PECULIARITIES OF ELECTROMAGNETIC WAVE REFRACTION ON THE SURFACE OF AN ABSORBING MEDIUM

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The expressions for the refraction angle of electromagnetic waves in absorptive media based on the classical vector Maxwell equations for a continuum have been obtained. It has been found that the propagation direction of the electromagnetic field energy coincides with a normal to the plane of equal phases only in the case of TM-polarization of the incident wave, and under the incidence being near to a perpendicular one. In this case, the refraction angle can be determined using only the real part of the refraction index. For all other polarizations of an incident wave, the complicated dependence of the electromagnetic field propagation direction on the polarization of an incident wave in the absorptive medium has been demonstrated. In the case of the mixed polarization of an incident wave, a deviation of the propagation plane of the electromagnetic field energy from the incidence plane has been established.

Introduction

An absorption of electromagnetic waves by the environment is traditionally taken into account in classical electrodynamics by the replacement of the real refraction index with the complex one $\widetilde{n} = n + i\chi$ in the formulas for dielectric media. But such a replacement in the relation to the sines of angles of the incident and refracted waves (Snell's law) results in the complex angle of refraction that has no simple physical meaning. For the real part of Snell's law, it can be put in the standard form by the formal introduction of the refraction index depending on the incidence angle of light falling on the absorptive medium. Therewith, the planes of stationary phase do not coincide with the planes of stationary amplitude, forming an angle equal to the real part of the refraction angle with them. As far as the amplitude changes along the surface of stationary phase in this case, waves cease to be homogeneous.

The formal replacement of the parameter $\widetilde{n}$ in Fresnel's formulas for transparent media also results in complexity while solving the inverse problem of optics, namely in the case of the determination of the parameters n, χ from absorption/reflection spectra. It was under close inspection in works [1], but the incorrect interpretation of complex quantities which appeared at the description of absorptive media led to the results devoid of physical meaning. In particular, in works [1] in the case of the derivation of expressions for the refraction angle and coefficients of admission/reflection, the electric field in the absorptive medium was considered to be real in the case of the use of the complex refraction index and the complex electric induction. Consequently, the displacement in phases of the electric field strength of incident and refracted waves was not taken into account [2]. In addition, a consistent analysis of the problem on the basis of the Maxwell equations and boundary conditions to them in [2] results in dependences on the absorption mechanism (band-to-band, exciton, etc.), that seems queer.

Most authors determine the refraction angle as an angle between the normal to an interface and the direction of the normal to the plane of equal phases and obtain Snell's law from the condition of invariability of wave vector projections on the interface plane during the transition from one medium into another [2—4]. Meanwhile, for many physical phenomena and for recording devices, the plane of equal phases is not substantial. Instead of it, direction of the electric field vector, which always is perpendicular to the direction of the time-averaged Poynting's vector (hereinafter referred to as Poynting's vector) is important. Such problems do not appear in the case of transparent isotropic media, where the directions of a normal to the plane of equal phases and Poynting's vector coincide, but, in absorptive media, the direction of Poynting's vector and the direction of a normal to the plane of equal phases are not the same. The similar situation has the place in crystal optics for transparent, but anisotropic media [4—6]. So, in a uniaxial crystal, two refracted waves appear, and a biaxial one is characterized by conical refraction. For their description, the notions of ray surface (surface which is described by the vectors of ray (group) velocity that coincide with the direction of non-averaged Poynting's vector) and the surface of normals (surface which is described by the vectors of phase velocity) are used. But, for absorptive media, the problem of introducing the group velocity $v_{gr} = \partial\omega/\partial k$ [7] appears, so far as, in this case, the wave vector (k) is a complex quantity, while the group velocity must be a real one. In this case, introducing the group speed as $v_{\rm gr} = \partial\omega/\partial \mathrm{Re}(k)$ or $v_{\rm gr} = |\partial\omega/\partial k|$, one gets non-physical results ($v_{\rm gr}$ becomes more than the velocity of light in vacuum). Moreover, in the case of absorptive media, a usage of Fresnel's formulas at the determination of the transmission and reflection coefficients results in a violation of the energy conservation law, while using their determination with the help of Poynting's vectors immediately ensures the implementation of this law. For this reason, we consider that it is necessary to define the angle of refraction in accordance with the direction of time-averaged Poynting's vector (in relation to the normal to the interface), so far as it determines the direction of energy propagation of an electromagnetic wave, and its value determines the wave intensity. In this case, Snell's law cannot be longer determined from the condition of invariability of projections of the wave vectors on the interface plane at the transition from one medium to another.

