EXACT COSMOLOGICAL SOLUTIONS OF THE EINSTEIN — CARTAN EQUATIONS

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In the framework of the Einstein—Cartan theory, spatially flat cosmological models with a nonminimally coupled scalar field, ultrarelativistic gas, and stiff fluid are considered. Exact partial solutions of the gravitational and massless scalar field equations are obtained for an arbitrary coupling constant. It is shown that nonsingular models are possible in some cases. For the obtained solutions, restrictions on the coupling constant are found. The influence of sources on the character of the evolution of models is elucidated.

Introduction

In the framework of the problem of existence of exact cosmological solutions in the Einstein—Cartan theory (ECT) with a nonminimally coupled scalar field [1 — 6], we continue our explorations of the possibilities of exact integrability of cosmological models in space-time with torsion, begun in [4, 5, 7, 8].

Here, we consider the multicomponent models, which are filled not only with a nonminimally coupled scalar field, but with an ultrarelativistic gas and a stiff fluid, that is, a perfect fluid with the equation of state $P_{\mathrm{f}1} = \varepsilon_{\mathrm{f}1}$ ($P_{\mathrm{f}1}$ and $\varepsilon_{\mathrm{f}1}$ are, respectively, the pressure and energy density of the fluid).

In [7], the cosmological models of flat space with a nonminimally coupled scalar field and an ultrarelativistic gas were studied within of ECT. Exact general solutions were derived for two-component models and those containing only a material scalar field ($\alpha_s = +1$) and a “gravitational” one ($\alpha_s = -1$) for an arbitrary coupling constant ξ.

It was shown that, for $\alpha_s = +1$, $\xi > 0$ and $\alpha_s = -1$, $\xi < 0$, the countable number of nonsingular models is possible.

It was demonstrated that, for $\alpha_s = +1$, $\xi > 0$ and $\alpha_s = -1$, $\forall\xi$, the models are nonsingular, while the model is singular for $\alpha_s = +1$, $\xi < 0$.

It was proved that, for $\alpha_s = +1$ and $\xi > 0$, three types of cosmological models exist, and there is one type for $\xi < 0$. Whereas only one type of the cosmological model exists for $\alpha_s = -1$ and $\xi > 0$, there are six types for $\xi < 0$.

For the obtained solutions, the special values of ξ and the restrictions on ξ were found.

In this article, we investigate the spatially flat models and the influence of sources of the gravitational field on the cosmological evolution.

1. Field Equations

The Lagrangian Ł of the model is chosen as follows :

$$Ł = -\frac{R(\Gamma)}{2æ} + \frac{\alpha_s}{8\pi}\left[\Phi_{,k}\Phi^{,k} + \xi R(\Gamma)\Phi^2\right] + Ł_p + Ł_{\mathrm{f}1}, \qquad (1)$$

where $R(\Gamma)$ is the curvature scalar obtained from the full connection $\Gamma^k_{ij} = \{^{\,k}_{ij}\} + S_{ij\cdot}{}^{k} + S^{k}_{\cdot ij} + S^{k}_{\cdot ji}$; $\{^{\,k}_{ij}\}$ are Christoffel symbols of the second kind; $S_{ij\cdot}{}^{k} = \Gamma^k_{[ij]}$ is the torsion tensor; $æ = 8\pi G/c^4$ is the Einstein constant; $Ł_p$ and $Ł_{fl}$ are, respectively, the Lagrangian of the ultrarelativistic gas and the stiff fluid.

The metric g_{ik} has signature $(-,-,-,+)$, the Riemann and Ricci tensors are defined as $R^m_{ijk\cdot} = \Gamma^m_{jk,i} - \Gamma^m_{ik,j} + \Gamma^m_{ip}\Gamma^p_{jk} - \Gamma^m_{jp}\Gamma^p_{ik}$, $R_{jk} = R^i_{ijk\cdot}$. We note that, in the framework of ECT, a scalar field nonminimally coupled to the gravitational field gives rise to torsion, even thought the scalar field has a zero spin. It follows from (1) that the torsion can interact with a scalar field only through its trace: $S_i = S^k_{ik\cdot}$ [6]. Hence, the curvature scalar can be presented in the form [6]

$$R(\Gamma) = R(\{\}) + 4\nabla_k S^k - \frac{8}{3} S_k S^k, \tag{2}$$

where $R(\{\})$ is the Riemannian part of the curvature built from Christoffel symbols; ∇_k is the covariant derivative of the Riemannian space.

