
**SPATIAL ANALYSIS OF ISOTROPIC AND ANISOTROPIC
DIFFRACTIONS OF LIGHT BY TRANSVERSE ACOUSTIC
WAVES IN BARIUM BETA-BORATE CRYSTALS****A.S. ANDRUSHCHAK¹, YA.V. BOBITSKI^{1,2}, M.V. KAIDAN¹,
B.V. TYBINKA¹**UDC 535.551:534
© 2005¹National University "L'vivs'ka Politekhnik" (12, S. Bandera Str., L'viv 79013, Ukraine),²Institute of Technics, Rzeszow University (16a, Rejtana Str, Rzeszow 35-959, Poland)

The indicative surfaces of the effective photoelastic (PE) constant p_{eff} and the acousto-optic (AO) figure of merit M_2 have been constructed, and their extreme values have been determined for both isotropic and anisotropic diffractions of light by transverse acoustic waves in β -BaB₂O₄ crystals. The relevant spatial analysis of the AO interaction has been carried out. The largest value of the AO figure of merit $M_2 = 40.1 \times 10^{-15} \text{ s}^3/\text{kg}$ for β -BaB₂O₄ crystals has been found in the case of anisotropic diffraction of light by a transverse acoustic wave with lower velocity. The specimen geometry and the corresponding positional relationship of interacting AO components have been determined for the most efficient use of β -BaB₂O₄ crystals in AO devices.

1. Introduction

A necessary prerequisite for the practical use of new AO materials is large values of their AO figure of merit. From the viewpoint of the determination of such coefficients [1, 2], perspective are materials with large values of the PE coefficients and low velocities of acoustic wave propagation. The latter reason stimulates the consideration of the AO interaction of light just by transverse acoustic waves, to which lower velocities are inherent.

For the AO materials, in particular low-symmetry anisotropic crystals, to be used more effectively [3–5], it is necessary that the analysis of the spatial distribution of the AO parameters should be carried out, which may help to determine such a geometry of the interaction between light and an acoustic wave, where the AO parameters take on values close to the maximum and

satisfy other necessary criteria of practical application [2]. According to work [6], the best means for a geometrical representation of the spatial distribution of induced optical effects, which are described by the tensors of the third and higher ranks, are indicative surfaces [6–9]. We suggest to carry out also researches of the AO-effect anisotropy and the relevant spatial analysis of the AO interaction by constructing the indicative surfaces and their stereographic projections for the AO figures of merit, taking into account the corresponding surfaces for the effective PE constant p_{eff} and for velocities. The proposed approach was testified making use of β -BaB₂O₄ crystals perspective for practical application which are considered as a good nonlinear-optical material [10–12] and, according to [13], are predicted as a perspective AO material.

This work aims at carrying out the 3D analysis of spatial distributions in the case of the AO interaction of light with transverse acoustic waves in β -BaB₂O₄ crystals and at estimating the prospects of practical applications of those crystals.

**2. Basic Relations for the Spatial Analysis of
the AO Interaction of Light**

Depending on the posed technological task, various AO figures of merit are considered [1]. To estimate the potential of applying a specific material in AO devices,

the following AO figure of merit is determined:

$$M_2 = \frac{n_\mu^3 n_\nu^3 p_{\text{eff}}^2}{\rho V^3} \cos \beta_\mu \cos \beta_\nu \cos \gamma, \quad (1)$$

where $\mu, \nu = 1$ and 2 , ρ is the crystal density, n_μ and n_ν are the refractive indices of the incident and diffracted light, respectively, V is the velocity of an acoustic wave, β_μ, β_ν , and γ are the walkoff angles between the propagation and wave front directions for the incident and diffracted light and an acoustic wave, respectively. It is assumed [2] that $\mu = \nu$ corresponds to isotropic diffraction and $\mu \neq \nu$ does to anisotropic one.

Each of the parameters in Eq. (1), except for the density ρ , has its own spatial distribution, i.e. the own anisotropy, and is described by the corresponding relation, namely:

1) The effective PE constant is determined according to the relation [5]

$$p_{\text{eff}} = \mathbf{i}_\mu \mathbf{i}_\nu \hat{p} \mathbf{a} \mathbf{f}_q, \quad (2)$$

where the interacting AO components \mathbf{i}_μ and \mathbf{i}_ν are the unit vectors along the polarization directions of the incident and diffracted light waves, respectively, \mathbf{a} and \mathbf{f}_q are the unit vectors pointing out the propagation direction and the polarization of the acoustic wave, respectively; \hat{p} is the tensor of PE coefficients. Hereafter, the non-coordinate form of notation [2, 6] is used.

2) The velocity of the acoustic wave, V , is determined from the solutions of the Christoffel equation of elastodynamics for plane waves [6, 14]

$$\mathbf{a} \hat{c} \mathbf{a} \mathbf{f}_q = \rho V_q^2 \mathbf{f}_q, \quad (3)$$

where $\mathbf{f}_q \mathbf{f}_{q'} = \delta_{qq'}$, $\delta_{qq'}$ is the Kronecker symbol, and \hat{c} is the elastic constant tensor. According to [14], the index q defines which of the three polarizations of the acoustic wave is examined: the case $q = 1$ corresponds to a transverse wave with lower velocity, $q = 2$ to a transverse wave with higher velocity, and $q = 3$ to a longitudinal acoustic wave.

