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## FROM SYMMETRIC SPIN GLASSES TOWARD THE PERCOLATION THRESHOLD: DIMENSIONAL CROSSOVER

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The concept of the effective dimensionality  $d$  has been attracted to explain the violation of universality rules for critical exponents in spin glasses. Using a simple qualitative model of the magnetic host with free embedded clusters, we presume that the effective dimension of short-range spin glasses with a strong ferromagnetic bias may differ from the Euclidean value  $d_E = 3$  and tend to  $d_f = 2.48$  corresponding to the fractal dimension at the percolation threshold. Since the lowest critical dimension in spin glasses  $d_L \approx 2.5$  was theoretically predicted to be comparable to  $d_f$ , the violation is expected the more clear, the closer a spin glass to the percolation threshold (upto the break-down of the phase transition to a spin glass state). Experimental evidences in favor of this viewpoint have been obtained. Other scenarios for the violation of universality rules are also discussed.

### Introduction

Renormalization-group theory predicts that experimentally accessible critical exponents ( $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , etc.) at standard second-order phase transitions depend only on a few basic arguments: the number of order parameter components  $n$ , the symmetry and the range of the Hamiltonian, and the space dimension  $d$  [1]. It is worth to note that the latter value connects the “geometric” exponents,  $\nu$  and  $\eta$ , with the others through the hyperscaling relations

$$\alpha = 2 - d\nu, \quad \beta = \frac{\nu}{2}(d - 2 + \eta), \quad \gamma = \nu(2 - \eta),$$

<sup>1</sup>The exponent  $\eta$  depends upon the coupling distribution sharpness. For instance, in [14], the author predicts  $\eta = -0.22 \pm 0.05$  for the  $\pm J$  distribution. But if the Gaussian distribution is assumed,  $\eta$  equals  $-0.45 \pm 0.05$  [19].

$$\delta = \frac{d + 2 - \eta}{d - 2 + \eta}, \quad \phi = \frac{\nu}{2}(d + 2 - \eta), \quad \text{etc.} \quad (1)$$

But it was recently argued [2] that  $d$  from Eqs. (1) is the “effective” dimension and may differ from the Euclidean quantity represented by integers  $d_E = 1, 2, 3, \dots$ . This concept seems very attractive to revise critical properties of those systems, wherein both the usual universality rule (identical values of all exponents for all members of a family with fixed basic arguments, mentioned above) and even the “weak” one (invariable value of  $\eta$  and, therefore, constant ratios of  $\beta/\nu$ ,  $\beta/\gamma$ , etc. [3]) are violated.

The most prominent amongst them are the bulk samples ( $d_E = 3$ ) of spin glasses (SG). These are the systems with random and frustrated interactions [4], which exhibit freezing transitions at a nonzero temperature  $T_F$  with a nearly constant exponent  $\alpha \approx -2$ , but rather scattered indices  $2.1 \leq \gamma \leq 3.4$ ,  $0.38 \leq \beta \leq 1.1$ , and, especially,  $3.3 \leq \delta \leq 10$  [5–9]. These peculiarities are reasonably explained in terms of the effective dimension approach [2, 10]. Since the lowest critical dimension  $d_L$  (i.e., dimension at which  $T_F = 0$ ,  $\beta = 0$ ,  $\delta \rightarrow \infty$ ) in SGs lies between 2 and 3 [4, 11] ( $d_L = 2.64 \pm 0.10$  [12] and  $2.5 \pm 0.2$  [13]), even a small deviation of the effective dimension  $d$  (from the Euclidean dimensionality  $d_E = 3$  toward  $d_L \approx 2.5$ ) can noticeably change the critical behavior in bulk SGs. In addition,  $\eta$  is usually a small negative value,  $-0.2 \leq \eta \leq -0.5$ <sup>1</sup>, so these changes can be primarily

manifested [through the term  $[d - 2 + \eta]$  in Eqs. (1)] in the dispersions of  $\beta$  and  $\delta$ .

Unfortunately, in [2], one proposes no ideas how to measure the effective dimension. Nevertheless, some arguments in favor of this viewpoint may be obtained by the following manner. One should, at first, search for a physical parameter that is related to  $d$  and easy for an experimental check. Then varying this parameter, one should try to suppress the paramagnet (PM) to SG phase transition. The most suitable for this purpose seems the ferromagnetic (FM) bias  $\varepsilon$  at the center of the coupling constant  $J_{ij}$  distribution. In the simplest case of the Gaussian distribution with a maximum at  $J_0$  and a width  $J$  [4] the value  $\varepsilon = J_0/J$  allows one to determine: To what magnetic state does PM transform at low temperatures? When  $\varepsilon > 1$ , FM occurs below the Curie temperature  $T_C \approx J_0$ , otherwise ( $0 < \varepsilon < 1$ ) SG does. But more significant now is another distinction whereby this parameter becomes coherent with the effective dimension. Along the boundary PM–SG line ( $T_F \approx J$ ) in the discussed interval  $0 < \varepsilon < 1$ , the bias  $\varepsilon$  may serve as an yardstick for the finite length of thermal FM correlations

$$\xi_{\text{FM}}(T = T_F) \sim \left[ \frac{T_F}{J_0} - 1 \right]^{-\nu'} \sim \left[ \frac{1 - \varepsilon}{\varepsilon} \right]^{-\nu'}, \quad (2)$$

where  $\nu' \approx 0.7$  [16]. Since this length vanishes in the so-called “symmetric” SGs (i.e. systems wherein the exchange distribution is symmetric to the zero point  $\varepsilon \approx 0$ ), all spins take part in the freezing processes and, hence, the Euclidean space is completely available for the SG phase:  $d(\varepsilon = 0) \approx d_E = 3$ . But the closer the studied SG to the percolation threshold ( $\varepsilon_p = 1$ ) of the long-range FM order, the larger extraneous FM inclusions [see Eq. (2)] it contains. If there is a non-magnetic interface *isolating* these inclusions from spins of the SG host, the whole magnetic structure becomes porous. There is no importance what kind of magnetic order does appear inside these pores — FM, any other, or none at all. The main point is that, due to a non-magnetic interface, the pores act like a magnetic isolator between the rest of spins participating in the PM to SG phase transition. The samples become similar to multilayered “SG — magnetic isolator — SG...” sandwiches tailor-made to study the dimensional  $3D \rightarrow 2D$  crossover [11]. But, in contrast to this laborious technology, we propose to explore natural SGs and to monitor “isolator” thicknesses through the FM bias,  $\varepsilon$ .

