LIGHT PRESSURE ON NON-SPHERICAL METALLIC PARTICLE

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UDC 535:214

The time-averaged force of light pressure on an ellipsoidal metallic particle has been considered. Under the action of this force, the particle polarizability becomes a tensor quantity. The expressions for the averaged force vector components in the cases of plane-polarized and circularly polarized light have been derived. We have demonstrated that the force of light pressure can depend substantially on the shape of a non-spherical particle and its orientation with respect to the directions of light propagation and light polarization.

1. Introduction

The advent of lasers made the development of researches in the field of microparticle trapping, confinement, and manipulation possible. In 1970, Arthur Ashkin [1] demonstrated, for the first time, the trapping and the manipulating of a micron-sized dielectric spherical particle in the field of two opposing laser beams. Later [2], it has been shown that even a single focused laser beam can trap such a particle above the focal point. For recent years, intensively developed have been both the researches of the peculiarities inherent to the mechanisms of light pressure action upon microparticles and the implication of this action in the tasks of small particle manipulation (optical tweezers). Such applications meet a wide usage in biology, medicine, and microelectronics. A review of some relevant problems can be found, e.g., in work [3].

In work [4], a theoretical study of the time-averaged force exering upon a spherical particle in a time-harmonic-varying electromagnetic field has been carried out. The expression for the force components obtained there depends on the gradient of the electromagnetic wave intensity and on the particle polarizability. The particle was considered spherical, so that its polarizability was characterized by a scalar parameter.

In this work, we consider metallic nanoparticles of the ellipsoidal form. In this case, the particle polarizability becomes a tensor and can depend rather strongly on the particle's form [5]. Moreover, the high-frequency (optical) conductivity, which is connected to the imaginary part of particle's polarizability and defines its absorption, also becomes a tensor. The dependence of the polarizability of a metallic nanoparticle on its form becomes especially appreciable in the infra-red range of frequencies. Under such conditions, the expression for the components of the force vector, which affects the particle in the electromagnetic wave field, would differ substantially from those obtained in work [4].

2. Formulation of the Problem

For particles, whose dimensions are considerably smaller than the length of the electromagnetic wave, we apply the Rayleigh approximation, i.e. the particle is considered as a dipole in a non-uniform field. The force affecting such a particle equals

$$\vec{F} = (\vec{p}\cdot\vec{\nabla})\vec{E} + \frac{1}{c}\dot{\vec{p}}\times\vec{B}, \qquad (1)$$

where $\vec{p}$ is the dipole moment of the particle, $\vec{E}$ the electric and $\vec{B}$ the magnetic fields, and c the speed of light. In Eq. (1), all quantities are real. It is convenient to use complex ones, passing in Eq. (1) to the variables

$$\vec{p} \Rightarrow \frac{1}{2}(\vec{p}+\vec{p}^*), \quad \vec{E} \Rightarrow \frac{1}{2}(\vec{E}+\vec{E}^*), \quad \vec{B} \Rightarrow \frac{1}{2}(\vec{B}+\vec{B}^*). \qquad (2)$$

Let us consider the complex quantities to depend harmonically on time:

$$\vec{E} = \mathrm{Re}(\vec{E}_0 \exp(-i\omega t)), \quad \vec{B} = \mathrm{Re}(\vec{B}_0 \exp(-i\omega t)),$$

$$\vec{p} = \mathrm{Re}(\vec{p}_0 \exp(-i\omega t)). \tag{3}$$

Here, ω is the frequency of the electromagnetic wave. Now, we can introduce the force averaged over the period T:

$$\langle \vec{F} \rangle = \frac{1}{4T} \int\limits_{-T/2}^{T/2} dt \Big\{ ((\vec{p} + \vec{p}^*) \cdot \vec{\nabla})(\vec{E} + \vec{E}^*) +$$

$$+ \frac{1}{c} (\dot{\vec{p}} + \dot{\vec{p}}^*) \times (\vec{B} + \vec{B}^*) \Big\}. \tag{4}$$