3) The refractive indices are determined according to the formula [2]

$$n_\omega^{-2} = \mathbf{i}_\omega \hat{\eta} \mathbf{i}_\omega, \quad (4)$$

where $\hat{\eta}$ is the tensor of reciprocal dielectric permittivity and the index $\omega = \mu$ or ν for the incident or diffracted wave, respectively.

4) The walkoff angles between the propagation directions and the wave fronts of light and the acoustic wave, according to works [2, 6], have the forms

$$\text{tg} \beta_\omega = n_\omega^{-2} (\mathbf{k} \hat{\eta} \mathbf{i}_\omega),$$

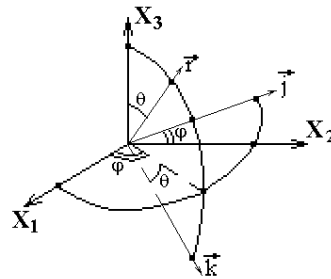


Fig. 1. Geometrical arrangement of the interacting vectors \mathbf{k} , \mathbf{j} , and \mathbf{r} in the case of a uniaxial crystal

$$\cos \gamma = \rho V_q^2 (\mathbf{a} (\mathbf{f}_q \hat{c} \mathbf{f}_q) (\mathbf{f}_q \hat{c} \mathbf{f}_q) \mathbf{a})^{-1/2}, \quad (5)$$

where \mathbf{k} is the unit vector along the direction of light propagation.

We note that all the tensors introduced above are considered in the common principal crystallophysical system of coordinates [6].

3. Equations of Indicative Surfaces for the AO Effect

With the purpose to reduce the number of additional conditions, which are necessary to be taken into account in the AO interaction when studying a spatial anisotropy, we confine ourselves, similarly to [2, 15], to the case where $\mathbf{k} \mathbf{a} = -\cos(90^\circ - \theta^*) = -\sin \theta^* = 0$, i.e. $\theta^* = 0$, where θ^* is the diffraction angle. Then, when analyzing the AO properties of materials [2, 15], we should not consider the lengths of light, λ , and elastic, Λ , waves which are related to the angles of incidence, θ_μ^* , and diffraction, θ_ν^* , by the known relations [2] for anisotropic ($\sin \theta_\mu^* = -1/2n_\mu [\lambda/\Lambda + \Lambda/\lambda(n_\mu^2 - n_\nu^2)]$, $\sin \theta_\nu^* = -1/2n_\nu [\lambda/\Lambda - \Lambda/\lambda(n_\mu^2 - n_\nu^2)]$) and isotropic ($\sin \theta_\mu^* = -\sin \theta_\nu^* = \lambda/2n_\mu \Lambda$) diffraction of light.

Let us write down the equations and construct the indicative surfaces for the effective PE constant p_{eff} and the AO figure of merit M_2 , taking advantage of the method similar to that used for induced optical effects, in particular the piezooptical one developed by us in [7–9, 13]. Then, the following indicative surfaces can be considered for each type of diffraction: 1) a longitudinal surface, where $\mathbf{i}_\mu \parallel \mathbf{a} \parallel \mathbf{r}$, 2) a transverse surface of light polarization, where $\mathbf{i}_\mu \parallel \mathbf{r}$ and $\mathbf{a} \parallel \mathbf{j}$, and 3) a transverse surface of the acoustic wave, where $\mathbf{a} \parallel \mathbf{r}$ and $\mathbf{i}_\mu \parallel \mathbf{j}$, where \mathbf{r} is the radius-vector of the indicative surface, $\mathbf{j} \perp \mathbf{r}$, and $\mathbf{j} \perp \mathbf{k}$. For uniaxial crystals, the vectors \mathbf{r} and \mathbf{j} coincide with the polarization directions of the ordinary and extraordinary light waves, respectively (see Fig. 1).

To write the equations of indicative surfaces, it is necessary to determine the angles θ_w and φ_w of the spherical coordinate system with respect to the axes $X1, X2, X3$ of the principal crystallophysical coordinate systems for each of the vectors $\mathbf{i}_\mu, \mathbf{i}_\nu, \mathbf{a}$, and \mathbf{f}_q . Then, the corresponding direction cosines

$$\begin{aligned} \alpha_{w1} &= \sin \theta_w \cos \varphi_w, & \alpha_{w2} &= \sin \theta_w \sin \varphi_w, \\ \alpha_{w3} &= \cos \theta_w, \end{aligned} \quad (6)$$

can be determined, where the subscript w is equal to μ, ν, \mathbf{a} , or \mathbf{f} , depending on the vector under consideration. Substituting Eq. (6) into Eq. (2), the general equation of indicative surfaces for the effective PE constant p_{eff} for the crystals of the symmetry class $3m$ including $\beta\text{-BaB}_2\text{O}_4$ crystals looks as