1. Basic Relations

Let us consider a magnetic absorptive medium with isotropic properties. In this case, the system of Maxwell's equations for electromagnetic waves in the medium in absence of free charges and currents is [5]

$$[\nabla \times \mathbf{E}] = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \quad \nabla \cdot \mathbf{H} = 0; \tag{1}$$

$$[\nabla \times \mathbf{H}] = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{D}}{\partial t}, \quad \nabla \cdot \mathbf{D} = 4\pi\rho, \tag{2}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the interatomic-distance-averaged strengths of the electric and magnetic fields, $\mathbf{D}$ is the electric induction, ρ and $\mathbf{j}$ are the induced charge and current. In the case of a linear response of the medium to an external excitation, we have $\mathbf{D} = \epsilon\mathbf{E}$, $\mathbf{j} = \sigma\mathbf{E}$, where ϵ is the dielectric permeability and σ is the medium conductivity.

Let us seek asolution of the system of equations (1), (2) in the form of plane transverse waves (for the magnetic field, it will be in a similar manner):

$$\mathbf{E}_{\mathbf{R},t} = \sum_{\mathbf{k},\alpha} \mathbf{C}_{\mathbf{k}}^{(\alpha)} e^{i(\mathbf{kR}-\omega t)} + \text{c.c.}, \tag{3}$$

where $\mathbf{R}$ is the space coordinate vector, and the vector $\mathbf{C}_{\mathbf{k}}^{(\alpha)}$ determines the polarization of the $\mathbf{k}$-th wave ($\mathbf{C}_{\mathbf{k}}^{(\alpha)} \cdot \mathbf{k} = 0$, $\alpha = 1,2$). The notation *c.c.* denotes the complex conjugation, since the field is a real quantity. Furthermore, we drop *c.c.* and the factor that defines the time dependence $e^{-i\omega t}$ for the sake of convenience. The summation in (3) is over two polarizations (α) and all $\mathbf{k}$. The substitution of (3) in system (1), (2) results in the dispersion equation

$$\mathbf{k}^2 = \frac{\omega^2}{c^2}\left(\epsilon + i\frac{4\pi}{\omega}\sigma\right) \equiv \tilde{\epsilon}. \tag{4}$$

In this case, the absorption is determined by the imaginary part of the dielectric permeability $\tilde{\epsilon}$ with dimensionality $(\omega/c)^2$.

Let us choose polarization vectors being relevant to s- and p-polarizations, which we will denote conditionally by $\mathbf{C}_{\mathbf{k}}^{(1)} \equiv \mathbf{A}_{\mathbf{k}}$ and $\mathbf{C}_{\mathbf{k}}^{(2)} \equiv \mathbf{B}_{\mathbf{k}}$, in the form

$$\mathbf{A}_{\mathbf{k}} = \frac{\{-k_y; k_x; 0\}}{\sqrt{k_x^2 + k_y^2}}, \ \mathbf{B}_{\mathbf{k}} = \frac{\{-k_z k_y; -k_z k_x; k_x^2 + k_y^2\}}{|\mathbf{k}|\sqrt{k_x^2 + k_y^2}}, \tag{5}$$

where the complex components of the wave vector $\mathbf{k} = \{k_x; k_y; k_z\}$ are introduced. Separating the real and imaginary parts in the wave vector, $\mathbf{k} = \mathbf{k}' + i\mathbf{k}''$ and replacing the sum in (3) with integrals over both vector spaces $\mathbf{k}'$ and $\mathbf{k}''$, we get the vector of electric field strength in accordance with the general solution of (2) in the form

$$\mathbf{E}_{\mathbf{R}} = \int d\mathbf{k}\prime \int d\mathbf{k}'' \delta(\mathrm{Re}((\mathbf{k}\prime + i\mathbf{k}'')^2 - \tilde{\epsilon}))\times$$

$$\times\delta(\mathrm{Im}((\mathbf{k}\prime + i\mathbf{k}'')^2 - \tilde{\epsilon})))\mathbf{E}_{\mathbf{k}\prime,\mathbf{k}'',\mathbf{R}}, \tag{6}$$

where condition (4) is taken into account with the use of delta functions.