At last, one can also find some similarities between the porous magnetic structure of SGs with  $\varepsilon \rightarrow 1$  and fractal aggregates [17]. In view of this, the following is

worth to be mentioned. The fractal dimension  $d_f \approx 2.48 \pm 0.09$  of the critical cluster, that occurs exactly at the percolation threshold  $\varepsilon_P$  [17], is comparable with the theoretical limit for the lowest critical dimension  $d_L \approx 2.5$  [13] in SGs. This fact instills some optimism that the genuine PM to SG phase transition may be broken-down ( $\beta \rightarrow 0, \delta \rightarrow \infty$ ) as the FM bias approaches unity:  $\varepsilon \rightarrow \varepsilon_p = 1$ .

## 1. Samples and Experimental Details

Under selecting the bulk SG samples suitable to check the effective dimension concept [2, 10], we went by two requirements. The prime one was for their FM and SG subsystems to be *isolated* one from another. This property restricted our search to, mainly, the amorphous materials wherein long-range interactions, e.g., dipolar forces, superexchange mediated by metalloid atoms, Ruderman–Kittel–Kasuya–Yosida (RKKY) interactions, etc., if present, have a negligible effect as compared with the nearest-neighbor exchange. This choice is also confirmed by the fact that the model of a magnetic host with embedded extraneous clusters [19] is widely applied in random FMs with quenched disorder [16], i.e. in systems with  $\varepsilon \geq 1$ .

The second condition was for the SGs to have various FM biases that cover the interval  $0 < \varepsilon < 1$  evenly enough. In general, this task could be resolved within a single system of quasibinary alloys whose compositions are close to the percolation threshold. But physical systems in this compositional region are extremely sensitive to the FM element doping. Let us consider, for instance, amorphous FM alloys of Fe–Ni with  $\varepsilon \rightarrow 1_+$ . A loss of 1 at.% Fe decreases their Curie temperatures  $T_C \approx J_0$  by approximately 20 K [16]. One can also expect a similar influence ( $\approx 20$  K/at.% Fe) in those samples where  $\varepsilon \rightarrow 1$ , i.e. in SGs. Since their freezing points  $T_F \approx J$  barely exceed 20 K [9], a strong FM bias may remain only in the very *narrow* interval (less than 1 at.% Fe) adjacent to the percolation limit. The rest of SGs possesses a nearly symmetric coupling distribution,  $\varepsilon \approx 0$ . Because almost the same holds true in other systems (Fe–Mn, Fe–Cr, Co–Ni, etc. [16]), one should not be astonished that, excepting the values  $\beta \approx 0.38$  and  $\delta \approx 10$  reported for the  $(\text{Fe}_{0.15}\text{Ni}_{0.85})_{75}\text{P}_{16}\text{B}_6\text{A}_{13}$  sample with  $\varepsilon = 0.55 \pm 0.15$  [9], the exponents usually cluster around  $\beta \approx 1, \delta \approx 3.5$  [6–8].

For this reason, before selecting the apposite samples, we checked the alloys with various compositions. These alloys were prepared by the melt-spinning technique and were shaped as long thin ( $15 \div 20 \mu\text{m}$ ) ribbons. We

tested the ribbons by X-ray diffraction and, then, cut and packed into samples whose typical sizes were  $2 \times 2 \times 10$  mm. To estimate the ratio  $\varepsilon = J_0/J$ , the width  $J$  and the center  $J_0$  of the exchange distribution were studied. In terms of the mean-field approach (see review [4]) both values may be extracted from the temperature dependence of the linear AC-susceptibility  $\chi_1(H, T)$  measured in the absence ( $H = 0$ ) of a DC-magnetic field. The cusp points at  $T = T_G$  (marked by arrows in Fig.1, inset) are close to the genuine critical temperatures  $T_F = J$ . Although the further scaling procedure may give a more precise value  $T_F$ , the difference  $[T_G - T_F]$  seems small as compared to the  $J_0$ 's extrapolation error (see Fig.1). The largest problem is that short-range magnets with a quenched disorder do not obey the well-known Curie–Weiss law  $\chi_1(H = 0, T) \approx (T - J_0)^{-\gamma'}$ ,  $\gamma' = 1$ . In these systems,  $\gamma'$  depends on their composition as well as the reduced temperature [16]. This peculiarity makes the extrapolation procedure very ambiguous. Fortunately, in our case  $\varepsilon \approx 1$ ,  $(T/J_0 - 1) \gg 10^{-3}$  this exponent was predicted to be constant  $\gamma' = 2$  [18]. Experimental data in [16] confirm this value and so do our results (see Fig. 1). Following the described procedure, we inspected a lot of samples, but only two of them satisfied us. The amorphous  $(\text{Fe}_{0.65}\text{Mn}_{0.35})_{75}\text{P}_{16}\text{B}_6\text{A}_{13}$  and  $(\text{Fe}_{0.07}\text{Ni}_{0.93})_{77}\text{B}_{13}\text{Si}_{10}$  alloys with biases  $\varepsilon = 0.35 \pm 0.15$  and  $0.80 \pm 0.06$ , respectively, were chosen so that our investigations could complete disembodied data obtained for SGs with  $\varepsilon \approx 0$  [7, 8] and  $\varepsilon = 0.55 \pm 0.15$  [9].