$$\begin{aligned} p_{\text{eff}} &= 0.5 \sin \theta_\mu \sin \theta_\nu \sin \theta_a \sin \theta_f (\sin(\varphi_a + \varphi_\mu) \times \\ &\times \sin(\varphi_f + \varphi_\nu) + \sin(\varphi_f + \varphi_\mu) \sin(\varphi_a + \varphi_\nu)) p_{11} + \\ &+ 0.5 \sin \theta_\mu \sin \theta_\nu \sin \theta_a \sin \theta_f (\sin(\varphi_a - \varphi_\mu) \times \\ &\times \sin(\varphi_f - \varphi_\nu) + \sin(\varphi_f - \varphi_\mu) \sin(\varphi_a - \varphi_\nu)) p_{12} + \\ &+ \sin \theta_\mu \sin \theta_\nu \cos \theta_a \cos \theta_f \cos(\varphi_\mu - \varphi_\nu) p_{13} + \\ &+ \cos \theta_\mu \cos \theta_\nu \sin \theta_a \sin \theta_f \cos(\varphi_a - \varphi_f) p_{31} + \\ &+ \cos \theta_\mu \cos \theta_\nu \cos \theta_a \cos \theta_f p_{33} + \\ &+ (\sin \theta_\mu \cos \theta_\nu (\sin \theta_a \cos \theta_f \cos(\varphi_\mu - \varphi_a) + \cos \theta_a \sin \theta_f \times \\ &\times \cos(\varphi_\mu - \varphi_f)) + \cos \theta_\mu \sin \theta_\nu (\cos \theta_a \sin \theta_f \cos(\varphi_\nu - \end{aligned}$$

$$\begin{aligned} & - \varphi_f) + \sin \theta_a \cos \theta_f \cos(\varphi_\nu - \varphi_a)) p_{44} + \\ & + \sin \theta_\mu \sin \theta_\nu (\sin \theta_a \cos \theta_f \sin(\varphi_\mu + \varphi_\nu + \varphi_a) + \\ & + \cos \theta_a \sin \theta_f \sin(\varphi_\mu + \varphi_\nu + \varphi_f)) p_{14} + \\ & + \sin \theta_a \sin \theta_f (\sin \theta_\mu \cos \theta_\nu \sin(\varphi_\mu + \varphi_a + \varphi_f) + \\ & + \cos \theta_\mu \sin \theta_\nu \sin(\varphi_\nu + \varphi_a + \varphi_f)) p_{41}, \end{aligned} \quad (7)$$

where p_{11}, p_{21}, \dots are the available coefficients of the PE effect for the $3m$ class of symmetry. Using the last equation, the equations of indicative surfaces of the effective PE constant p_{eff} and the corresponding equation for the AO figure of merit M_2 can be written down (see the schematic notation of those equations in Table 1) for all possible cases of isotropic and anisotropic diffraction, taking into account the relative arrangement of the vectors $\mathbf{i}_\mu, \mathbf{i}_\nu$, and \mathbf{a} .

Below, we give some explanations to Table 1. The superscripts ($\mathbf{i}, \mathbf{a}, \mathbf{r}$) indicate which of the vectors $\mathbf{i}_\mu, \mathbf{a}$, or the both (see Fig. 1) are collinear to the radius-vector \mathbf{r} , and the subscripts specify the type of AO diffraction (isotropic or anisotropic). The quantities n_r, n_j and β_r, β_j are determined according to formulae (4) and (5), provided that $\mathbf{r} \parallel \mathbf{i}_w$ or $\mathbf{j} \parallel \mathbf{i}_w$, respectively. Analogously, the quantities V_r, V_j and γ_r, γ_j are determined according to formulae (3) and (5), provided that $\mathbf{r} \parallel \mathbf{a}$ or $\mathbf{j} \parallel \mathbf{a}$, respectively. Since the PE factor is symmetrical with respect to its first two indices, it is obvious that Eqs. (11) and (17) coincide with Eqs. (13) and (19), respectively,

Table 1. Basic relations for constructing the indicative surfaces of the effective PE constant p_{eff} and the AO figure of merit M_2 for various types of diffraction in uniaxial crystals