We note that Lagrangian (1) is a covariant generalization of its counterpart in General Relativity (GR) and, for $\xi = 1/6$, we get the so-called conformal coupling (in the torsionless theory). As shown in [6], when $\alpha_s = -1$, $\xi = -1/6$, the scalar field corresponding to Lagrangian (1) is the axion field in GR. In cosmology, the axion field is a cold dark matter candidate (see, e.g., [6, 9] and references therein).

Varying the action with Lagrangian (1) in g_{ij}, S_k, and Φ, we obtain the following set of equations for the gravitational fields and matter:

$$G_{ij}(\{\}) = æ(T^p_{ij} + T^{fl}_{ij} + T^s_{ij}) + \Lambda_{ij}, \tag{3}$$

$$S^k = (3/2)\xi\Psi\Phi\Phi^{,k}, \tag{4}$$

$$\Box\Phi - \xi\Phi R(\Gamma) = 0, \tag{5}$$

where

$$T^p_{ij} = (\varepsilon_p + P_p)u_i u_j - P_p g_{ij}, \tag{6}$$

$$T^{\mathrm{f1}}_{ij} = (\varepsilon_{\mathrm{f1}} + P_{\mathrm{f1}})u_i u_j - P_{fl} g_{ij}, \tag{7}$$

$$T^s_{ij} = \frac{\alpha_s}{4\pi}\Big\{\Phi_{,i}\Phi_{,j} - \frac{1}{2}\Big[\Phi_{,m}\Phi^{,m} + \xi R(\{\})\Phi^2\Big]g_{ij} +$$

$$+\xi\Big[-2S_i\nabla_j - 2S_j\nabla_i + 2g_{ij}S^n\nabla_n -$$

$$-\nabla_i\nabla_j + g_{ij}\Box + R_{ij}(\{\}) - \Lambda_{ij}\Big]\Phi^2\Big\}, \tag{8}$$

$$\Lambda_{ij} = \frac{8}{3}S_i S_j - \frac{4}{3}S_k S^k g_{ij}. \tag{9}$$

Here, ε_p is the ultrarelativistic gas energy density; P_p is its pressure; u_i is the four-velocity; $\Box$ is the D'Alembertian operator of the Riemannian space, and $\Psi = æ(4\pi\alpha_s - \xi æ\Phi^2)^{-1}$.

For spatially-flat homogeneous isotropic models with the metric

$$ds^2 = a^2(\eta)[-dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) + d\eta^2], \tag{10}$$

Eqs. (3) and (5) take the form

$$2\frac{a''}{a} - \frac{a'^2}{a^2} = \Psi\Big[2\xi\Phi\Phi'' + 2\xi\frac{a'}{a}\Phi\Phi' +$$

$$+(-\frac{1}{2} + 2\xi + 3\xi^2\Phi^2\Psi)\Phi'^2\Big] - 4\pi\alpha_s a^2\Psi(P_p + P_{\mathrm{f1}}), \tag{11}$$

$$\frac{a'^2}{a^2} = \Psi\Big[(\frac{1}{6} - \xi^2\Phi^2\Psi)\Phi'^2 + 2\xi\frac{a'}{a}\Phi\Phi'\Big] +$$

$$+\frac{4}{3}\pi\alpha_s a^2\Psi(\varepsilon_p + \varepsilon_{\mathrm{f1}}), \tag{12}$$

$$(1 - 6\xi^2\Phi^2\Psi)(\Phi'' + 2\frac{a'}{a}\Phi') +$$

$$+6\xi\frac{a''}{a}\Phi - 24\pi æ^{-1}\alpha_s\xi^2\Psi^2\Phi\Phi'^2 = 0, \tag{13}$$

where the prime denotes the differentiation with respect to η.