The second term in the integrand in expression (4) can be integrated by parts, and the equation

$$-\frac{1}{c} \frac{d\vec{B}}{dt} = \vec{\nabla} \times \vec{E} \equiv \mathrm{rot} \vec{E} \tag{5}$$

may be applied. Then, instead of Eq. (4), we obtain

$$\langle \vec{F} \rangle = \frac{1}{4T} \int\limits_{-T/2}^{T/2} dt \Big\{ ((\vec{p} + \vec{p}^*) \cdot \vec{\nabla})(\vec{E} + \vec{E}^*) +$$

$$+ \frac{1}{c} (\vec{p} + \vec{p}^*) \times [\vec{\nabla} \times (\vec{E} + \vec{E}^*)] \Big\}. \tag{6}$$

Now, taking advantage of the explicit dependence on time (see Eqs. (3)), it is easy to carry out the integration in Eq. (6). We obtain the expression

$$\langle \vec{F} \rangle = \frac{1}{4} \Big\{ (\vec{p}_0 \cdot \vec{\nabla}) \vec{E}_0^* + (\vec{p}_0{}^* \cdot \vec{\nabla}) \vec{E}_0 +$$

$$\vec{p}_0 \times [\vec{\nabla} \times \vec{E}_0^*] + \vec{p}_0{}^* \times [\vec{\nabla} \times \vec{E}_0] \Big\}. \tag{7}$$

We will use formula (7) to calculate the force exerting upon the particle embedded into the electromagnetic field.

Below, we consider a metallic nanoparticle which possesses the ellipsoid-of-revolution form. In the reference frame connected to the principal axes of this ellipsoid, the dipole moment of such a particle looks like [5]

$$P_{oj} = \frac{V}{4\pi} \frac{(\varepsilon_{jj} - 1) E_{oj}}{1 + L_j (\varepsilon_{jj} - 1)}, \quad j = x, y, z. \tag{8}$$

Here, V is the volume of the particle, L_j are the depolarization factors,

$$\varepsilon_{jj} = \varepsilon'_{jj} + \varepsilon'' = \varepsilon' + i \frac{4\pi}{\omega} \sigma_{jj}, \tag{9}$$

ε' is the real part of the dielectric constant which has the form

$$\varepsilon' = 1 - \frac{\omega_p^2}{\omega^2}, \tag{10}$$

ω_p is the plasma oscillation frequency, and σ_{jj} are the diagonal elements of the tensor of high-frequency (optical) conductivity.

We admit the characteristic dimension of the metallic particle to be smaller than the mean free path of an electron in the course of its scattering by phonons. Provided such dimensions and the asymmetric form of the particle, the conductivity becomes, as was demonstrated in work [5], a tensor quantity. In such a case, the conductivity and, therefore, dissipation are influenced by both the electric field E (electric absorption) and the magnetic field B (magnetic absorption) of the wave. In the case of the ellipsoid of revolution, the following components of the tensor σ_{jj} are distinct from zero in the reference frame connected to the principal axes of this ellipsoid:

$$\sigma_{xx} = \sigma_{yy} \equiv \sigma_\perp, \quad \sigma_{zz} \equiv \sigma_\parallel, \tag{11}$$

while the depolarization factors equal

$$L_x = L_y = \frac{1}{2}(1 - L_z) \equiv L_\perp,$$

$$L_z \equiv L_\parallel = \begin{cases} \frac{1-e_p^2}{2e_p^3}[\ln \frac{1+e_p}{1-e_p} - 2e_p], & R_\parallel > R_\perp, \\ \frac{1+e_p^2}{e_p^3}[e_p - \mathrm{arctg}\, e_p], & R_\parallel < R_\perp. \end{cases} \tag{12}$$

In expressions (13), the notation

$$e_p^2 \equiv \left| 1 - \frac{R_\perp^2}{R_\parallel^2} \right|, \tag{13}$$

where $R_\parallel$ and $R_\perp$ are the corresponding semi-axes of the ellipsoid of revolution, is introduced.

Presenting the components of the polarization vector in the form

$$P_{0i} = \sum_j \alpha_{ij} E_{0j}, \tag{14}$$

Eqs. (8) and (14) yield the following expressions for nonzero components of the polarization tensor α_{jj}:

$$\alpha_{xx} = \alpha_{yy} \equiv \frac{V}{4\pi} \frac{(\varepsilon_\perp - 1)}{1 + L_\perp (\varepsilon_\perp - 1)},$$

$$\alpha_{zz} \equiv \alpha_\parallel = \frac{V}{4\pi}\frac{(\varepsilon_\parallel - 1)}{1 + L_\parallel(\varepsilon_\parallel - 1)}, \quad (15)$$

where

$$\varepsilon_\perp = \varepsilon' + i\frac{4\pi}{\omega}\sigma_\perp, \quad \varepsilon_\parallel = \varepsilon' + i\frac{4\pi}{\omega}\sigma_\parallel. \quad (16)$$