In order to extract critical exponents, we used the AC-technique that was *ad hoc* developed [20] to explore SGs with a strong FM bias. Instead of the generalized susceptibility

$$\chi(H, \tau) = \chi(0, \tau) - \chi_{NL}(H, \tau) = \chi(0, \tau) - |\tau|^{\beta} F_{\pm}(x), \quad (3)$$

where  $x = H^2 / |\tau|^{\beta\delta}$  is the usual variable for the scaling function  $F_{\pm}(x)$  which suffixes “+” or “-” refer to the positive or negative reduced temperature  $\tau = (T - T_F)/T_F$ , the magnetic field derivative

$$\frac{\partial \chi(H, \tau)}{\partial H} \equiv \chi_2(H, \tau) = -2H |\tau|^{\beta(1-\delta)} F'_{\pm}(x) \quad (4)$$

was studied. This value was directly measured by lockin-amplifier operating at the double frequency ( $2f$ ) of the sin-wave exciting magnetic field  $h = h_0 \sin(2\pi ft)$ , where  $f = 75$  Hz and  $h_0 = 6$  Oe. More details may be found in [20].

## 2. Experimental Results

At first, let us briefly discuss the common methods to extract critical exponents. Since  $\chi_2(H, \tau)$  is the function

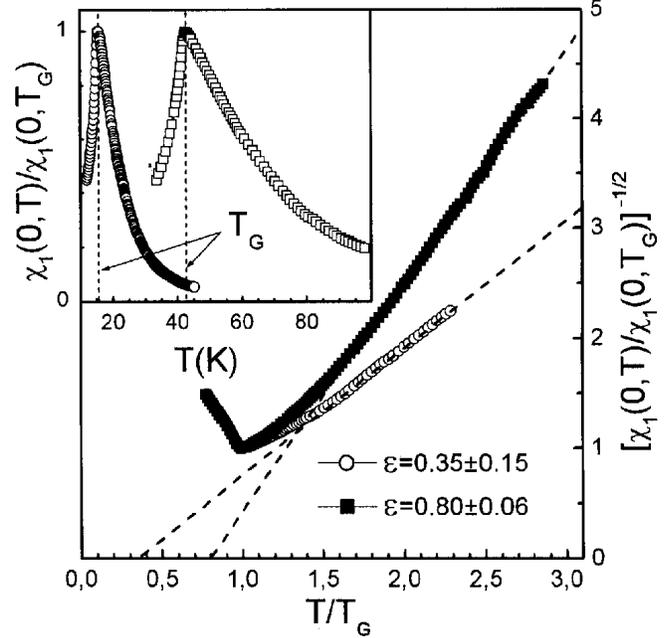


Fig. 1. FM bias estimation procedure for amorphous  $(\text{Fe}_{0.65}\text{Mn}_{0.35})_{75}\text{P}_{16}\text{B}_6\text{A}_{13}$  and  $(\text{Fe}_{0.07}\text{Ni}_{0.93})_{77}\text{B}_{13}\text{Si}_{10}$  alloys. Inset: original temperature dependences of AC-susceptibility  $\chi_1(H = 0, T)$  normalized, for the sake of convenience, to unity. Dashed lines mark the first approximations  $T_G = 15.6$  K and  $T_G = 42.5$  K for the critical temperatures  $T_F$  in the studied samples. The amplitude of the exciting magnetic field is  $h_0 = 0.3$  Oe, the frequency is  $f = 75$  Hz

of two arguments, in the usual coordinate system with three mutually perpendicular axes,  $\chi_2$ ,  $H$  and  $\tau$ , it looks like a surface. Within the scaling approach, there is a unique couple of critical indices,  $\beta$  and  $\delta$ , which provides a collapse of this surface onto the function  $F'_{\pm}(x)$ . So, the most evident method is the direct reconstruction of the scaling curve,  $F'_{\pm}(x)$ . On the other hand, each point of this curve corresponds, on the surface  $\chi_2(H, \tau)$  to the so-called “crossover line”, wherealong a scaling parameter  $x$  remains constant and so does the factor  $F'_{\pm}(x)$  in Eq. (4). Thus, being projected onto the mutually perpendicular planes wherein  $\tau$ ,  $H$  or  $\chi_2$  is constant, these crossover lines must obey simple asymptotic laws, respectively

$$\chi_2 \sim H^{2/\delta-1}, \quad (5)$$

$$\chi_2 \sim \tau^{\beta(1-\delta/2)}, \quad (6)$$

$$\text{and } \tau \sim H^{2/\beta\delta}. \quad (7)$$

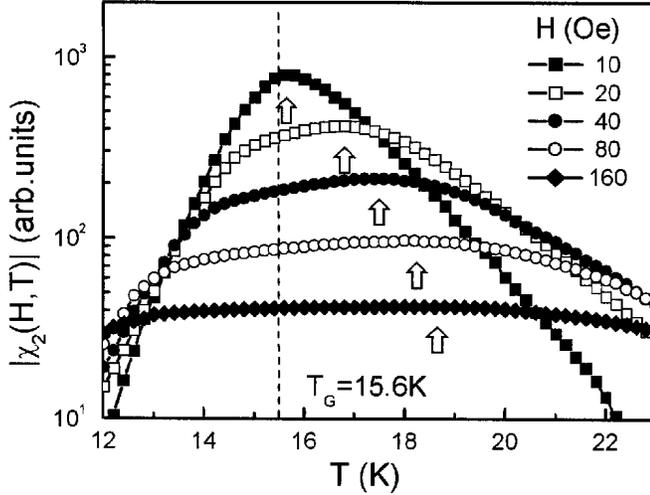


Fig. 2. Temperature variation of the nonlinear susceptibility  $\chi_2(H, T)$  measured in various DC magnetic fields for amorphous  $(\text{Fe}_{0.07}\text{Ni}_{0.93})_{77}\text{B}_{13}\text{Si}_{10}$  alloy. The amplitude of the exciting magnetic field is  $h_0 = 6$  Oe, the frequency is  $f = 75$  Hz. The arrows denote the temperatures  $T_{\max}(H)$  where the curves reach the maximum values. The vertical dashed line marks the temperature  $T_G$  of the linear susceptibility cusp (see Fig. 1, inset)