For the ultrarelativistic gas with metric (10), the following relations are true :

$$P_p = \varepsilon_p/3, \quad \varepsilon_p = C_p a^{-4}, \quad C_p = \mathrm{const}. \tag{14}$$

For the stiff fluid we have

$$\varepsilon_{\mathrm{f1}} = P_{\mathrm{fl}} = C_{\mathrm{f1}} a^{-6}, \qquad C_{\mathrm{f1}} = \mathrm{const}. \tag{15}$$

Note that, to solve the field equations, the requirement of positivity of the Einstein's effective constant was imposed:

$$æ_{\mathrm{eff}} = æ\{1 - (\alpha_s\xi/4\pi)æ\Phi^2\}^{-1} > 0. \tag{16}$$

2. Solutions with Material Scalar Field

2.1. Solution for $\xi > 0$

The exact partial solution is obtained by quadrature :

$$a(\eta) = \beta F^{-1/2}\cosh u, \quad \Phi(\eta) = B\tanh u\ ,$$

$$\int\frac{dF}{\sqrt{s_1F^2 + s_2F}} = 2n\int\frac{du}{W_1(u)\cosh u},$$

$$t = \frac{\beta}{c}\int\frac{du}{W_1(u)F^{3/2}}. \tag{17}$$

Here, $\beta = (æ|\xi|)^{1/4}/(4\pi)^{1/2}$, $B = (4\pi/æ|\xi|)^{1/2}$; $s_1 = (C_1 + 32\pi^2 C_{\mathrm{f1}})/6\xi$; C_1 is an integration constant; $s_2 = 4\pi(æC_p/3)/(æ|\xi|)^{1/2}$; $n = \pm 1$; $W_1(u) = (C_1 + 32\pi^2 C_{\mathrm{f1}}\tanh^2 u)^{1/2}$; $C_1 + 32\pi^2 C_{\mathrm{fl}}\tanh^2 u > 0$.

Let us consider the possible models that admits solution (17).

1. $\mathbf{C_1 > 0}$. The analysis has shown that solution (17) exists only when the ultrarelativistic gas is taken into account and describes nonsingular cosmological models of three types:

1) $a|_{t\to-\infty} \sim (-t)^{1/2}, \qquad \Phi|_{t\to-\infty} \sim (-t)^{-1/2},$

$$a|_{t\to+\infty} \sim t^{2/3}, \qquad \Phi|_{t\to+\infty} \simeq B, \tag{18}$$

2) $a|_{t\to-\infty} \sim (-t)^{1/2}, \qquad \Phi|_{t\to-\infty} \sim (-t)^{-1/2},$

$$a|_{t\to+\infty} \sim \exp(H_1 t), \qquad \Phi|_{t\to+\infty} \simeq B, \tag{19}$$

3) $a|_{t\to-\infty} \sim t^{2/3}, \qquad \Phi|_{t\to-\infty} \simeq -B,$

$$a|_{t\to+\infty} \sim \exp(H_1 t), \qquad \Phi|_{t\to+\infty} \simeq B, \tag{20}$$

where t is the cosmological time $(a(\eta)d\eta = cdt) : H_1 = c(C_1 + 32\pi^2 C_{\mathrm{f}1})(\text{æ}\xi)^{1/2}/4\pi a_0^3$, $a_0 = a(t=0)$.

It is necessary to point out that, for all three types of models, reverse asymptotics are possible.

It is not difficult to see that Hubble's constant H_1 can take great values at $\xi \gg 1$. As the models are considered in the framework of the classical theory of gravitation, they will be physically permissible provided that $\varepsilon < \varepsilon_{\mathrm{pl}}$, where $\varepsilon_{\mathrm{pl}} = c^7/\hbar G^2$. As a result, we get the restriction on $\xi : \xi < a_0^6 \varepsilon_{\mathrm{pl}}(4\pi)^2/3(C_1 + 32\pi^2 C_{\mathrm{f}1})^2$.