The expressions for $\sigma_\perp$ and $\sigma_\parallel$ under various specific conditions are presented in work [5]. In particular, if the electric absorption dominates (see work [5]), simple analytical expressions for the components $\sigma_\perp$ and $\sigma_\parallel$ can be obtained in the cases of strongly prolate ($R_\parallel \gg R_\perp$) and strongly oblate ($R_\parallel \ll R_\perp$) ellipsoids:

$$\sigma_\parallel \approx \frac{3}{2}\sigma_\perp \approx \frac{9\pi}{64}\frac{v_\mathrm{F}}{R_\perp}\frac{ne^2}{m\omega^2} \quad (R_\parallel \gg R_\perp),$$

$$\sigma_\parallel \approx \frac{1}{2}\sigma_\perp \approx \frac{9\pi}{16}\frac{v_\mathrm{F}}{R_\parallel}\frac{ne^2}{m\omega^2} \quad (R_\parallel \ll R_\perp). \quad (17)$$

Here, v_F is the Fermi velocity, n the concentration of electrons, m the electron mass, and e the electron charge.

For spherical particles ($R_\parallel = R_\perp = R$), we obtain

$$\sigma_\parallel = \sigma_\perp = \frac{3}{4}\frac{v_\mathrm{F}}{R}\frac{ne^2}{m\omega^2}. \quad (18)$$

Formulae (17) and (18) are valid in the case of high-frequency fields, when the frequency of light is higher than the transit-time ones ($\omega > v_\mathrm{F}/R_\perp, v_\mathrm{F}/R_\parallel$).

Starting from formulae (14) and (15), the polarization vector can be written down in the form

$$\vec{p}_0 = \alpha_\perp \vec{E}_0 + (\alpha_\perp - \alpha_\parallel)(\vec{q}\vec{E}_0)\vec{q}. \quad (19)$$

Here, $\vec{q}$ is a unit vector directed along the axis of rotation of the ellipsoid. Formulae (7) and (19) will serve as the basic ones for studying the force of light pressure.

3. Force of Light Pressure

In order to obtain the explicit expression for the time-averaged force (7), it is necessary to establish the coordinate dependence of the field $\vec{E}_0$. As the first example of such a dependence, we take this field in the form accepted in work [4]. In terms of the dimensionless variables, it looks like

$$\vec{E}_0 = (E_0, 0, 0); \quad E_0 = e^{-(x^2+y^2)/2}e^{ikz}. \quad (20)$$

Substituting expression (20) into Eqs. (7) and (19), we obtain the expressions for nonzero components of the time-averaged force:

$$\langle F_x \rangle = -\frac{x}{2}\{|E_0|^2\mathrm{Re}\alpha_\perp + |(\vec{E}_0\vec{q})|^2\mathrm{Re}(\alpha_\parallel - \alpha_\perp)\},$$

$$\langle F_y \rangle = -\frac{y}{2}\{|E_0|^2\mathrm{Re}\alpha_\perp + |(\vec{E}_0\vec{q})|^2\mathrm{Re}(\alpha_\parallel - \alpha_\perp)\},$$

$$\langle F_z \rangle = \frac{k}{2}\{|E_0|^2\mathrm{Im}\alpha_\perp + |(\vec{E}_0\vec{q})|^2\mathrm{Im}(\alpha_\parallel - \alpha_\perp)\}. \quad (21)$$

From Eqs. (21), one can see that, contrary to the particles with a spherical form (when $\alpha_\perp = \alpha_\parallel$), the force components for the nanoparticles with the ellipsoid-of-revolution geometry acquire the dependence on the angle between the field direction and the revolution axis of the ellipsoid. Moreover, the dependence of the force on the particle's form is incorporated into the depolarization factors (13) which enter into the expressions for $\alpha_\perp$ and $\alpha_\parallel$ (see Eqs. (15)).