Diffraction type	Surface type	Positional relation of vectors	Relation for indicative surfaces	
			the effective PE constant	the AO figure of merit
isotropic	longitudinal	$\mathbf{i}_\mu \parallel \mathbf{a} \parallel \mathbf{r}$	$p_{\text{eff}(\text{iso})}^{(r)} = \mathbf{r} \hat{\rho} \mathbf{r} \mathbf{f}_q$ (8)	$M_{2(\text{iso})}^{(r)} = \frac{n_r^6 (p_{\text{eff}(\text{iso})}^{(r)})^2}{\rho V_r^3} \cos^2 \beta_r \cos \gamma_r$ (14)
isotropic	transverse	$\mathbf{i}_\mu \parallel \mathbf{r}, \mathbf{a} \parallel \mathbf{j}$	$p_{\text{eff}(\text{iso})}^{(i)} = \mathbf{r} \hat{\rho} \mathbf{j} \mathbf{f}_q$ (9)	$M_{2(\text{iso})}^{(i)} = \frac{n_r^6 (p_{\text{eff}(\text{iso})}^{(i)})^2}{\rho V_j^3} \cos^2 \beta_r \cos \gamma_j$ (15)
isotropic	transverse	$\mathbf{a} \parallel \mathbf{r}, \mathbf{i}_\mu \parallel \mathbf{j}$	$p_{\text{eff}(\text{iso})}^{(a)} = \mathbf{j} \hat{\rho} \mathbf{r} \mathbf{f}_q$ (10)	$M_{2(\text{iso})}^{(a)} = \frac{n_r^6 (p_{\text{eff}(\text{iso})}^{(a)})^2}{\rho V_r^3} \cos^2 \beta_j \cos \gamma_r$ (16)
anisotropic	longitudinal	$\mathbf{i}_\mu \parallel \mathbf{a} \parallel \mathbf{r}, \mathbf{i}_\nu \parallel \mathbf{j}$	$p_{\text{eff}(\text{an})}^{(r)} = \mathbf{r} \hat{\rho} \mathbf{r} \mathbf{f}_q$ (11)	$M_{2(\text{an})}^{(r)} = \frac{n_r^3 n_j^3 (p_{\text{eff}(\text{an})}^{(r)})^2}{\rho V_r^3} \cos \beta_r \cos \beta_j \cos \gamma_r$ (17)
anisotropic	transverse	$\mathbf{i}_\mu \parallel \mathbf{r}, \mathbf{i}_\nu \parallel \mathbf{a} \parallel \mathbf{j}$	$p_{\text{eff}(\text{an})}^{(i)} = \mathbf{r} \hat{\rho} \mathbf{j} \mathbf{f}_q$ (12)	$M_{2(\text{an})}^{(i)} = \frac{n_r^3 n_j^3 (p_{\text{eff}(\text{an})}^{(i)})^2}{\rho V_j^3} \cos \beta_r \cos \beta_j \cos \gamma_j$ (18)
anisotropic	transverse	$\mathbf{i}_\nu \parallel \mathbf{a} \parallel \mathbf{r}, \mathbf{i}_\mu \parallel \mathbf{j}$	$p_{\text{eff}(\text{an})}^{(a)} = \mathbf{j} \hat{\rho} \mathbf{r} \mathbf{f}_q$ (13)	$M_{2(\text{an})}^{(a)} = \frac{n_r^3 n_j^3 (p_{\text{eff}(\text{an})}^{(a)})^2}{\rho V_r^3} \cos \beta_r \cos \beta_j \cos \gamma_r$ (19)

i.e. the corresponding surfaces are identical; so the case of anisotropic diffraction will be represented below by only two indicative surfaces.

It should be emphasized that, for the polarization direction of the acoustic wave \mathbf{f}_q to be determined as a function of its propagation direction \mathbf{a} , the cubic Eq. (3) is to be solved.

4. 3D Analysis of Spatial Distribution for the AO interaction in Barium Beta-Borate Crystals

In our researches, the following values of the PE coefficients of barium beta-borate crystals, taken from work [16], were used: $p_{11} = -0.195$, $p_{12} = -0.197$, $p_{13} = -0.059$, $p_{31} = -0.112$, $p_{33} = 0.039$, $p_{14} = -0.005$, $p_{41} = -0.007$, and $p_{44} = -0.078$. The density was taken $\rho = 3840 \text{ kg/m}^3$ [10]. The elastic-stiffness coefficients (in units of GPa) $C_{11} = 105.3$, $C_{12} = 78.6$, $C_{13} = 49.6$, $C_{14} = -1.8$, $C_{33} = 53.7$, and $C_{44} = 3.3$ were found through the inversion $C_{ij} = S_{ij}^{-1}$ of the matrix of the elastic-compliance coefficients S_{ij} , the latter being taken from [10] in view of the remarks made in [16].

First, according to formulae (8–19) and making use of the developed software program, the maximal absolute value for each indicative surface of the effective PE constant p_{eff} or the AO figure of merit M_2 was determined (see Table 2). The values that correspond to the calculation of p_{eff} or M_2 for the lower acoustic velocity (the case $q = 1$) are displayed in the top line of each indicative surface item, and for the higher velocity ($q = 2$) in the bottom one.

It is unexpected that, in the case of isotropic diffraction, the extreme values of p_{eff} and M_2 are reached when the transverse acoustic wave is faster (the case $q = 2$). In this work, we give examples of the indicative surfaces (see Figs. 2 and 3) for only those largest AO parameters which are emphasized in Table 2 by bold face, being interesting from the practical aspect.

When plotting the indicative surfaces, the sign of the effective PE constant p_{eff} was taken positive, i.e. we used the absolute value of p_{eff} because it was impossible to set unequivocally the directions for the vectors \mathbf{i}_μ , \mathbf{i}_ν , and \mathbf{f}_q . It has no basic importance, since the quantity p_{eff} is squared in all formulae for the AO figure of merit [1].