It is easy to verify that models (18)—(20) are regular due to the violation of the strong energy condition with a scalar-torsion field. It is not difficult to show that, for the minima of the scale factor $a(t)$ and for asymptotics $a \sim t^{2/3}$, $a \sim \exp(H_1 t)$, the contribution of the scalar-torsion field dominates. For $a \sim t^{1/2}$, the contribution of the ultrarelativistic gas dominates.

2. $\mathbf{C_1 < 0}$. In this case, we have the same qualitative picture of the evolution of models as for $C_1 > 0$.

3. $\mathbf{C_1 = 0}$. The analysis has shown that, in this case, the evolution of models depends on the parameter s_2.

a) For $s_2 \neq 0$, it is possible to express $F = F(u)$ from (17) explicitly as

$$F^{1/2} = (1/2)(s_2/s_1)^{1/2}(V(u) - V^{-1}(u)), \tag{21}$$

where $V(u) = C_2(|\tanh(u/2)|)^{n/\sqrt{6\xi}}$, $V^2 \geq 1$; C_2 is an integration constant.

Note that there exist variable solutions for the scale factor a according to the value of the parameter n.

With $n = +1$, a solution exists only for $C_2 > 1$, $u \geq \ln[(1 + C_2^{-\sqrt{6\xi}})/(1 - C_2^{-\sqrt{6\xi}})]$, and $u \leq -\ln[(1 + C_2^{-\sqrt{6\xi}})/(1 - C_2^{-\sqrt{6\xi}})]$. This solution describes a nonsingular model with the asymptotics

$$a|_{t\to-\infty} \sim (-t)^{1/2},\ \Phi|_{t\to-\infty} \simeq 2B(C_2^{\sqrt{6\xi}} + C_2^{-\sqrt{6\xi}})^{-1},$$

$$a|_{t\to+\infty} \sim \exp(H_2 t), \qquad \Phi|_{t\to+\infty} \simeq B. \tag{22}$$

Here, $H_2 = c(\text{æ}\xi)^{1/2} C_p^{3/2}(C_2^2 - 1)^3/2^{5/2} C_2^3 C_{\mathrm{f}1}$. Note that reverse asymptotics are possible, and solution (22) exists for $\xi < 32 C_2^6 C_{\mathrm{f}1}^2 \varepsilon_{\mathrm{pl}}/3(C_2^2 - 1)^6 C_p^3$.

With $n = -1$, solutions exist for $C_2 \geq 1$ at $u \in (-\infty, \infty)$ and for $C_2 < 1$ at $-\ln[(1 + C_2^{\sqrt{6\xi}})/(1 - C_2^{\sqrt{6\xi}})] \leq u \leq \ln[(1 + C_2^{\sqrt{6\xi}})/(1 - C_2^{\sqrt{6\xi}})]$.

The analysis has shown that, for $n = -1$, the singular models of three types are possible. For all the types of models at early times, the scale factor and the scalar field behave in a similar way:

$$a|_{t\to0} \sim t^{1/3}, \qquad \Phi|_{t\to0} \sim t^{\sqrt{6\xi}/3}.$$

But, at the late times, the expansion laws are

1) $C_2 < 1$

$$a|_{t\to+\infty} \sim t^{1/2},\ \Phi|_{t\to+\infty} \simeq 2B(C_2^{\sqrt{6\xi}} + C_2^{-\sqrt{6\xi}}), \tag{23}$$

2) $C_2 = 1$

$$a|_{t\to+\infty} \sim t^{2/3}, \qquad \Phi|_{t\to+\infty} \simeq B, \tag{24}$$

3) $C_2 > 1$

$$a|_{t\to+\infty} \sim \exp(H_2 t), \qquad \Phi|_{t\to+\infty} \simeq -B, \tag{25}$$

where the expression for H_2 coincides with that from (22). Since solutions (23)—(25) have been obtained in two cards : $t \in (-\infty, 0)$ and $t \in (0, +\infty)$, reverse asymptotics are possible. It is not difficult to show that the contribution of the perfect fluid dominates as $t \to 0$.