This allows one to extract critical indices even without a knowledge of the scaling function  $F_{\pm}(x)$ . The same idea is in a daily use when critical isotherms ( $T = T_F$ ) and zero-field dependences ( $H = 0$ ) are explored. In the proposed terms, the first case corresponds to the crossover line with  $x = \infty$ , the second does to  $x = 0$ . But both lines belong to basic planes, either  $\tau = 0$  or  $H = 0$ . It gives a single projection, either (5) or (6), instead of three possible ones, (5)–(7), which are available when  $0 < x < \infty$ . For this reason, any concept suitable for the line with  $0 < x < \infty$  to be selected is worthy of the detailed discussion.

Fig. 2 represents the temperature dependences of the nonlinear susceptibility  $\chi_2(H, T)$  measured after the  $(\text{Fe}_{0.07}\text{Ni}_{0.93})_{77}\text{B}_{13}\text{Si}_{10}$  sample's cooling in various magnetic fields<sup>2</sup>. In other words, these curves are the cross-sections of the surface,  $\chi_2(H, \tau)$ , by isofield planes  $H = 10, 20, 40,$  and  $80$  Oe. Let us focus our attention on extrema (arrows in Fig. 2) at  $T_{\max}(H)$ , i.e. at  $\tau_{\max}(H) =$

<sup>2</sup>In order to escape overloading this manuscript by original experimental curves which look similar for both studied samples, we will report these data only for the alloy  $(\text{Fe}_{0.07}\text{Ni}_{0.93})_{77}\text{B}_{13}\text{Si}_{10}$ . Of necessity, the omitted information about another sample,  $(\text{Fe}_{0.65}\text{Mn}_{0.35})_{75}\text{P}_{16}\text{B}_6\text{A}_{13}$ , may be found in [20].

<sup>3</sup>Other crossover line(s) may be localized by the points

$$\frac{\partial \chi_2}{\partial H}(H, \tau) = -2\tau^{\beta(1-\delta)} [F'_{\pm}(x) + 2xF''_{\pm}(x)] = 0,$$

where the  $\chi_2(H, \tau)$  vs  $H$  curves exhibit their extrema. These points obey the same Eqs. (5)–(7) and are well applicable for critical exponents to be estimated [20].

$1 - T_{\max}(H)/T_F$ , which satisfy the equation

$$\frac{\partial \chi_2}{\partial \tau}(H, \tau) = 2\beta H |\tau|^{\beta(1-\delta)-1} \times \\ \times [(\delta - 1)F'_{\pm}(x) + \delta x F''_{\pm}(x)] = 0, \quad (8)$$

Since both  $H$  and  $\tau_{\max}(H)$  are different from 0, Eq. (8) is valid when  $[\dots] = 0$ . Thus, these extremum points may serve as markers for such *constant* value of the scaling argument  $x$  that causes the expression in brackets to vanish<sup>3</sup>. It gives us an opportunity to use only these points, when critical exponents will be estimated, and *pro tem* to forget about the rest of experimental data in Fig. 2.

The first exponent,  $2/\delta - 1$ , is readily calculated from the slope of projection (5) plotted on the “log-log” scale. But since the  $\delta$  values are expected for both samples ( $\varepsilon = 0.35$  and  $\varepsilon = 0.80$ ) to much exceed 2, the power exponents in Eq. (5) may be close to  $-1$ . To visualize the FM bias influence onto the critical properties of studied SGs, instead the initial “log-log” scale,  $\chi_2(H, \tau_{\max})$  vs  $H$ , we have to use another,  $\chi_2(H, \tau_{\max}) \times H$  vs  $H$ . In this case, the obtained dependences would remain to be straight lines, but their slopes would differ one from another more distinctly. Owing to this trick, Fig. 3 shows a convincing argument in favor of the effective dimension approach. It was originally assumed that the closer SG to the percolation threshold, the smaller is to be  $d$  and, hence, the larger is to be the exponent  $\delta$  [see Eqs. (1)]. In fact, when  $H \geq 15$  Oe, the regular linear fit gives the slope  $2/\delta = 0.334 \pm 0.005$  (i.e.  $\delta = 6.0 \pm 0.1$ ) for the sample with  $\varepsilon = 0.35$ . In the other case,  $\varepsilon = 0.80$ , the value  $\chi_2(H, \tau_{\max}) \times H$  appears almost independent of  $H$ . The slope, at least in the interval  $15 \leq H \leq 100$  Oe, equals  $2/\delta = 0.020 \pm 0.015$ . Since an application of the absolute value  $\delta \geq 57$  looks somewhat senseless, we will hereafter refer to the reciprocal exponent,  $1/\delta$ .