Furthermore, we consider a system with a heterogeneous change of dielectric permeability along the z axis (it determines the direction of a normal to the interface). For convenience, we present the wave and coordinate vectors as $\mathbf{k} = \{\boldsymbol{\kappa}; k_z\}$ and $\mathbf{R} = \{\mathbf{r}; z\}$, where $\boldsymbol{\kappa}$ and $\mathbf{r}$ are two-dimensional vectors in the XOY plane. In this case, $\boldsymbol{\kappa}$ must be real according to the condition of field finiteness. Consequently, according to (6), the Fourier transformation of the electric field strength over the two-dimensional coordinate $\mathbf{r}$ ($\mathbf{E_R} = \int d\boldsymbol{\kappa}\mathbf{E}_{\boldsymbol{\kappa},z}e^{i\boldsymbol{\kappa}\mathbf{r}}$) takes the form

$$\mathbf{E}_{\boldsymbol{\kappa},z} = \left(\alpha_{\boldsymbol{\kappa}}^{(+)}\mathbf{A}_{\boldsymbol{\kappa}} + \beta_{\boldsymbol{\kappa}}^{(+)}\mathbf{B}_{\boldsymbol{\kappa}}\right)e^{ik_z(\kappa)z} +$$

$$+\left(\alpha_{\boldsymbol{\kappa}}^{(-)}\mathbf{A}_{\boldsymbol{\kappa}} + \beta_{\boldsymbol{\kappa}}^{(-)}\mathbf{B}_{\boldsymbol{\kappa}}^{(-)}\right)e^{-ik_z(\kappa)z}. \qquad (7)$$

Here, $k_z(\kappa) = \sqrt{\tilde{\epsilon} - \kappa^2}$ is a complex wave vector which is determined by the delta function in (6):

$$k_z(\kappa) = \sqrt{\frac{\sqrt{(\tilde{\epsilon}' - \kappa^2)^2 + \tilde{\epsilon}''^2} + \tilde{\epsilon}' - \kappa^2}{2}} +$$

$$+i\sqrt{\frac{\sqrt{(\tilde{\epsilon}' - \kappa^2)^2 + \tilde{\epsilon}''^2} - \tilde{\epsilon}' + \kappa^2}{2}}, \qquad (8)$$

where the real $\tilde{\epsilon}'$ and imaginary $\tilde{\epsilon}''$ parts of $\tilde{\epsilon}$ are introduced. The coefficients $\alpha_{\boldsymbol{\kappa}}^{(+)}$, $\beta_{\boldsymbol{\kappa}}^{(+)}$ in (7) determine the waves which spread from the interface $z = 0$ (the direction of the z axis is defined as the transition direction from the first medium to the second one), and $\alpha_{\boldsymbol{\kappa}}^{(-)}$, $\beta_{\boldsymbol{\kappa}}^{(-)}$ correspond to the spreading in the opposite direction (vector $\mathbf{B}_{\boldsymbol{\kappa}}^{(-)}$ is determined by (5) under the replacement of k_z by $-k_z$). In addition, in accordance with the selected vectors (5), the coefficients α correspond to TE-waves (s-polarization), and β correspond to TM-waves (p-polarization).