Note that, for $n = -1$, $C_2 = 1$, and $\xi = 1/6$, the solution with asymptotics of the type (24) can be expressed in elementary functions as

$$a(t) = \beta(s_1/s_2)^{1/2}(1 + (\lambda t)^{2/3})^{1/2}(\lambda|t|)^{1/3},$$

$$\Phi(t) = B(\lambda|t|)^{1/3}(1 + (\lambda t)^{2/3})^{-1/2}, \tag{26}$$

where $\lambda = 3c\beta^{-1}(s_2/s_1)^{3/2}(32\pi^2 C_{\mathrm{fl}})^{1/2}$.

b) For $s_2 = 0$, the exact solution may be written as follows:

$$a(\eta) = \beta(C_2\gamma)^{-1/2}(1 + \varphi(\eta))(1 - \varphi(\eta))^{-1}(-n_1\eta)^{1/2},$$

$$\Phi(\eta) = 2n_1 B\varphi^{1/2}(\eta)(1 + \varphi(\eta))^{-1}. \tag{27}$$

Here, $\gamma = (6\xi)^{1/2}/2C_2(32\pi^2 C_{\mathrm{f}1})^{1/2}$, $n_1 = \pm1$, and $\varphi(\eta) = (-n_1\gamma\eta^{-1})^{n\sqrt{6\xi}}$.

With $n = +1$, solution (27) exists for $\eta \leq -\gamma$ ($n_1 = +1$) and $\eta \geq \beta$ ($n_1 = -1$) and describes a nonsingular model with the asymptotics ($n_1 = +1$)

$$a|_{t\to-\infty} \sim (-t)^{1/3}, \qquad \Phi|_{t\to-\infty} \sim (-t)^{-\sqrt{6\xi}/3},$$

$$a|_{t\to+\infty} \sim \exp(H_3 t), \qquad \Phi|_{t\to+\infty} \simeq B, \tag{28}$$

where $H_3 = c(4\pi C_2)^{3/2}(2C_{f1})^{1/2}(æ\xi)^{-1/4}$, $\xi^{1/2} > 6 \times (4\pi C_2)^3 C_{f1}/æ^{3/2}\varepsilon_{pl}$. Note that reverse asymptotics are possible ($n_1 = -1$).

With $n = -1$, solution (27) exists for $-\gamma \le \eta < 0$ ($n_1 = +1$), $0 < \eta \le \gamma$ ($n_1 = -1$) and describes singular models (in two cards) with asymptotics of the type (25) ($H = H_1$).

Thus, the analysis of case $C_1 = 0$ has shown that, for $s_2 \neq 0$, there are four types of cosmological models, while there are two types for $s_2 = 0$.

2.2. Solution for $\xi < 0$

An exact partial solution has the form of quadratures:

$$a(\eta) = \beta F^{-1/2} W_2^{-1/2}(u), \qquad \Phi = B\tanh u,$$

$$\int \frac{dF}{\sqrt{s_1 F^2 + s_2 F}} = 2n \int \frac{du}{W_2^{1/2}(u) W_3^{1/2}(u)\cosh^2 u},$$

$$t = \frac{\beta}{c}\int \frac{du}{W_2(u) W_3^{1/2}(u) F^{3/2}\cosh^2 u}. \tag{29}$$

Here, $W_2(u) = 1 + \tanh^2 u$, $W_3(u) = C_1 - 32\pi^2 C_{\text{fl}}\tanh^2(u)$, $W_3(u) > 0$.

