x) \left|W'(x)\right|^2 \exp\left(l \frac{W(x) - \zeta}{kT}\right) dx - \right.$$

$$\left. - \int\limits_{W(x)\ge\zeta} \tau(x) \left|W'(x)\right|^2 \exp\left(l \frac{\zeta - W(x)}{kT}\right) dx \right]. \qquad (22)$$

In expressions (20)—(22), the integration is fulfilled only over the positive values of x. In addition, the following notations are introduced:

$$f_l^\sigma = \frac{\pi^2 lkT/\mu^* B}{\mathrm{sh}\left(\pi^2 lkT/\mu^* B\right)}, \qquad (23)$$

$$h_l^\sigma = \frac{\mu^* Bl/kT}{\sinh\left(\mu^* Bl/kT\right)}. \qquad (24)$$

These expressions also allow making transitions to the zero magnetic field and to the case of a nondegenerate gas of current carriers. In addition, as becomes clear from Eq. (21), the oscillations of conductivity exist only in a degenerate electron gas, since the integral in (21) taken over x turns into zero in the case of a nondegenerate one.

Utilizing the dispersion law (2), we perform detailed calculations in two cases: (i) relaxation time is invariable in the quantum numbers, and (ii) it is proportional to the longitudinal electron velocity. The former approximation is often used for the description of the scattering of carriers by acoustic phonons in the crystals with a superlattice [11, 12] in the approximation of the strong quasi-two-dimensionality of the energy spectrum of current carriers at high temperatures, $\Delta < kT$. The utilization of the latter one is more pertinent in the case of low temperatures ($\Delta \gg kT$), i.e. where the Shubnikov—de Haas effect is strongly pronounced even in the quasiclassical approximation. It is approximation (ii) that explicitly takes into account the anisotropy of scattering, particularly the fact that, in the anisotropic case, the small phonon vectors that are tangent to the Fermi surface (FS) don't form a small angle with that axis of FS which is parallel to the direction of the magnetic field. In addition, near the ultraquantum limit, where the contribution of the transitions between Landau subbands is small, this approximation is valid for both the induced and spontaneous scatterings of carriers by acoustic phonons, as well as for the scattering by impurities and static defects [13], for which the relaxation time is inversely proportional to the density of states per one Landau subband. In the case where the magnetic field is weak, the modeled relaxation time, being invariable in quantum numbers, will be considered as isotropic and obeying the T^{-5} law [13]. At the same time, only the σ_0 and $\sigma_{os}(B)$ terms will be accounted for in what follows, since, under the conditions where the Shubnikov — de Haas effect is strongly pronounced, the $\sigma_{mr}(B)$ term is expected to be exponentially small. In the case where $\tau(x) = \tau_0$, the components of the total conductivity read

$$\sigma_0 = \frac{8\pi^2 e^2 m^* a \tau_0 \Delta^2}{h^4} \left(C_0 - C_2\right), \qquad (25)$$

$$\sigma_{os}(B) = \frac{16\pi^2 e^2 m^* a \tau_0 \Delta^2}{h^4} \sum_{l=1}^{\infty} (-1)^l f_l^\sigma \times$$

$$\times \left\{ \cos\left(\pi l \frac{\zeta - \Delta}{\mu^* B}\right) \left[(C_0 - C_2) J_0\left(\frac{\pi l \Delta}{\mu^* B}\right) + \right.\right.$$

$$\left. + \sum_{r=1}^{\infty} (-1)^r (2C_{2r} - C_{2r+2} - C_{2r-2}) J_{2r}\left(\frac{\pi l \Delta}{\mu^* B}\right) \right] -$$

$$- \sin\left(\pi l \frac{\zeta - \Delta}{\mu^* B}\right) \sum_{r=0}^{\infty} (-1)^r (2C_{2r+1} - C_{2r+3} -$$

$$-C_{|2r-1|})J_{2r+1}\left(\frac{\pi l\Delta}{\mu^*B}\right)\Bigg\}. \quad (26)$$