In the case of a transparent medium, the integration over $\boldsymbol{\kappa}$ in (7) is usually limited in order to take into account the total reflection from the plane interface (requirement of the imaginary part absence in (8)). However, as it will be shown below, the consideration of solutions with completely imaginary wave vectors in the transparent medium gives the zero z-component of Poynting's vector. It actually means the absence of the penetration of energy into the medium and consequently takes into account the condition of total reflection. In this case, it is no sense to limit the integration over $\boldsymbol{\kappa}$. Using (1) and (7) for the magnetic field, we get

$$\mathbf{H}_{\boldsymbol{\kappa},z} = \sqrt{\epsilon}\Bigg[\left(\alpha_{\boldsymbol{\kappa}}^{(+)}\mathbf{B}_{\boldsymbol{\kappa}} - \beta_{\boldsymbol{\kappa}}^{(+)}\mathbf{A}_{\boldsymbol{\kappa}}\right)e^{ik_z(\kappa)z} +$$

$$+\left(\alpha_{\boldsymbol{\kappa}}^{(-)}\mathbf{B}_{\boldsymbol{\kappa}}^{(-)} - \beta_{\boldsymbol{\kappa}}^{(-)}\mathbf{A}_{\boldsymbol{\kappa}}\right)e^{-ik_z(\kappa)z}\Bigg]. \qquad (9)$$

Here, the polarization vectors (5) are given by the expressions

$$\mathbf{A}_{\boldsymbol{\kappa}} = \frac{[\mathbf{e_z} \times \boldsymbol{\kappa}]}{\kappa}, \quad \mathbf{B}_{\boldsymbol{\kappa}}^{(\pm)} = \frac{\mp\boldsymbol{\kappa}k_z(\kappa) + \mathbf{e}_z\kappa^2}{\kappa\sqrt{\tilde{\epsilon}}}. \qquad (10)$$

Using (1) and (2), after the integration over the superthin transition layer δ near the interface, we get the boundary conditions at the interface of two media. In the case where electric and magnetic fields are time-dependent as $e^{-i\omega t}$, the last equations in (1) and (2) become unnecessary (they follows from the first ones). Consequently,

$$[\mathbf{n} \times \mathbf{E_R}]\Big|_{\mathbf{R}_s - \delta\cdot\mathbf{n}}^{\mathbf{R}_s + \delta\cdot\mathbf{n}} = 0, \quad [\mathbf{n} \times \mathbf{H_R}]\Big|_{\mathbf{R}_s - \delta\cdot\mathbf{n}}^{\mathbf{R}_s + \delta\cdot\mathbf{n}} = 0, \qquad (11)$$

where $\delta \to 0$, $\mathbf{R}_s$ is the coordinate vector describing the interface, and $\mathbf{n}$ is the unit vector of a normal to the surface.

Poynting's vector, which defines the value and direction of the electromagnetic wave energy propagation (and, consequently, the refraction angle), takes the form [5]

$$\mathbf{S} = \frac{c}{8\pi}\overline{[\mathbf{E}_{\mathbf{R},t} \times \mathbf{H}_{\mathbf{R},t}]}_t = \frac{c}{4\pi}\mathrm{Re}[\mathbf{E}_{\mathbf{R}}^* \times \mathbf{H}_{\mathbf{R}}], \qquad (12)$$

where $\overline{(...)}_t$ means the averaging over the time.

2. Plane Interface

Furthermore, we consider the plane interface between two media (denoting them by 1 and 2) which is described by the equation $z = 0$ ($\mathbf{n} = \partial z/\partial\mathbf{R} = \mathbf{e}_z$; $\mathbf{e}_z$ is a unit vector of the coordinate axis OZ). We suppose that, in medium 1, a plane wave with polarizations α, β and with the projection of the wave vector on the interface plane $\boldsymbol{\kappa}_i$ comes from infinity to the interface. Then

$$\alpha_{\boldsymbol{\kappa}}^{(1,+)} = \alpha\delta(\boldsymbol{\kappa} - \boldsymbol{\kappa}_i), \quad \beta_{\boldsymbol{\kappa}}^{(1,+)} = \beta\delta(\boldsymbol{\kappa} - \boldsymbol{\kappa}_i). \qquad (13)$$

For medium 2, the radiation condition requires

$$\alpha_{\boldsymbol{\kappa}}^{(2,-)} = \beta_{\boldsymbol{\kappa}}^{(2,-)} = 0. \qquad (14)$$