Table 2. Maximal absolute values and corresponding directions \mathbf{i}_μ , \mathbf{i}_ν , \mathbf{a} , and \mathbf{f}_q for the indicative surfaces of p_{eff} or M_2 of $\beta\text{-BaB}_2\text{O}_4$ crystals

N	Indicative surfaces	Light wave						Acoustic wave						p_{eff}	$M_2, 10^{-15} \text{ s}^3/\text{kg}$
		\mathbf{i}_μ , degree			\mathbf{i}_ν , degree			\mathbf{a} , degree		\mathbf{f}_q , degree		γ	V_q , m/s		
		θ_μ	φ_μ	β_μ	θ_ν	φ_ν	β_ν	θ_a	φ_a	φ_f	φ_f				
1	$p_{\text{eff}}^{(r)}(\text{iso})$	11	-90	1	11	-90	1	11	-90	78	90	57	1058	0.056	5.2
		26	-90	3	26	-90	3	26	-90	121	-90	47	1562	0.094	6.1
2	$p_{\text{eff}}^{(i)}(\text{iso})$	0	—	0	0	—	0	90	90	11	-90	16	923	0.002	0.018
		45	0	4	45	0	4	90	90	90	0	8	1872	0.007	0.033
3	$p_{\text{eff}}^{(a)}(\text{iso})$	90	0	0	90	0	0	52	-90	24	90	29	1706	0.033	1.1
		90	-1	0	90	-1	0	33	-91	49	88	30	1728	0.047	2.1
4	$p_{\text{eff}}^{(r)}(\text{an})$	4	90	1	90	0	0	4	90	90	0	0	898	0.079	36
		0	90	0	90	0	0	0	90	90	0	0	927	0.078	30
5	$p_{\text{eff}}^{(i)}(\text{an})$	3	88	0	90	-2	0	90	-2	80	89	3	882	0.078	40
		16	90	2	90	0	0	90	0	80	-90	0	1893	0.007	0.03
6	$M_2^{(r)}(\text{iso})$	11	-90	1	11	-90	1	11	-90	78	90	57	1058	0.056	5.2
		21	30	3	21	30	3	21	30	114	30	55	1400	0.089	6.3
7	$M_2^{(i)}(\text{iso})$	0	—	0	0	—	0	90	90	11	-90	16	923	0.002	0.018
		48	0	4	48	0	4	90	90	90	0	8	1872	0.007	0.033
8	$M_2^{(a)}(\text{iso})$	90	0	0	90	0	0	52	-90	24	90	29	1706	0.033	1.1
		90	120	0	90	120	0	33	30	49	-120	30	1728	0.047	2.1
9	$M_2^{(r)}(\text{an})$	9	90	1	90	0	0	9	90	10	90	0	882	0.078	39.9
		0	90	0	90	0	0	0	90	90	0	0	927	0.078	30
10	$M_2^{(i)}(\text{an})$	4	90	1	90	0	0	90	0	10	90	0	882	0.078	40.1
		19	90	2	90	0	0	90	0	80	-90	0	1893	0.007	0.03