Here, the coefficients C_m are determined as

$$C_0 = \arccos(1-\gamma), \quad (27)$$

$$C_m = \frac{\sin mC_0}{m}, \text{ for } m \neq 0. \quad (28)$$

The above expressions are valid under the condition that FSs are closed, i.e. that $0 \leq \gamma \leq 2$. In the case where FSs are open, expressions (25) and (26) turn into the following ones:

$$\sigma_0 = \frac{8\pi^3 e^2 m^* a\tau_0\Delta^2}{h^4}, \quad (29)$$

$$\sigma_{os}(B) = \frac{16\pi^3 e^2 m^* a\tau_0\Delta^2}{h^4}\sum_{l=1}^{\infty}(-1)^l f_l^\sigma \times$$

$$\times\cos\left(\pi l\frac{\zeta-\Delta}{\mu^*B}\right)\left(J_0\left(\frac{\pi l\Delta}{\mu^*B}\right)+J_2\left(\frac{\pi l\Delta}{\mu^*B}\right)\right). \quad (30)$$

When $\gamma = 1$, we have:

$$\sigma_0 = \frac{4\pi^3 e^2 m^* a\tau_0\Delta^2}{h^4}, \quad (31)$$

$$\sigma_{os}(B) = \frac{8\pi^3 e^2 m^* a\tau_0\Delta^2}{h^4}\times$$

$$\times\sum_{l=1}^{\infty}(-1)^l f_l^\sigma\left(J_0\left(\frac{\pi l\Delta}{\mu^*B}\right)+J_2\left(\frac{\pi l\Delta}{\mu^*B}\right)\right). \quad (32)$$

In the case where $\tau(x) = C|W'(x)|$, i.e. the relaxation time is proportional to the longitudinal velocity of an electron, the components of the total conductivity determined from the general expressions (20) and (21) read

$$\sigma_0 = \frac{16\pi^2 e^2 m^* aC}{h^4}\left(\zeta^2\Delta - \frac{\zeta^3}{3}\right), \quad (33)$$

$$\sigma_{os}(B) = \frac{64\pi^2 e^2 m^* aC}{h^4}\sum_{l=1}^{\infty}(-1)^l f_l^\sigma\left[\left(\frac{\mu^*B}{\pi l}\right)^3\times\right.$$

$$\times\sin\left(\frac{\pi l\zeta}{\mu^\cdot B}\right) - \Delta\left(\frac{\mu^*B}{\pi l}\right)^2\cos\left(\frac{\pi l\zeta}{\mu^\cdot B}\right)+$$

$$\left.+(\Delta-\zeta)\left(\frac{\mu^*B}{\pi l}\right)^2\right]. \quad (34)$$

provided that $0 \leq \gamma \leq 2$. Otherwise, when $\gamma \geq 2$, σ_0 and σ_{os} are expressed as

$$\sigma_0 = \frac{64\pi^2 e^2 m^* aC\Delta^3}{3h^4}, \quad (35)$$

$$\sigma_{os}(B) = \frac{64\pi^2 e^2 m^* aC}{h^4}\sum_{l=1}^{\infty}(-1)^l f_l^\sigma\left\{\left(\frac{\mu^*B}{\pi l}\right)^3\times\right.$$

$$\times\left[\sin\left(\frac{\pi l\zeta}{\mu^*B}\right)-\sin\left(\pi l\frac{\zeta-2\Delta}{\mu^*B}\right)\right]-$$

$$\left.-\Delta\left(\frac{\mu^*B}{\pi l}\right)^2\left[\cos\left(\frac{\pi l\zeta}{\mu^*B}\right)+\cos\left(\pi l\frac{\zeta-2\Delta}{\mu^*B}\right)\right]\right\}. \quad (36)$$

If the condition of applicability of the quasiclassical approach, which turns in the considered case into the inequality $\Delta/\mu^*B \gg 1$, is fulfilled, then Eqs. (26) and (30), with only the first uncompensated terms in the asymptotic expansion of the Bessel functions being kept, can be rewritten as a single simpler expression

$$\sigma_{os}(B) = \frac{16\pi\sqrt{2}\tau_0 e^2 m^* a\Delta^{1/2}(\mu^*B)^{3/2}}{h^4}\times$$

$$\times\sum_{l=1}^{\infty}(-1)^1 l^{-3/2} f_l^\sigma\left[\sin\left(\frac{\pi l\zeta}{\mu^*B}-\frac{\pi}{4}\right)-\right.$$

$$\left.-\Theta(\zeta-2\Delta)\sin\left(\frac{\pi l(\zeta-2\Delta)}{\mu^*B}+\frac{\pi}{4}\right)\right], \quad (37)$$