Consequently, for the determination of the electromagnetic field in such a system, it is necessary to define $\alpha_{\boldsymbol{\kappa}}^{(1,-)}$, $\beta_{\boldsymbol{\kappa}}^{(1,-)}$ and $\alpha_{\boldsymbol{\kappa}}^{(2,+)}$, $\beta_{\boldsymbol{\kappa}}^{(2,+)}$. Substituting the coefficients in expressions (7), (9) and boundary conditions (11), we obtain

$$\alpha_{\kappa_i}^{(1,-)} = \frac{\sqrt{\tilde{\epsilon}_1 - \kappa_i^2} - \sqrt{\tilde{\epsilon}_2 - \kappa_i^2}}{\sqrt{\tilde{\epsilon}_1 - \kappa_i^2} + \sqrt{\tilde{\epsilon}_2 - \kappa_i^2}}\alpha,$$

$$\alpha_{\kappa_i}^{(2,+)} = \frac{2\sqrt{\tilde{\epsilon}_1 - \kappa_i^2}}{\sqrt{\tilde{\epsilon}_1 - \kappa_i^2} + \sqrt{\tilde{\epsilon}_2 - \kappa_i^2}}\alpha,$$

$$\beta_{\kappa_i}^{(1,-)} = \frac{(\tilde{\epsilon}_2/\tilde{\epsilon}_1)\sqrt{\tilde{\epsilon}_1 - \kappa_i^2} - \sqrt{\tilde{\epsilon}_2 - \kappa_i^2}}{(\tilde{\epsilon}_2/\tilde{\epsilon}_1)\sqrt{\tilde{\epsilon}_1 - \kappa_i^2} + \sqrt{\tilde{\epsilon}_2 - \kappa_i^2}}\beta,$$

$$\beta_{\kappa_i}^{(2,+)} = \frac{2\sqrt{\tilde{\epsilon}_2/\tilde{\epsilon}_1}\sqrt{\tilde{\epsilon}_1 - \kappa_i^2}}{(\tilde{\epsilon}_2/\tilde{\epsilon}_1)\sqrt{\tilde{\epsilon}_1 - \kappa_i^2} + \sqrt{\tilde{\epsilon}_2 - \kappa_i^2}}\beta. \qquad (15)$$

Coefficients (15) are of the Fresnel's formulas type for a transparent medium. For convenience, when wrighting coefficients (15), the multiplier $\delta(\boldsymbol{\kappa} - \boldsymbol{\kappa}_i)$ was ommited. This multiplier is a mathematical formulation of Snell's law: the equality of projections of wave vectors on the plane interface of the different media. Consequently, as was foreseen, in the case of a plane interface and a plane incident wave, we obtain two plane waves (reflected and refracted ones). In the case of an absorptive medium, coefficients (15) become complex that means the appearance of a phase shift of the reflected and refracted waves in relation to the incident one.

In accordance with (7), (9), and (15), the direction of a normal to the planes of equal phases for the refracted wave does not depend on the polarization of the incident wave and is determined by the real part of the vector $\boldsymbol{\kappa}_i + \mathbf{e}_z\sqrt{\tilde{\epsilon}_2 - \kappa_i^2}$, while its imaginary part determines the direction of the planes of equal amplitudes, which, as one can see, always coincides with $\mathbf{e}_z$. However, coming from definition (12) and using (7), (9), and coefficients (15), Poynting's vector in another medium at $z = 0$ is

$$\mathbf{S} = \frac{c^2}{4\pi\omega}\mathrm{Re}\Bigg[|\alpha_{\kappa_i}^{(2,+)}|^2\left(\boldsymbol{\kappa}_i + \mathbf{e}_z\sqrt{\tilde{\epsilon}_2 - \kappa_i^2}\right) +$$

$$+\frac{\tilde{\epsilon}_2}{|\tilde{\epsilon}_2|}|\beta_{\kappa_i}^{(2,+)}|^2\left(\boldsymbol{\kappa}_i + \mathbf{e}_z\sqrt{\tilde{\epsilon}_2^* - \kappa_i^2}\right) -$$

$$-2i\frac{\sqrt{\tilde{\epsilon}_2}}{|\tilde{\epsilon}_2|}\beta_{\kappa_i}^{(2,+)^*}\alpha_{\kappa_i}^{(2,+)}[\mathbf{e}_z \times \boldsymbol{\kappa}_i]\mathrm{Im}\sqrt{\tilde{\epsilon}_2 - \kappa_i^2}\Bigg], \qquad (16)$$