In order to answer the question: To what extent may this data be relied on? — one needs to discuss another problem: What reasons are resulted in the scaling hypothesis (4) to be a good approximation for experimental values  $\chi_2(H, \tau)$  only inside a certain range

$H_{\min} \leq H \leq H_{\max}$ ? Let us begin with the lower limits  $H_{\min}$  which are nearly equivalent,  $\approx 15$  Oe, for both samples. This equality confirms the conclusion [20] that the low-field deviation from the scaling behavior is just resulted from the experimental error of AC-methods, namely the inevitable contribution  $E_{rr}$  of high-order harmonics into the signal  $\chi_2(H, \tau)$ . Having no other information about the scaling function  $F_{\pm}(x)$  except for the well-known asymptotic law  $F_{\pm}(x \rightarrow \infty) \sim x^{1/\delta}$ , one can estimate only the upper bond for this contribution (see [19], Appendix)

$$E_{rr} \leq 2 \left( \frac{h_0}{H} \right)^2 \frac{(\delta - 1)(\delta - 2)}{\delta^2}. \quad (9)$$

Since, under  $\delta \gg 1$ , this mainly depends on the ratio  $h_0/H$ , the same (for both samples) AC-field amplitudes  $h_0 = 6$  Oe have to cause almost the same  $H_{\min}$ . Although along the proposed boundary,  $H_{\min} \approx 15$  Oe, Eq. (9) overestimates the errors by rather large values, namely 18% ( $\delta = 6$ ) and 32% ( $\delta \rightarrow \infty$ ), the genuine errors are much smaller and the accuracy proliferates, the stronger magnetic field  $H$  is applied. Another situation is observed for the high-field boundary  $H_{\max}$ . When  $\varepsilon = 0.35$ , the presented data show that  $H_{\max} \gg 300$  Oe, otherwise ( $\varepsilon = 0.80$ )  $H_{\max}$  is well defined as  $\approx 100$  Oe. This behavior is in agreement with numerical calculations of mean-field SGs with a strong FM bias,  $\varepsilon \rightarrow 1$ . Considering, for instance, SG with  $\varepsilon = 0.7$  that is close enough to our  $\varepsilon = 0.8$ , Roshko et al. [21] revealed critical isotherm violations when the reduced variable  $\mu_B g H / k_B T_F \geq 10^{-2}$ . Substituting  $T_F = 15$  K, one can obtain  $H_{\max} \approx 10^3$  Oe. Whereas  $H_{\max}$  is dramatically decreased with approaching the threshold  $\varepsilon \rightarrow 1$  [21], different remotenesses from this point,  $(1 - \varepsilon) = 0.3$  and  $(1 - \varepsilon) = 0.2$ , still may account for the disagreement by one order of magnitude between theoretical and experimental values of  $H_{\max}$ . But this difference is hardly enough to result in two orders or more that are required to mistrust the data in the selected range  $15 \leq H \leq 100$  Oe.

The next step is to estimate another exponent, for example,  $2/(\beta\delta)$  [see Eq. (7)]. But this responds for the *reduced* temperature dependence, whose processing on the common-used “log-log” plot requires a predefined  $T_F$ . In addition, ones usually believe that  $T_F$  remains constant, whatever  $H$  is applied. Meantime, a magnetic field is well known [22–24] to affect the freezing temperature

$$\tau_{AT}(H) = 1 - \frac{T_F(H)}{T_F(0)} = \left[ \frac{H}{H_{AT}} \right]^{2/\phi}$$

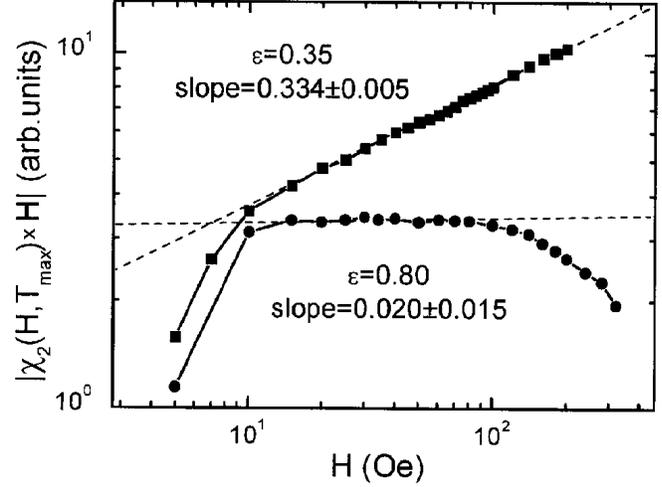


Fig. 3. The values  $\chi_2(H, T_{\max}) \times H$  plotted against magnetic field  $H$  on the “log-log” scale. Squares and circles correspond to the samples with  $\varepsilon = 0.35$  and  $\varepsilon = 0.80$ , respectively. The slopes are equal to  $2/\delta$

$$\text{with } H_{AT} = \frac{2k_B T_F}{\mu_B \sqrt{5}} (1 - \varepsilon) \quad (10)$$

the stronger, the closer is  $\varepsilon$  to 1. Considering the sample  $(\text{Fe}_{0.07}\text{Ni}_{0.93})_{77}\text{B}_{13}\text{Si}_{10}$  wherein  $\varepsilon \approx 0.8$  and  $T_F \approx T_G = 15.6$  K (see Fig. 1, inset), one can readily calculate that  $H_{AT} = 3.9 \times 10^4$  Oe. After substituting the typical values,  $H = 80$  Oe and  $2/\phi \approx 0.6$  [5–7], one can estimate  $\tau_{AT} \approx 0.025$ . This value seems no longer negligible. Experimental results are still less optimistic. Since  $T_F(H)$  are usually related to temperatures  $T_{irr}(H)$  where the abnormal magnetic absorption occurs, we monitored magnetic losses through the phase angle  $\varphi(H, T) = \arctg[\chi''_1/\chi'_1]$  between the exciting magnetic field and the sample response (see Fig.4). Under the same condition  $H = 80$  Oe, the temperature  $T_{irr} \approx 13.7$  K, corresponding to the threshold criteria  $\varphi_{irr} = 0.1$  (angle degrees), appears much lower than that predicted theoretically. The experimental shift approximately equals  $\tau_{AT} \approx 0.15$  that is, at least, five times larger than that obtained from Eq. (10). This discrepancy is usually explained in terms of the observation time scale  $t \approx 1/f \approx 10^{-2}$  s that influences  $T_{irr}$ , but does not change the power exponent,  $2/\phi$  [24]. Because  $\phi = \beta\delta$  within the scaling approach [1], the irreversibility line must exhibit the same asymptotic behavior as  $\tau_{\max}(H)$  from Eq. (7). Hence, the same exponent,  $2/\phi$ , has to remain for the difference