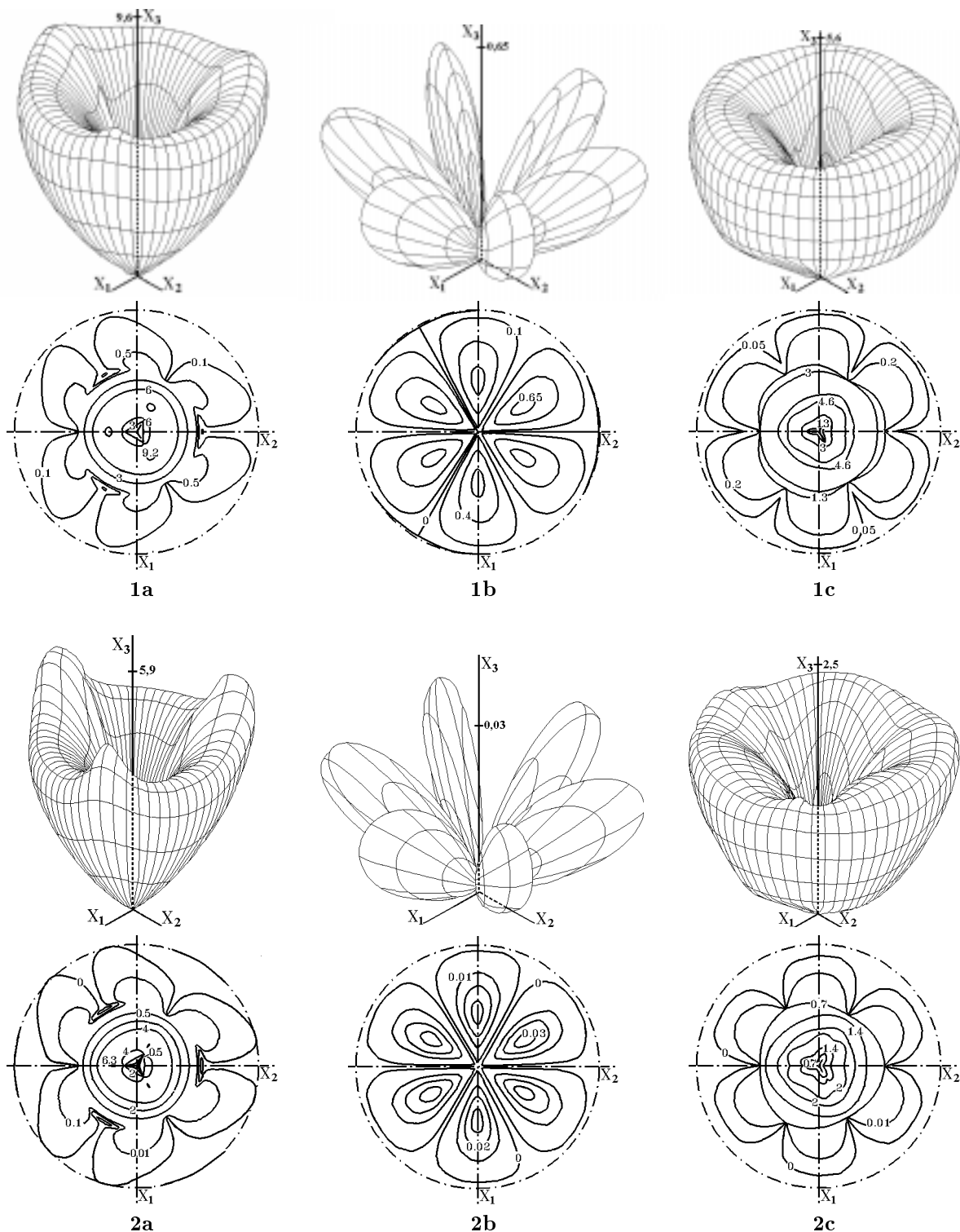


Fig. 2. Indicative surfaces for the effective PE constants p_{eff} (1) and the AO figures of merit M_2 (2) together with their stereographic projections (shown below) for the case of isotropic diffraction of light by the transverse acoustic wave with a higher velocity: (a) $\mathbf{i}_\mu \parallel \mathbf{a}$ and \mathbf{r} , (b) $\mathbf{i}_\mu \parallel \mathbf{r}$ and $\mathbf{a} \parallel \mathbf{j}$, and (c) $\mathbf{a} \parallel \mathbf{r}$ and $\mathbf{i}_\mu \parallel \mathbf{j}$. Numerical data for $100p_{\text{eff}}$ are shown, and for M_2 in units of $10^{-15} \text{ s}^3/\text{kg}$

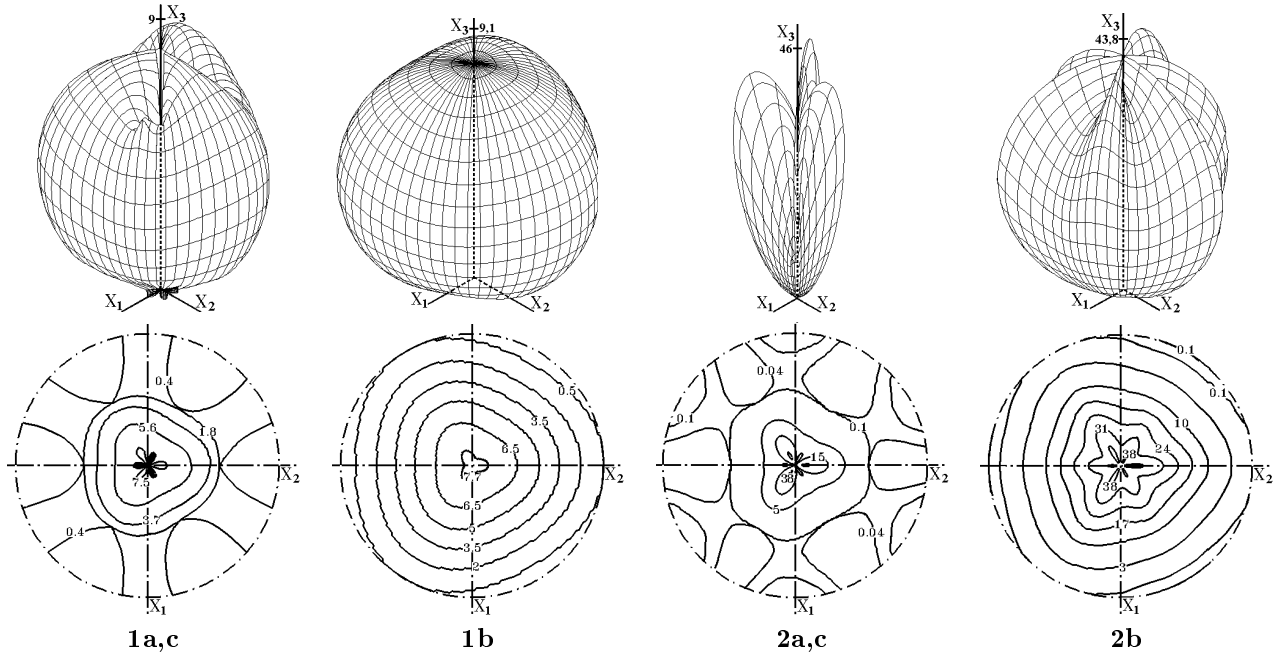


Fig. 3. Indicative surfaces and their stereographic projections in the case of anisotropic diffraction of light by the transverse acoustic wave with a lower velocity. All notations are the same as in Fig. 2

In order to eliminate the accumulation of lines, only the face and top parts of the surfaces are shown. These surfaces can be imagined in full, if one reflects the presented figures with respect to the origin of the coordinate system. We note that all the surfaces were constructed by the method of their meridional ($\varphi = \text{const}$) and equatorial ($\theta = \text{const}$) sections. For quantitative estimation of the AO effect, we constructed also the stereographic projections for all the surfaces (the bottom images).