One can see from (29) that, for $\forall C_1$, this solution exists only when the ultrarelativistic gas is taken into account ($s_2 \neq 0$) and describes nonsingular symmetric cosmological models: $a|_{t\to\pm\infty} \sim (|t|)^{1/2}$. The asymptotics of the scalar field may be of the following forms: $\Phi|_{t\to\pm\infty} \sim \pm(|t|)^{-1/2}$ for $\forall C_1$, $\Phi|_{t\to\pm\infty} \simeq \pm B$ for $C_1 \ge 32\pi^2 C_{\text{fl}}$ and $\Phi|_{t\to\pm\infty} \simeq \pm B\sqrt{C_1/32\pi^2 C_{f1}}$ for $C_1 < 32\pi^2 C_{f1}$.

3. Solutions with "Gravitational" Scalar Field

3.1. Solution for $\xi > 0$

An exact partial solution is obtained by quadrature as

$$a(\eta) = \beta F^{-1/2}\cosh^{-1} u, \qquad \Phi(\eta) = B\sinh u,$$

$$\int \frac{dF}{\sqrt{s_3 F^2 + s_2 F}} = 2n \int \frac{du}{W_4^{1/2}(u)},$$

$$t = \frac{\beta}{c}\int \frac{du}{W_4^{1/2}(u) F^{3/2}\cosh u}, \tag{30}$$

where $s_3 = (-C_1 + 32\pi^2 C_{\text{fl}})(6\xi)^{-1}$, $W_4(u) = C_1 + 32\pi^2 C_{\text{fl}}\sinh^2 u$.

Solution (30) yields that the integration constant C_1 may take both zero and nonzero values. Let us consider both variants.

1. $\mathbf{C_1 = 0}$. For $s_2 \neq 0$, it is possible to present $F = F(u)$ as

$$F^{1/2} = (1/2)(s_2/s_3)^{1/2}(V(u) - V^{-1}(u)). \tag{31}$$

We note that, in this case, the restrictions on C_2 and u are such that have been written out for the models with a material scalar field ($\xi > 0, C_1 = 0, n = \pm 1$).

With $n = +1$, the solution admits only a singular model with the asymptotics

$$a|_{t\to0} \sim t^{1/2}, \qquad \Phi|_{t\to0} \sim t^{-1/2},$$

$$a|_{t\to+\infty} \sim t^{1/2}, \Phi|_{t\to+\infty} \simeq 2B(C_2^{\sqrt{6\xi}} - C_2^{-\sqrt{6\xi}})^{-1}. \tag{32}$$

It is necessary to note that reverse asymptotics are possible.

As in the previous variant of the material scalar field ($\alpha_s = +1$), singular models of three types are possible for $n = -1$:

1) $C_2 < 1$

$$a|_{t\to0} \sim t^{1/3}, \qquad \Phi|_{t\to0} \sim t^{\sqrt{6\xi}/3},$$

$$a|_{t\to+\infty} \sim t^{1/2}, \Phi|_{t\to+\infty} \simeq 2B(C_2^{-\sqrt{6\xi}} - C_2^{\sqrt{6\xi}}). \tag{33}$$

2) $C_2 = 1$

$$a|_{t\to0} \sim t^{1/3}, \qquad \Phi|_{t\to0} \sim t^{\sqrt{6\xi}/3},$$

$$a|_{t\to+\infty} \simeq A_1, \qquad \Phi|_{t\to+\infty} \sim t, \tag{34}$$

where $A_1 = 4(6C_{\text{fl}}\xi/C_p)^{1/2} = \text{const}$.

It is seen from (34) that the scalar-torsion field slows down the cosmological evolution as compared to the similar evolution in GR. We'd like to note that an analogous effect in the affine-metric theory of gravity for the non-Riemannian geometric space-time objects was obtained in [6]. A salient feature of the model is the growth of a scalar field at great t. It is necessary to remark that a similar behaviour of the scalar field was typical of the exactly integrable two-dimensional cosmological model with torsion [10].