where $\Theta(...)$ is a Heaviside unit function. For expressions (34) and (36), however, the quasiclassical approach implies the reduction only to cosinusoidally oscillating terms. Thus, it is seen that the relative contribution of oscillations to the total LC depends to a large extent on how the relaxation time is simulated. It follows from Eqs. (34), (36), and (37) that, within the scope of applicability of the quasiclassical approach, the relative contribution of oscillations to the total conductivity is proportional to $(\mu^*B/\Delta)^{3/2}$ in the case of isotropic scattering and to $(\mu^*B/\Delta)^2$ for anisotropic scattering. This difference originates from the following. In the latter case, the relaxation time on the extreme sections of FS turns to zero. Thus, in the neighborhood of extreme points, $x_{\rm ex}$, the proportionality $\tau(x)(W'(x))^2 \propto (x-x_{\rm ex})^3$ is valid. In the former case, however, another relation, namely $\tau(x)(W'(x))^2 \propto (x-x_{\rm ex})^2$, is true. These dependences can be used for a preliminary examination of the validity of both the ways used to simulate the relaxation times. The oscillation frequencies are the same in both cases, and they depend on the areas of extreme sections of FS which are made by the planes perpendicular to the field.

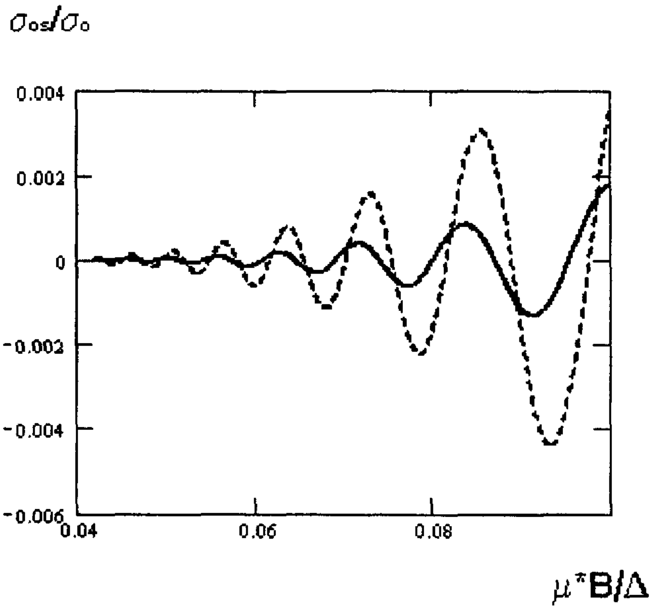

Fig. 2. Field dependence of the oscillating term of LC in weak (quasiclassical) magnetic fields (the dashed line describes the case where the relaxation time is constant)

In particular, the frequencies

$$F_l = \zeta/2\mu^* \tag{38}$$

are related to the maximal section of FS made by the plane $k_z = 0$, whereas the frequencies

$$F_l = |\zeta - 2\Delta|/2\mu^* \tag{39}$$

are associated with the minimal sections made by the planes $k_z = \pm\pi/a$. However, according to the general expression (21), $\sigma_{os}(B)$ depends on ζ smoothly. Thus, since no extreme section corresponding to frequencies (39) exists in the case where $\zeta < 2\Delta$, the oscillations interfere in such a way that this results in the appearance of an additional monotonous component. As follows from expression (34), this component is present explicitly in the case of anisotropic scattering. It is this component that turns into an additional oscillating term at $\gamma > 2$, which is not taken into account in the traditional quasiclassical approach.

It is evident from Fig. 2 that, within the range of the validity of the quasiclassical approach, the oscillation frequencies coincide for both the ways of simulation of the relaxation time, but their phases differ from each other by $\pi/4$, and, in the case where the relaxation time is constant, a the relative contribution of oscillations is greater.

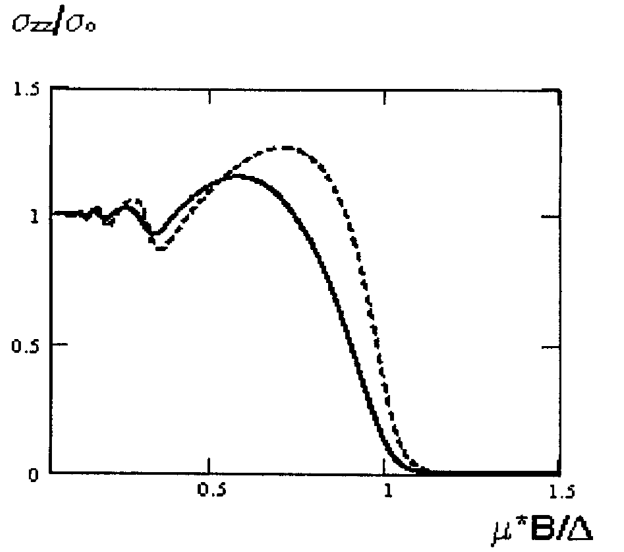

Fig. 3. Field dependence of the total conductivity in strong (ultraquantum) fields at $\gamma = 1$ calculated without taking account of the field dependence of the chemical potential (the dashed line corresponds to the case where relaxation time is constant)