Consider now a circularly polarized Gaussian beam:

$$\vec{E}_0 = (\vec{b}_1 + i\vec{b}_2)e^{-(x^2+y^2)/2}e^{ikz}, \quad (22)$$

$$\vec{b}_1 = (b_1, 0, 0), \quad \vec{b}_2 = (0, b_2, 0).$$

In this case, after having substituted Eqs. (22) into expressions (7) and (19), we obtain

$$\langle F_x \rangle = -\frac{x}{2}e^{-(x^2+y^2)}\{(b_1^2 + b_2^2)\mathrm{Re}\alpha_\perp +$$

$$+[(\vec{q}\vec{b}_1)^2 + (\vec{q}\vec{b}_2)^2]\mathrm{Re}(\alpha_\parallel - \alpha_\perp)\},$$

$$\langle F_y \rangle = -\frac{y}{2}e^{-(x^2+y^2)}\{(b_1^2 + b_2^2)\mathrm{Re}\alpha_\perp +$$

$$+[(\vec{q}\vec{b}_1)^2 + (\vec{q}\vec{b}_2)^2]\mathrm{Re}(\alpha_\parallel - \alpha_\perp)\},$$

$$\langle F_z \rangle = \frac{k}{2}e^{-(x^2+y^2)}\{(b_1^2 + b_2^2)\mathrm{Im}\alpha_\perp +$$

$$+[(\vec{q}\vec{b}_1)^2 + (\vec{q}\vec{b}_2)^2]\mathrm{Im}(\alpha_\parallel - \alpha_\perp)\}. \quad (23)$$

One should bear in mind that $\vec{q}$ is a unit vector directed along the rotation axis of the ellipsoid. We see that, in this case, the force depends on two angles – between $\vec{q}$ and $\vec{b}_1$ vectors and between $\vec{q}$ and $\vec{b}_2$ ones). In the case of circular polarization, $\vec{b}_1 = \vec{b}_2 = \vec{b}$, so that

$$(\vec{q}\vec{b}_1)^2 + (\vec{q}\vec{b}_2)^2 = (q_x^2 + q_y^2)b^2 = (1 - q_z^2)b^2.$$

That is, in this case, only the dependence on the angle between the vector $\vec{q}$ and the direction of beam propagation survives.

Thus, similarly to the cases of plane-polarized and circularly polarized light beams, the time-averaged force affecting a non-spherical metallic nanoparticle becomes angle-dependent. In addition, this force depends on the particle's form through the components $\alpha_\perp$ and $\alpha_\parallel$ of the polarization tensor; this dependence manifests itself to the maximal extent in the infra-red range of the spectrum (in the vicinity of the CO_2-laser frequency). For example, taking $\omega_p \approx 5 \times 10^{15}$ s^{-1} for gold and $\omega = 2 \times 10^{14}$ s^{-1} for the CO_2-laser frequency, we obtain $\varepsilon' \approx -600$. Therefore, the combinations $L_{\perp,\parallel}(\varepsilon_{\perp,\parallel} - 1)$ that enter into the denominators of formula (15) approximately equal

$$L_{\perp,\parallel}(\varepsilon_{\perp,\parallel} - 1) \approx -600 L_{\perp,\parallel}. \quad (24)$$

Since the quantities $L_\perp$ and $L_\parallel$ may vary from 0 to 1 (provided that $2L_\perp + L_\parallel = 1$), it is clear to which extent quantity (24) and, respectively, the quantities $\alpha_\perp$ and $\alpha_\parallel$ can be sensitive to the form of a metallic particle within this range of frequencies.

Thus, in this work, we have obtained the expressions for the averaged (over a period) force which is exerted by a laser beam on a non-spherical metallic nanoparticle. We have shown that this force can depend substantially on the form of the particle and on the angles that define the orientation of the non-spherical particle relative to the propagation direction and the polarization of the beam.

The work was financially supported in part within the framework of the target program "Nanophysics and nanoelectronics" of the National Academy of Sciences of Ukraine.

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Received 23.03.05.
Translated from Ukrainian by O.I. Voitenko

СВІТЛОВИЙ ТИСК НА НЕСФЕРИЧНУ МЕТАЛІЧНУ ЧАСТИНКУ

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Р е з ю м е

Розглянуто силу лазерного тиску, що діє на металічну частинку еліпсоїдальної форми. Під дією цієї сили поляризовність частинки стає тензорною величиною. Знайдено вирази для компонент вектора усередненої сили у випадку плоскополяризованого і циркулярно поляризованого світла. Показано, що сила тиску світла може істотно залежати від форми частинки, а також кутів, які визначають орієнтацію несферичної частинки відносно напрямку поширення променя і його поляризації.