3) $C_2 > 1$

$$a|_{t\to0} \sim t^{1/2}, \qquad \Phi|_{t\to0} \sim t^{-1/2},$$

$$a|_{t\to t_0} \sim (t_0 - t)^{1/3}, \Phi|_{t\to t_0} \sim (t_0 - t)^{\sqrt{6\xi}/3}. \tag{35}$$

It is worth to note that, in this case, the scalar-torsion field creates the effect similar to a curvature [11], since the evolution of model (35) is characteristic of one of the closed type.

For $s_2 = 0$, the dependence $a(\eta)$ and $\Phi(\eta)$ is the same as (27), with the only difference that, in both the expressions for $a(\eta)$ and $\Phi(\eta)$, it is necessary to change the sign before $\varphi(\eta)$, namely from "+" to "–". In this case, we have the singular models (in two cards) with the asymptotics

$$a|_{t\to\pm0} \sim (\pm t)^{1/2}, \qquad \Phi|_{t\to\pm0} \sim (\pm t)^{-1/2},$$

$$a|_{t\to\pm\infty} \sim (\pm t)^{1/3}, \qquad \Phi|_{t\to\pm\infty} \sim (\pm t)^{-\sqrt{6\xi}/3}. \qquad (36)$$

Thus, the analysis of the solutions with $C_1 = 0$ has shown that the case $s_2 \neq 0$ admits four types of cosmological models, whereas there is one type for $s_2 = 0$.

2.$\mathbf{C_1 \neq 0}$. In this case, the models exist only for $s_2 \neq 0$, and their two types are possible:

1) $a|_{t\to\pm0} \sim (\pm t)^{1/2}, \qquad \Phi_{t\to\pm0} \sim \mp(\pm t)^{-1/2},$

$$a|_{t\to\pm\infty} \simeq A_2, \qquad \Phi|_{t\to\pm\infty} \sim \pm t, \qquad (37)$$

2) $a \sim (mt)^{1/2}, \qquad \Phi \sim -m(mt)^{-1/2}, \qquad (38)$

where $A_2 = \mathrm{const}$, $m = +1$ for $t \in (0, +\infty)$, $m = -1$ for $t \in (-\infty, 0)$.

3.2. Solution for $\xi < 0$

An exact partial solution has the following form :

$$a(\eta) = \beta F^{-1/2}\cosh u, \qquad \Phi(\eta) = B\tanh u,$$

$$\int \frac{dF}{\sqrt{s_3F^2 + s_2F}} = 2n\int \frac{du}{W_3^{1/2}(u)\cosh u},$$

$$t = \frac{\beta}{c}\int \frac{du}{W_3^{1/2}(u)F^{3/2}}. \qquad (39)$$

The investigation of solution (39) has demonstrated that physically permissible models exist for $C_1 \neq 32\pi^2 C_{f1}$ only, and the behaviour of $a(t)$ is the same and coincides with that in the models for $\alpha_s = +1$, $\xi < 0$, $s_2 \neq 0$, $C_1 \neq 32\pi^2 C_{f1}$. In addition, for $C_1 > 32\pi^2 C_{fl}$, $s_2 \neq 0$, there exists an additional nonsymmetric and nonsingular model with the asymptotics

$$a|_{t\to-\infty} \sim (-t)^{1/2}, \qquad \Phi|_{t\to-\infty} \simeq -B,$$

$$a|_{t\to+\infty} \sim \exp(H_4 t), \qquad \Phi|_{t\to+\infty} \simeq B, \qquad (40)$$

where $H_4 = c(æ|\xi|)^{1/2}(C_1 - 32\pi^2 C_{fl})/4\pi a_0^3$. Note that solution (40) exists for $|\xi| < a_0^6(4\pi^2) \times \varepsilon_{fl}/3(C_1 - 32\pi^2 C_{f1})^2$, and reverse asymptotics are possible.

Conclusion

Let's formulate the main results that have been obtained in this work.

1. In the framework of ECT, exact partial solutions for spatially flat cosmological models with a stiff fluid, ultrarelativistic gas, and nonminimally coupled scalar field have been obtained and analyzed in the case of an arbitrary coupling constant ξ.