Fig. 3 shows the total conductivity, which is normalized to σ_0, of a layered crystal at high magnetic fields, calculated under the condition that the field dependence of the chemical potential is neglected. In this case where the scattering is isotropic (dashed line), the total conductivity depends on magnetic field weaker, as compared to the case of anisotropic scattering (solid line). As follows from the general expressions (19)—(24), the total conductivity goes to zero at a sufficiently strong magnetic field.

However, it is in the case of strong magnetic fields that a decrease in the degree of the filling of a single Landau subband should be taken into account. To do this, the substitution $\gamma \to \gamma' = \arccos\left[1 - (\zeta - \mu^* B)/\Delta\right]$ should be made in the modulation coefficients C_m in the field range where $\zeta - \mu^* B \leq 2\Delta$. Making such a change of variables keeps Eqs. (30), (35), and (36) inalterable, since they deal with the open FSs, but affects Eqs. (33) and (34) which eventually acquire the forms:

$$\sigma_0(B) = \frac{16\pi^2 e^2 m^* aC}{h^4}\left[(\zeta - \mu^* B)^2 \Delta - \frac{(\zeta - \mu^* B)^3}{3}\right], \tag{40}$$

$$\sigma_{os}(B) = \frac{64\pi^2 e^2 m^* aC}{h^4}\sum_{l=1}^{\infty}(-1)^l f_l^{\sigma}\times$$

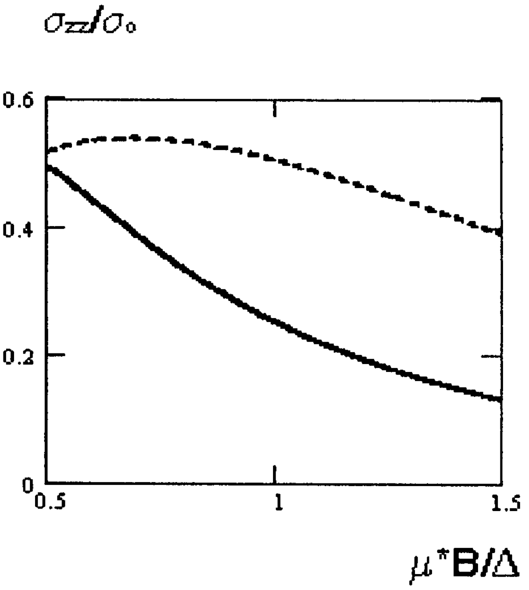

Fig. 4. Field dependence of the total conductivity at $\gamma = 1$ calculated with taking account of the FS contraction along the magnetic field direction

$$\times\left[\left(\frac{\mu^* B}{\pi l}\right)^3 \sin\left(\frac{\pi l\zeta}{\mu^* B}\right) - \Delta\left(\frac{\mu^* B}{\pi l}\right)^2 \cos\left(\frac{\pi l\zeta}{\mu^\cdot B}\right) + \right.$$

$$\left. +(\Delta - \zeta + \mu^* B)(-1)^l\left(\frac{\mu^* B}{\pi l}\right)^2\right]. \tag{41}$$

As a consequence of this, all components of the longitudinal conductivity turn into zero in both the cases at $\zeta = \mu^* B$. But the equality to zero is not the identical equality, but, as follows from Eq. (17), it is an asymptotic one. This is evident from Fig. 4, where the total conductivity of a layered crystal in a range of strong magnetic fields is shown for both the cases. The oscillating and non-oscillating parts of the conductivity can no longer be separated from each other, and the condition for a transition from closed to open surfaces acquires the form $\zeta - \mu^* B \geq 2\Delta$.