$$[T_{\max}(H) - T_{irr}(H)] \approx H^{2/\phi}. \quad (11)$$

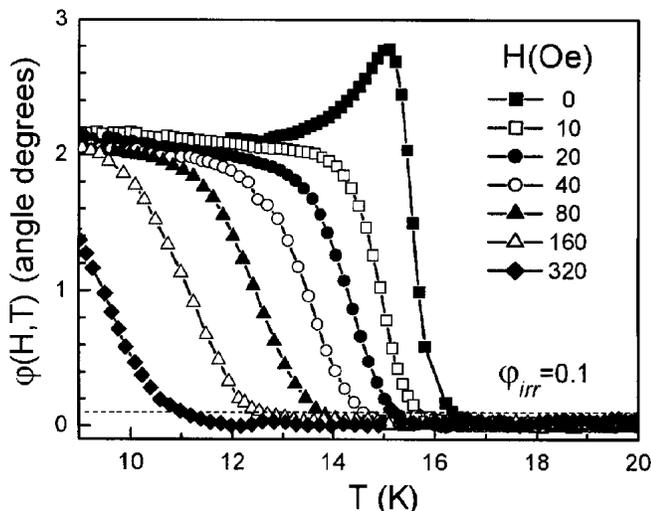


Fig. 4. Temperature dependences of the phase lag  $\varphi(H, T)$  between the exciting magnetic field  $h = h_0 \sin(2\pi ft)$ , where  $h_0 = 0.3$  Oe and  $f = 75$  Hz, and the response of the sample  $(\text{Fe}_{0.07}\text{Ni}_{0.93})_{77}\text{B}_{13}\text{Si}_{10}$  onto this excitation in various magnetic fields  $H=0, 10, 20, 40, 80,$  and  $320$  Oe. The dashed line marks the threshold  $\varphi_{irr} = 0.1$  used to determine the irreversibility temperatures  $T_{irr}(H)$

Eq. (11) allows us to ignore the field dependence  $T_F(H)$  and to avoid the pre-estimation of the critical temperature  $T_F(0)$ . In other words, due to this hint,  $\phi$  becomes available through the usual “log-log” technique. As compared with data in Fig. 3, this procedure (see Fig. 5) reveals only a nice distinction between the studied samples in the same interval  $15 \leq H \leq 100$  Oe. One can conclude that  $\phi$  appears to grow as  $\varepsilon$  is increased, but this growth seems not so evident as predicted in [23, 24]:  $\phi(\varepsilon = 1) = 7$ . The estimated values are  $\phi = 3.20 \pm 0.04$  and  $\phi = 3.60 \pm 0.09$  for, respectively,  $\varepsilon = 0.35$  and  $\varepsilon = 0.80$ .

Substituting the obtained values of  $2/\delta$ , we would calculate  $\beta$  as well as other exponents through simple scaling relations [1]:  $\beta = \phi/\delta$ ,  $\gamma = \phi(\delta - 1)/\delta$ ,  $\alpha = 2 - \phi(\delta + 1)/\delta$  (see Table). But in general, it requires the scaling hypothesis to be proved. This is usually confirmed in two different ways. The first method is to estimate the third exponent and to test, at least, one of the mentioned relations. Another is the direct reconstruction of the scaling function  $F'_{\pm}(x)$ , whose collapse with the experimental data in Fig. 2 is achieved by fitting three variables:  $\delta$ ,  $\phi$  and  $T_F$ . Since fitting parameters have to repeat the exponents describing crossover lines, this procedure seems to us more preferable now. Aside from a convincing argument in favor of assumption (3), it will allow us to compare the results obtained by various

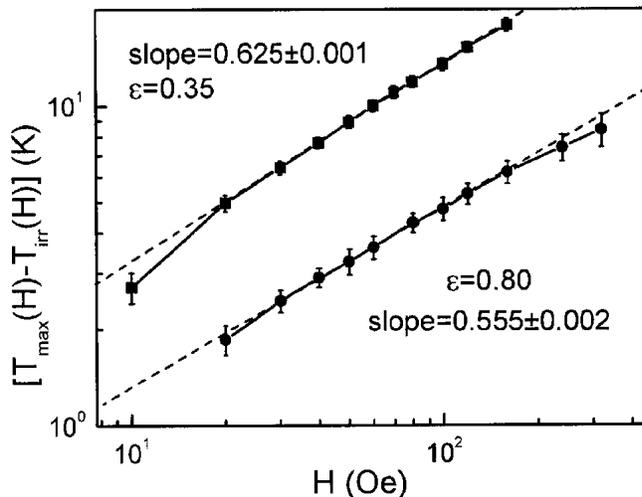


Fig. 5. The values  $[T_{\max}(H) - T_{irr}(H)]$  from Eq. (11) plotted against magnetic field  $H$  on the “log-log” scale. Squares and circles correspond to the samples with  $\varepsilon = 0.35$  and  $\varepsilon = 0.80$ , respectively. The slopes are equal to  $2/\phi$

methods and, thus, to estimate the *genuine* error gap, but not the leastsquare deviation from the optimum slopes as in Figs. 3, 5. Although experimental data for this standard routine (see Fig. 6) were chosen to be inside the admissible range of magnetic fields  $15 \leq H \leq 100$  Oe and temperatures  $T > T_{irr}(H)$ , the best-fit results do differ from the previous estimates. Being more evident for the sample  $\varepsilon = 0.80$ , this difference may be concerned with the inevitable fault of the fitting procedure that disregards the magnetic field influence on the critical temperature  $T_F$ . Despite a certain disaccord (see Table), both techniques nevertheless confirm the main tendency — an increase of the FM bias diminishes the values  $\beta$  and  $1/\delta$ . But this conclusion is just a forcible argument in favor of the effective dimension approach. The unappealable proof demands to find one of the “geometric” indices, either  $\eta$  or  $\nu$ , and to calculate the direct dependence of the effective dimension  $d$  on  $\varepsilon$  from the hyperscaling relations (1). Unfortunately, we have encountered no reliable experimental method that could allow one to estimate these indices. For this reason, we