2. As is well known, for flat Friedmann models with an ultrarelativistic gas and a stiff fluid, the following asymptotics are true :

$$a|_{t\to\pm0} \sim (\pm t)^{1/3}, \qquad a|_{t\to\pm\infty} \sim (\pm t)^{1/2}.$$

For the model with a stiff fluid only, the scale factor $a(t)$ behaves like $a \sim t^{1/3}$ with $t \in (0, +\infty)$ and $a \sim (-t)^{1/3}$ with $t \in (-\infty, 0)$.

Thus, the analysis of the exact solutions that have been obtained above shows that, as compared to similar models in GR, the presence of a scalar-torsion field leads to

- the slow-down of the late stages of the cosmological evolution : $a|_{t\to+\infty} \simeq \mathrm{const}$ for $\alpha_s = -1$, $\xi > 0$, $C_1 = 0$ ($C_2 = 1$); $C_1 \neq 0$.
- the slow–down of the initial cosmological evolution: $a|_{t\to0} \sim t^{1/2}$ for $\alpha_s = -1$, $\xi > 0$, $C_1 = 0$, ($C_2 > 1$); $C_1 \neq 0$.
- the creation of the effect similar to a curvature for $\alpha_s = -1$, $\xi > 0$, $C_1 = 0$ ($C_2 > 1$).
- the speeding–up of the late stages of the cosmological evolution:
  - $a|_{t\to+\infty} \sim t^{2/3}$ for $\alpha_s = +1$, $\xi > 0$, $C_1 = 0$ ($C_2 = 1$); $C_1 \neq 0$.
  - $a|_{t\to+\infty} \sim \exp(Ht)$ for $\alpha_s = +1$, $\xi > 0$, $\forall C_1$ and $\alpha_s = -1$, $\xi < 0$.
- the removal of the initial singularity for $\alpha_s = +1$, $\forall\xi$ and $\alpha_s = -1$, $\xi < 0$.

3. The models for $\alpha_s = +1$, $\xi > 0$, $C_1 > 0$ and $\alpha_s = +1$, $\xi < 0$, $\forall C_1$; also for $\alpha_s = -1$, $\xi > 0$, $C_1 \neq 1$ and $\alpha_s = -1$, $\xi < 0$, $C_1 \neq 32\pi^2 C_{f1}$ have been obtained only when the ultrarelativistic gas is taken into account.

4. For $\alpha_s = +1$, $\xi > 0$, $\forall C_1$ and $\alpha_s = -1$, $\xi < 0$, $C_1 > 32\pi^2 C_{f1}$, the models describe asymptotically the de Sitter universes. When Hubble's constant may grow up to Planck's values, the restrictions on ξ have been found.

5. As distinct from the two-component cosmological models with an ultrarelativistic gas and a nonminimally coupled scalar field [7], the account of a stiff fluid as an additional source of the gravitational field leads to

- singular models for $\alpha_s = \pm 1$, $\xi > 0$,
- a nonsingular one for $\alpha_s = +1$, $\xi < 0$,
- the absence of the specific values of the parameter ξ (for $\xi < 0$), i.e. those which would qualitatively change the character of the evolution of cosmological models at a fixed sign of the parameter α_s.

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ТОЧНІ КОСМОЛОГІЧНІ РОЗВ'ЯЗКИ РІВНЯНЬ ЕЙНШТЕЙНА—КАРТАНА

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Р е з ю м е

Досліджуються просторово плоскі космологічні моделі з немінімально зв'язаним скалярним полем, ультрарелятивістським газом та рідиною "жорсткого" типу у теорії Ейнштейна—Картана. Одержано точні частинні розв'язки системи рівнянь гравітаційного та безмасового скалярного полів для довільних значень сталої зв'язку. Показано, що у деяких випадках можливі несингулярні моделі. Для отриманих розв'язків знайдено обмеження на сталу зв'язку. Визначено вплив джерел на характер еволюції моделей.