We now estimate the order of a value of σ_0 at $\zeta = \Delta$ for some cases of the current carrier scattering by the deformation potential of acoustic phonons under the condition that σ_0 is nearly constant. For example, in the case where the quasiclassical approach is valid and $\tau(x) = \tau_0$, Eq. (31) takes the form

$$\sigma_0 = \frac{4h^3 \rho s_0^6 k_0^3 e^2 m^* a\Delta^2}{62\pi^3 \Gamma(5)\,\zeta(5)\, m_{es}^* \Xi^2 (kT)^5} \tag{42}$$

after the utilization of the expression for transport time adopted from [13]. Here, $\Gamma(...)$ is the gamma function ($\Gamma(5) = 24$), $\zeta(...)$ — the Riemann zeta function ($\zeta(5) \approx 1.037$), ρ — the crystal density, s_0 — the sound velocity in a crystal, Ξ — the effective constant of the deformation potential, k_0 — the equivalent radius of the Fermi sphere, and m_{es}^* — the equivalent effective mass of current carriers located on it. The last two parameters are introduced to allow the consideration of a case where the current carrier scattering is isotropic. They are determined from the condition that the concentration of current carriers and their Fermi energy are the same as in a real crystal, i.e. from the expressions

$$k_0 = \sqrt[3]{\frac{12\pi^2 m^* \Delta}{ah^2}}, \tag{43}$$

$$m_{es}^* = \frac{h^2 k_0^2}{8\pi^2 \Delta} \tag{44}$$

for $\zeta = \Delta$. Given $\Delta = 0.01$eV, $m^* = 0.01 m_0$, $a = 1$ nm, $\rho = 5 \cdot 10^3$ kg/m^3 , $s_0 = 5 \cdot 10^3$ m/s, $\Xi = 10$ eV, and $T = 3$ K, we obtain $\sigma_0 = 1.65 \cdot 10^5\ \Omega^{-1}\cdot$m^{-1}.

If the anisotropy of scattering is taken into account [13], then, according to (33), in the case where $\zeta = \Delta$ and

$$C = \frac{4h^5 k_f^2 \rho s_0^6 a}{31\Gamma(5)\,\zeta(5)\,(\pi kT)^5\, \Xi^2}, \tag{45}$$

we get

$$\sigma_0 = \frac{32e^2 h m^* \rho s_0^6 \Delta^3}{93\pi\Gamma(5)\,\zeta(5)\,(kT)^5\, \Xi^2}, \tag{46}$$

which equals $6.99\cdot 10^5\ \Omega^{-1}\cdot$m^{-1} for the same numerical parameters as used above. Thus, the conductivity is greater in the anisotropic case, and this mainly results from a sharp decrease in the transport factor caused by a strong elongation of FS along the field direction. It is noteworthy that, in Eq. (45), the notation $k_f = \arccos(1 - \zeta/\Delta)/a$ is used.

Consider the situation where the spontaneous scattering dominates in strong magnetic fields so that the transitions between Landau subbands are "turned off" [13]. In the case where

$$C = \frac{3h^3 a\rho s_0^4}{4\pi^2 \Xi^2 (kT)^3}, \tag{47}$$

we get

$$\sigma_0 = \frac{8e^2 \rho s_0^4 a^2 m^* a^2 \Delta^3}{h\Xi^2 (kT)^3}, \tag{48}$$

that equals $1.98\cdot 10^5\ \Omega^{-1}\cdot$m^{-1} for the above numerical parameters. In this case, the T^{-3} law can be explained by the fact that, under the conditions that the transitions between Landau subbands are "turned off", the phonons effective from the viewpoint of scattering don't fill up a three-dimensional volume in the k-space. Instead, they are located along the contours formed as a result of the sections of Landau tubes by the planes perpendicular to the direction of the magnetic field. Thus, their number is proportional to T, but not to T^3, as in the case of weak fields. What is more, for the spontaneous scattering, as well as in the case of weak fields, the transport factor is proportional to T^2. Thus, it is seen that, in the case of low temperatures and weak magnetic fields, the neglect of a scattering anisotropy results in the underestimation of the conductivity. However, when the field is sufficiently strong so that the magnetic field dependence of the chemical potential becomes noticeable, the neglect of a scattering anisotropy leads to a considerable overestimation of the absolute value of LC.

Finally, we determine the asymptotic laws for the approach of the total LC to zero. To do this, we write the asymptotic formula (15) as

$$\zeta = \mu^* B + \frac{\varphi(\gamma_0)\,\Delta^3}{(\mu^* B)^2} \tag{49}$$

and substitute it into expressions (25) and (26). Taking into account that, at high values of B, the relation $\sum_{l=1}^{\infty} f_l^\sigma \propto B$ is valid, we obtain $\sigma_{zz} \propto B^{-2}$ in the case of isotropic scattering. As directly follows from Eqs. (40) and (41), $\sigma_{zz} \propto B^{-3}$ in the case of anisotropic scattering.