#### Critical exponents in the studied SGs with various FM biases, $\varepsilon$

$\varepsilon$	$1/\delta$	$\phi$	$\beta$	$\gamma$	$\alpha$
0.35	$0.167^a$	$3.20^a$	$0.53^c$	$2.67^c$	$-1.73^c$
	$0.156^b$	$3.00^b$	$0.47^c$	$2.53^c$	$-1.47^c$
0.80	$0.010^a$	$3.60^a$	$0.04^c$	$3.56^c$	$-1.64^c$
	$0.055^b$	$3.80^b$	$0.21^c$	$3.59^c$	$-2.01^c$

$a$  — estimated along crossover lines,  $b$  — extracted from scaling plots,  $c$  — calculated from scaling relations.

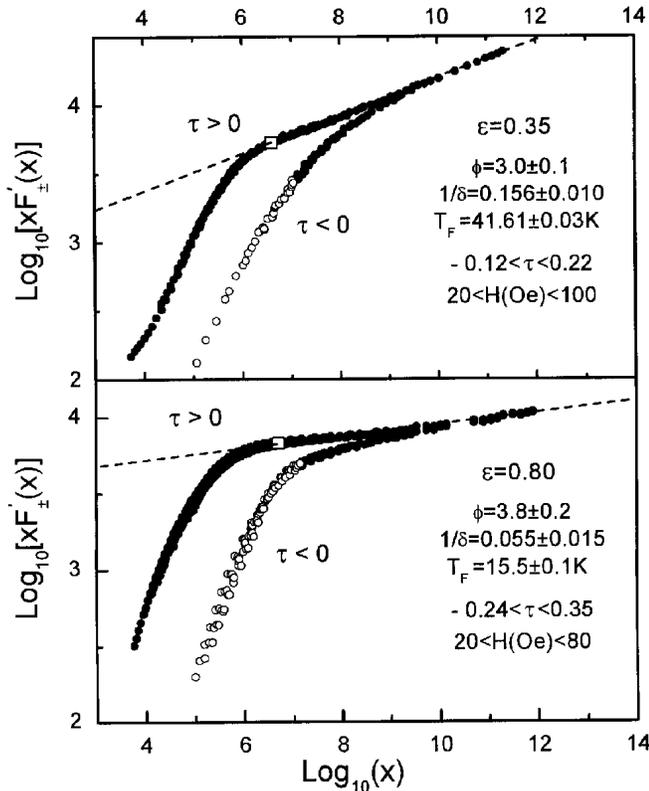


Fig. 6. Scaling plots of  $xF'_\pm(x)$  for SGs with a different FM biases,  $\varepsilon$ . When selecting the best exponents, we disregarded the unreliable experimental points [o], whose temperatures are lower than the irreversibility line  $T_{irr}(H)$ . Open squares restrict the area wherein crossover lines (8) are focused. The slopes of dashed lines are equal to  $1/\phi$

are now authorized to extract only the magnitudes  $dv$  and  $d/(2-\eta)$  presented in Fig. 7. But their sentence to the FM bias still may be explained within two different scenarios resulting in the break-down of the PM to SG phase transition:  $d/(2-\eta) \rightarrow 1$ . The first scenario ensues from the concept [2, 10] of effective dimension  $d \leq 3$ , the second occurs if one adheres to the condition  $d = 3$ . In the next chapter, we discuss both scenarios, their values and faults. This discussion will prompt us what physical parameter is linked to the non-universality.

### 3. Discussion

Let us begin with the traditional viewpoint,  $d = 3$ . In this case, the  $\nu$  values cluster around  $4/3$ , whereas the  $\eta$  values are dispersed so that  $\eta$  tends to  $-1$  as  $\varepsilon \rightarrow 1$ . This peculiarity contradicts the usual renorm-group predictions [1] and requires to revise the theory of the critical behavior in SGs. The attempt was undertaken

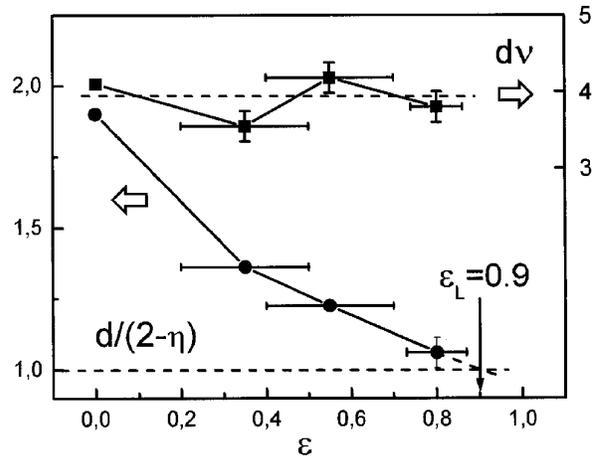


Fig. 7. The influence of the FM bias,  $\varepsilon$ , onto critical exponents  $d=(2-\eta)$  (circles) and  $d$  squares). The arrow marks the bias  $\varepsilon_L = 0.9 \pm 0.1$  which satisfies the condition  $d = (2-\eta) = 1$  of the lowest critical dimension. The points at  $\varepsilon = 0$  correspond to the best experimental estimates [4–8], the points at  $\varepsilon = 0.55$  are taken from [9]

by de Almeida [25]. In particular, he claimed that the *linear* renorm-group approach is inappropriate in Ising SGs, since the standard rescaling factor  $b$  has to be replaced by a set of factors  $b^{\alpha\beta}$ . Then, Bernardi and Campbell [26, 27] evolved this concept. Their computer simulations showed that the non-universality ( $\eta \neq \text{const}$ ) of bulk Ising SGs may be explained in terms of different sharpnesses  $R = \langle J_{ij}^4 \rangle / \langle J_{ij}^2 \rangle^2$  in the exchange distribution. It was briefly pointed out that other parameters, e.g., the type of lattice, the range of interactions as well as the FM bias  $\varepsilon$ , may also be pertinent. But no specific data on how  $\varepsilon$  influences the critical exponent  $\eta$  were obtained.