It is obvious that the study of a spatial distribution for acoustic velocities is rather important in researching the anisotropy of the AO interaction. Therefore, we constructed the corresponding velocity surfaces for two transverse acoustic waves with the lower ($q = 1$) and higher ($q = 2$) velocities (see Fig. 4). They served as the basis for the calculation of extreme values and the corresponding angular coordinates (in parentheses) for the propagation directions of those acoustic waves; namely, the minimal values are $V_{1\text{min}} = 881$ m/s ($\theta = 90^\circ$, $\varphi = m \times 60^\circ$, where $m = 0 \dots 5$) and $V_{2\text{min}} = 927$ m/s ($\theta = 0^\circ$), the maximal values are $V_{1\text{max}} = 1709$ m/s ($\theta = 51^\circ$, $\varphi = 30^\circ + m \times 120^\circ$, where $m = 0 \dots 2$) and $V_{2\text{max}} = 1893$ m/s ($\theta = 90^\circ$, $\varphi = m \times 60^\circ$, where $m = 0 \dots 5$).

Let us analyze the results obtained. According to the Neumann–Curie principle, all the surfaces preserve the

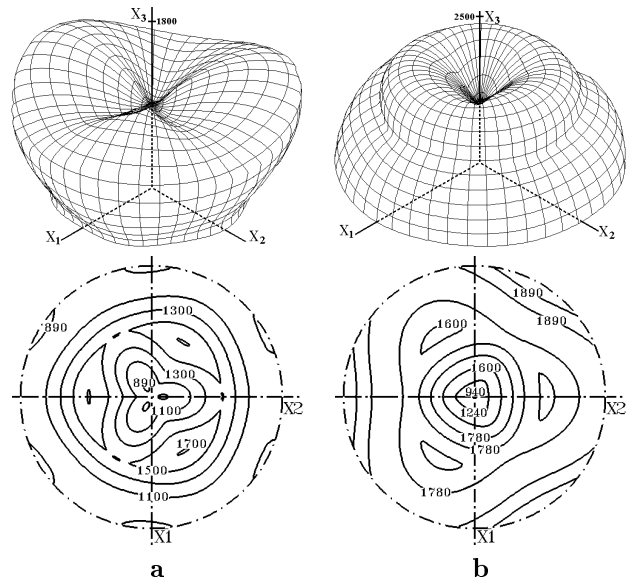


Fig. 4. Surfaces of velocities and their stereographic projections for the transverse acoustic waves with lower (a) and higher (b) velocities

basic elements of symmetry which are typical of the crystals of the $3m$ class of symmetry, i.e. the axis of 3-fold symmetry and three planes of symmetry perpendicular to the figure plane. By comparing the corresponding surfaces (see the stereographic projections

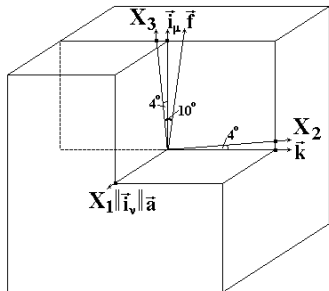


Fig. 5. Positional relationship of the AO components \mathbf{k} , \mathbf{i} , \mathbf{a} , and \mathbf{f} in the case of anisotropic diffraction of light by the transverse acoustic wave with the maximal value of the AO effect for a corresponding cross-section of β -BaB₂O₄ crystals

in Figs. 2–4), one can draw a conclusion that the influence of the effective PE constant p_{eff} on the distribution of anisotropy of the AO figure of merit M_2 is more significant as compared to acoustic velocities. From this viewpoint, the most illustrative are the data of Table 2, where the angular coordinates of the maximal values of p_{eff} are either completely identical (cf. the top values in rows N 1, 2, 3 and those in rows N 6, 7, 8, respectively) or close (in other cases) to the corresponding maximal values of M_2 .

The maximal values obtained for the AO figure of merit M_2 of β -BaB₂O₄ crystals amount to 6.3×10^{-15} for isotropic and $40.1 \times 10^{-15} \text{ s}^3/\text{kg}$ for anisotropic diffraction of light by transverse acoustic waves. Such comparatively large values of the AO figure of merit are a sound argument concerning the perspective for applying β -BaB₂O₄ crystals as a good AO material. Fig. 5 displays the geometry of the specimen and the corresponding directions of the interacting AO components \mathbf{i}_μ , \mathbf{i}_ν , \mathbf{a} , and \mathbf{f} , which are necessary for the realization of the largest value of the AO figure of merit $M_2 = 40.1 \times 10^{-15} \text{ s}^3/\text{kg}$.

5. Conclusions

1. On the basis of the 3D analysis of the spatial anisotropy of induced optical effects, the procedure of constructing the indicative surfaces for effective PE constants p_{eff} and AO figures of merit M_2 has been proposed for both isotropic and anisotropic diffraction of light by acoustic waves.

2. On the basis of this procedure, the corresponding 3D analysis of the spatial distribution of the AO effect in β -BaB₂O₄ crystals has been carried out.