It should be noted that such results cannot be obtained within the scope of the traditional approach, since this approach is inapplicable to non-quasiclassical magnetic fields, especially in the case where the magnetic field dependence of the chemical potential is accounted for. It is also noteworthy that the authors of work [7] considered only the conductivity of layered conductors in crossed magnetic fields in the quasiclassical approach in the case of open FSs.

4. Conclusions

It is shown in the present paper that, within the scope of applicability of the quasiclassical approach, the account of the anisotropy of the current carrier scattering by the deformation potential of acoustic phonons in layered crystals gives rise to both a reduction of the relative contribution of the oscillating term to the total LC along the c-axis and an increase in the absolute value of the total conductivity. At the same time, in the quasiclassical approach, the oscillation amplitude is mainly proportional to $B^{3/2}$ in the case of isotropic scattering and to B^2 for the anisotropic one. Near the ultraquantum limit, however, when the magnetic field dependence of the chemical potential becomes noticeable, the account of both the scattering anisotropy and the fact that the transitions between Landau subbands are "turned off" leads to a reduction of the absolute value of conductivity, in comparison with the isotropic case. In the case of ultraquantum fields, LC approaches zero according to the B^{-2} law when the scattering is isotropic and to the B^{-3} law when it is anisotropic, provided that quasielasticity conditions are not violated. These factors can be used to examine the validity of various models for a relaxation time.

1. *Shoenberg D.* Magnetic Oscillations in Metals. — Cambridge: Cambridge Univ. Press, 1984.
2. *Ebert G., von Klitzing K., Probst C., Ploog K.* // Solid State Communs. — 1982. — **44**, N2. — P.95—98.
3. *Kartsovnik M.V., Laukhin V.N., Nizhankovskii V.I., Ignat'ev A.A.* // Pis'ma Zh. Exp. Teor. Fiz. — 1988. — **47**, N6. — P.302—305.
4. *Kartsovnik M.V., Kononovich P.A., Laukhin V.N., Shchegolev I.F.* // Pis'ma Zh. Exp. Teor. Fiz. — 1988. — **48**, N9. — P.498—501.
5. *Kaganov M.I., Lifshitz I.M., Sinel'nikov K.D.* // Zh. Exp. Teor. Fiz. — 1957. — **32**, N3. — P.605—607.
6. *Fivaz R.F.* // J. Phys. Chem. Solids. — 1967. — **28**, N4. — P.839—852.
7. *Peshchanskii V.G.* // Zh. Exp. Teor. Fiz. — 2002. — **121**, N5. — P.1204—1212.
8. *Korn G.A., Korn T.M.*. Mathematical Handbook for Scientists and Engineers. — New York: McGraw-Hill, 1961.
9. *Gradshtein I.S., Ryzhyk I.M.* Table of Integrals, Sums, Series, and Products. — Moscow: Nauka, 1971 (in Russian).
10. *Isihara A.* Statistical Physics. — New York: Acad. Press, 1971.
11. *Borisenko S.I.* // Fiz. Techn. Polupr. — 2002. — **36**, N7. — P.861—867.
12. *Borisenko S.I.* // Ibid. — 2004. — **38**, N2. — P.207—212.
13. *Gantmakher V.F., Levinson I.B.* Scattering of Current Carriers in Metals and Semiconductors. — Moscow: Nauka, 1984 (in Russian).

Received 10.01.05.
Translated from Ukrainian by A.I. Tovstolytkin

ПОЗДОВЖНЯ ЕЛЕКТРОПРОВІДНІСТЬ ШАРУВАТИХ КРИСТАЛІВ У СИЛЬНОМУ МАГНІТНОМУ ПОЛІ

П.В. Горський

Р е з ю м е

Розраховано поздовжню провідність шаруватих кристалів в напрямку C-осі з врахуванням залежності хімічного потенціалу електронного газу від квантуючого магнітного поля для двох випадків: сталого часу релаксації і часу релаксації, пропорційного поздовжній швидкості носіїв струму. Показано, що у першому випадку відносний внесок осцилюючої частини провідності в квазікласичному наближенні в повну поздовжню провідність більший, ніж у другому випадку. Враховуючи залежності хімічного потенціалу від магнітного поля, показано, що у сильних полях повна поздовжня провідність прямує до нуля за законом B^{-2} при сталому часі релаксації і за законом B^{-3} при часі релаксації, пропорційному поздовжній швидкості носіїв струму (B — індукція магнітного поля).