More promising is the concept of effective dimension [2, 10]. Taking this assumption as the starting point and using a simple qualitative model of the magnetic host with free embedded clusters [19], we have already predicted and revealed the non-universal behavior  $\eta(\varepsilon) \neq \text{const}$  in amorphous SG systems wherein long-range interactions, if present, have a negligible effect as compared to the nearest-neighbor exchange [16]. Another question appears: Is this concept consistent with constant values  $dv$  (see Fig. 7)? To answer this, let us just recall that the expected reduction of the effective dimension ( $d_E - d$ ) barely exceeds 0.5 or 15%, that is comparable to an experimental error. In addition, the critical exponent  $\nu$  usually varies with  $d$  so that  $dv$  does remain nearly constant (see, for instance, the data compiled from different sources in Fig. 8 in [27]).

But there are more reasons why this viewpoint seems preferable. At first, if the *long-range* interactions (e.g. dipolar forces, RKKY-interactions, etc.) are responsible for the SG formation, their length may exceed the nonmagnetic interlayer separating the host from clusters. In this case, clusters are no longer free of the freezing of other spins at  $T = T_F$  in a SG state. Then, the effective dimension is constant irrespectively of the FM bias and so are the exponents. This conclusion well agrees with the experimental data by Williams [6], who found almost the same critical indices ( $\beta \approx 1$ ,  $\gamma \approx 2$ ) in diluted RKKY-alloys with a strong FM bias  $\varepsilon \geq 0.9$  as, for instance, Bouchiat [7] did in “symmetric” ( $0 \leq \varepsilon \leq 0.3$ ) SGs of AgMn.

It would be predicted that a similar influence of the dominating interaction length is also valid for random FMs ( $\varepsilon \geq 1$ ). But the recent studies [28] of concentrated FeNiCr alloys *already* demonstrated a certain difference with the critical behavior of amorphous samples [16]. In particular, the critical exponent  $\gamma$  was reported close to the mean-field value  $\gamma = 1$ , whereas amorphous systems obey the well-known tendencies  $\gamma \rightarrow 2$ ,  $\beta \rightarrow 0.5$  and  $\delta \rightarrow 5$  as  $\varepsilon \rightarrow 1_+$  [16, 18]. One should not be surprised why these tendencies are different from those ( $\beta \rightarrow 0$ ,  $\delta \rightarrow \infty$ ) in amorphous SGs. This dissemblance may be attributed to different values of lowest critical dimensions. In contrast to SGs [11], significant size effects do not occur until the FM film thickness reaches only a few monolayers [29].

In view of this, it would be especially interesting to use our experimental data for the estimation  $d_L$  in SGs and to compare this value with the theoretical ones given by MacMillan [12] ( $d_L = 2.64 \pm 0.10$ ) and Singh [13] ( $2.5 \pm 0.2$ ). Fig. 7 shows that the point  $\varepsilon_L = 0.9 \pm 0.1$ , wherein the condition  $d/(2 - \eta) = 1$  of the lowest critical dimension is satisfied, is very close to unity. So,  $d_L$  is expected a little bit higher than the fractal dimension  $d_f = 2.48 \pm 0.09$  of the critical cluster [17], i.e. an infinite SG host appearing exactly at the percolation threshold  $\varepsilon = 1$ . Proceeding from the *linear* dependence<sup>4</sup>  $d$  on  $\varepsilon$  between the points  $d(\varepsilon = 0) = 3$  and  $d(\varepsilon = 1) = 2.48$ , one can derive a more explicit value  $d_L = 2.53$ . To our knowledge, this is the first time when an *experimental* estimate has been reported. An agreement between our result and theoretical predictions [12, 13] also testifies the eligibility of the concept of effective dimension.

The above comparison between two mentioned scenarios for the destruction of the PM to SG phase transition shows that the effective dimension approach

[2, 10] is fruitful and, if anything, the most advanced than the others proposed hitherto to explain the violation of the universality rules.

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<sup>4</sup>This assumption looks very doubtful. But the distance between the extrapolated point  $d(\varepsilon_L = 0.9)$  and the fractal dimension  $d(\varepsilon = 1) = 2.48$  is small and so is the mistake resulted from this assumption.

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ВІД СИМЕТРИЧНИХ СПІНОВИХ СТЕКОЛ  
 ДО ПЕРКОЛЯЦІЙНОЇ МЕЖІ: КРОСОВЕР ВИМІРНОСТІ

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Резюме

Наведено нові результати з фундаментальної проблеми "нижча критична вимірність магнетиків". Це, зокрема, стосується

ся фазових переходів з парамагнітного стану у стан спінового скла. Аналіз проведених магнітних експериментів на масивних аморфних сплавах із залученням теорії фрактальних кластерів показав, що нижча критична вимірність спінових стекол  $d_L \approx 2,5$ , що фактично збігається з оцінками величини  $d_L$ , отриманими в результаті комп'ютерного моделювання. Цією роботою автори хотіли поставити крапку у проблемі нижчої критичної вимірності спінових стекол.