Making use of the developed software, the extreme values for the surfaces of velocities, effective PE constants, and AO figures of merit M_2 have been determined. Provided isotropic diffraction, the largest values of the effective PE constants and the AO figures of merit M_2 were observed for the transverse acoustic wave with higher velocity; and for the wave with lower velocity in the case of anisotropic diffraction. In both cases, the corresponding indicative surfaces have been constructed. It has been revealed that, for the crystals under consideration, the spatial anisotropy of the AO figure of merit M_2 is mostly affected by the effective PE constant.

3. β -BaB₂O₄ crystals have the maximal value of the AO figure of merit $M_2 = 40.1 \times 10^{-15} \text{ s}^3/\text{kg}$ in the case of anisotropic diffraction of light by transverse acoustic waves with the following angular coordinates of polarization directions: for incident light, $\theta = 4^\circ, \varphi = 90^\circ + m \times 120^\circ$; for diffracted light and for the propagation of the acoustic wave with lower velocity, $\theta = 90^\circ, \varphi = m \times 120^\circ$; here $m = 0, 1, 2$.

4. In the case of anisotropic diffraction of light by a transverse acoustic wave, the geometry of a specimen has been determined, and the corresponding positional relationship of the interacting AO components has been given for the most efficient application of β -BaB₂O₄ in AO devices.

This work has been supported by STCU-program (proj. N1712 and N3222).

1. Chang I.C. // Opt. Eng. — 1985. — **24**, N 1. — P. 132–137.
2. Balakshii V.I., Parygin V.N., Chirkov L.E. Physical Fundamentals of Acousto-Optics. — Moscow: Radio i Svyaz', 1985 (in Russian).
3. Zamkov A.V., Zaitsev A.I., Parshikov S.A., Sysoev A.M. // Inorg. Mater. — 2001. — **37**, N 1. — P. 82–83.
4. Kulakova L.A. // Phys. Solid State. — 2000. — **42**, N 1. — P. 55–58.
5. Hatayama Y., Miyazaki Y., Gotoo N. // Jpn. J. Appl. Phys. Part 1. — 1994. — **33**, N 5B. — P. 3226–3229.
6. Sirotin Yu.I., Shaskol'skaya M.P. Fundamentals of Crystal Physics. — Moscow: Nauka, 1979 (in Russian).
7. Andrushchak A.S., Mytsyk B.G. // Ukr. Fiz. Zh. — 1995. — **40**, N 11–12. — P. 1122–1126.
8. Vlokh O.G., Mytsyk B.G., Andrushchak A.S., Pryriz Ya.V. // Crystallogr. Repts. — 2000. — **45**, N 1. — P. 138–144.
9. Kaidan M.V., Andrushchak A.S., Klimash M.M. et al. // Ukr. Fiz. Zh. — 2003. — **48**, N 10. — P. 1104–1109.
10. Chen C., Wu Yi, Li Rukang J. // Cryst. Growth. — 1990. — **99**. — P. 790–793.

11. *Eimeri D., Davis L., Velsko S. et al.* // J. Appl. Phys. — 1987. — **62**. — P. 1968—1983.
12. *Chen C., Wu B., Jiang A., You G.* // Scientia Sinica. Ser. B. — 1985. — **28**. — P. 235—239.
13. *Andrushchak A.S., Adamiv V.T., Krupych O.M. et al.* // Ferroelectrics. — 2000. — **238**. — P. 299—305.
14. *Fedorov F.I.* Theory of Elastic Waves in Crystals. — Moscow: Nauka, 1965 (in Russian).
15. *Bondarenko V.S., Byshevskii O.A., Perelomova N.V., Chirkov L.E.* // Kristallogr. — 1985. — **30**, N 2. — P. 220—226.
16. *Andrushchak A.S., Bobitski Ya.V., Kaidan M.V. et al.* // Optica Applicata. — 2003. — **33**, N 2—3. — P. 345—357.

Received 27.07.04.

Translated from Ukrainian by O.I.Voitenko

ПРОСТОРОВИЙ АНАЛІЗ ІЗОТРОПНОЇ
ТА АНІЗОТРОПНОЇ ДИФРАКЦІЇ СВІТЛА
НА ПОПЕРЕЧНИХ АКУСТИЧНИХ ХВИЛЯХ
В КРИСТАЛАХ БЕТА-БОРАТУ БАРИЮ

A.C. Андрущак, Я.В. Бобицький, М.В. Кайдан, Б.В. Тибінка

Р е з ю м е

Для ефективної пружнооптичної сталої $p_{\text{еф}}$ та коефіцієнта акустооптичної (АО) якості M_2 на прикладі кристалів β -BaV₂O₄ побудовано вказівні поверхні та визначено їх екстремальні значення як для ізотропної, так і для анізотропної дифракції світла на поперечних акустичних хвилях. На цій основі проведено просторовий аналіз АО-взаємодії та знайдено, що для кристалів β -BaV₂O₄ найбільше значення АО-якості $M_2 = 40,1 \cdot 10^{-15} \text{ с}^3/\text{кг}$ спостерігається у випадку анізотропної дифракції світла на поперечній акустичній хвилі з меншою швидкістю. Визначено геометрію зразка та задано необхідне розміщення взаємодіючих АО-складових для найефективнішого використання кристалів β -BaV₂O₄ в АО-приладах.