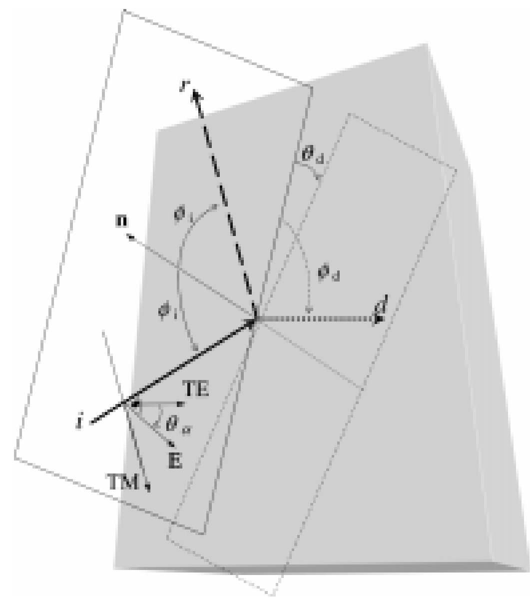


Fig. 1. Scheme which presents a deviation of the refraction plane of an absorptive medium as a function of the polarization angle of the incident wave θ_α. Here, i is the incident wave, r is the reflected wave, and d is the refracted one

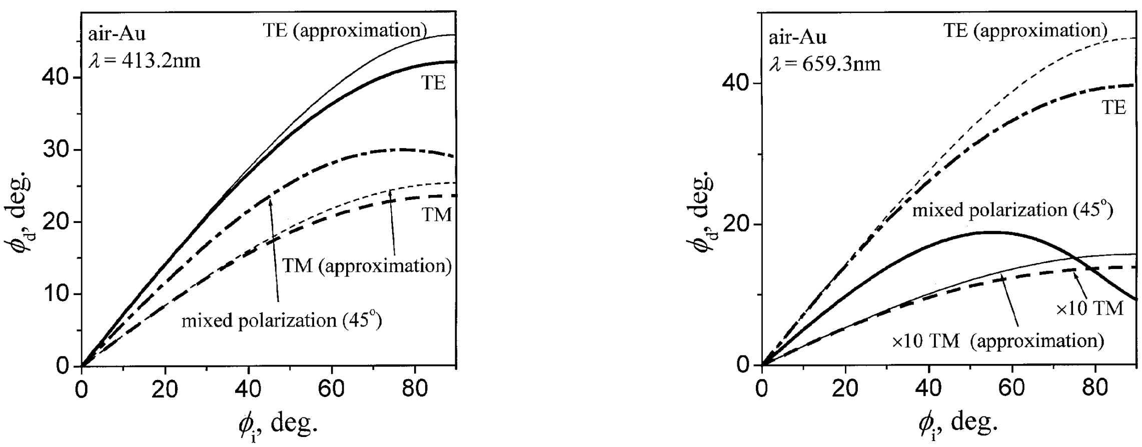
It is evident that the direction of Poynting's vector depends on polarization. In the case of mixed polarization, refraction takes place also in the direction perpendicular to the incident plane (the plane which is defined by the wave vector of an incident wave and by the normal to the interface), as shown in Fig. 1. Moreover, $S_z = 0$ in the case of a transparent medium at $\boldsymbol{\kappa}_i$ such that $\sqrt{\tilde{\epsilon}_2 - \kappa_i^2}$ is completely imaginary, which corresponds to the total internal reflection.

Let us define the angle of refraction in relation to the normal to the interface in the following way:

$$\cos\phi_d = (\mathbf{S}\cdot\mathbf{n})/|\mathbf{S}|. \qquad (17)$$

Denoting the angle of incidence ϕ_i, we obtain $\kappa_i = \sqrt{\epsilon_1}\sin\phi_i$ (in this case, the dielectric permeability of the first medium must be real because of the condition of the problem: a plane wave comes from infinity). For the cases of non-mixed polarizations TE and TM, separating the real part in (16) and taking (17) into account, we obtain the ratio of the sines of the angle of refraction and the angle of incidence in the form of classical Snell's law:

$$\frac{\sin\phi_d}{\sin\phi_i} = \frac{n_1}{\tilde{n}_2(\phi_i)} \qquad \text{(TE-polarization)}, \qquad (18)$$



a **b**

Fig. 2. Dependence of the angle of refraction ϕ_d on the angle of incidence ϕ_i for the air-gold interface at the wave-lengths $\lambda = 413.2$ (a) and 659.3 nm (b) [in (b), the angle for TM polarization is multiplied by 10]

$$\frac{\sin\phi_d}{\sin\phi_i} = \frac{n_2^2-\chi_2^2}{n_2^2+\chi_2^2}\frac{n_1}{\widetilde{n}_2(\phi_i)} \quad \text{(TM-polarization).} \tag{19}$$

Here, we introduced formally the refraction index for an absorptive medium $\widetilde{n}_2(\phi)$ (similarly to [2, 3]) which depends on the angle of incidence of the incident wave:

$$\widetilde{n}_2(\phi) \equiv \frac{1}{\sqrt{2}}\left(n_2^2-\chi_2^2+n_1^2\sin^2\phi+\right.$$

$$\left.+\sqrt{(n_2^2-\chi_2^2-n_1^2\sin^2\phi)^2+4n_2^2\chi_2^2}\right)^{1/2}. \tag{20}$$

When writing Eqs. (18) — (20), we introduced the refraction indices of the first and second media: $n_1 \equiv \sqrt{\epsilon_1}$ and $n_2 + i\chi_2 \equiv \sqrt{\epsilon_2}$, where n_1, n_2, and χ_2 are real quantites ($n_2, \chi_2 > 0$).

In the case of almost normal incidence where $\phi \approx 0$ $n_2(\phi) \approx n_2$, and the ratio of the sines of the incident and refracted waves is simplified:

$$\frac{\sin\phi_d}{\sin\phi_i} \approx \frac{n_1}{n_2} \quad \text{(TE-polarization),} \tag{21}$$

$$\frac{\sin\phi_d}{\sin\phi_i} \approx \frac{n_2^2-\chi_2^2}{n_2^2+\chi_2^2}\frac{n_1}{n_2} \quad \text{(TM-polarization).} \tag{22}$$

In spite of the fact that the difference between TM and TE disappears in the case of normal incidence (Poynting's vectors (16) become identical), Eqs. (21) and (22) are different. However, in the case of normal incidence, the ratio of sines (21) and (22) loses its meaning and these equations just show the rate, with which the refraction angle tends to zero when the incidence angle tends to zero for different polarizations.

3. Numerical Results and Discussion

The dependences of ϕ_d on the angle of incidence for the air-gold interface are calculated for TE and TM polarizations of the incident wave, using formulas (18) and (19) for wave lengths $\lambda = 413.2$ and 659.3nm. (Data for Au are taken from [8]). The calculations are presented in Fig. 2, where the slim lines that correspond to the simplified formulas (21), (22) are added for comparison. They demonstrate deviation from (18), (19) being not more 15% in the case of the use of approximate formulas. In addition, in Fig. 2, the dependence $\phi_d(\phi_i)$ for the polarization of 45° is depicted. It is calculated using expressions (17) and (16) and demonstrates the case of a mixed polarization of the electric field strength of an incident wave in relation to the incidence plane. The data for Au which are used while calculating the angle dependences shown in Fig. 2 are chosen to demonstrate two cases: when the real part of dielectric permeability is considerable (a), and when it is near 0 (b). In accordance with (19), one can observe the almost normal propagation of a wave (Fig. 2,b) in the case of completely imaginary dielectric permeability and TM polarization.

For a deeper understanding of the influence of absorption on the refraction angle, we presented the dependences $\phi_d(\phi_i)$ for three polarizations at a fixed value of the real part of the refraction coefficient $n_2 = 1.3$ and for different values of the imaginary part χ_2 in Fig. 3. Fig. 3 demonstrates a strong dependence of the refraction angle on absorption in the medium. From Figs. 3 and 2, one can see that, unlike the cases of TE and TM polarizations where the refraction angle is a single-valued function of the angle of incidence, the refraction angle can have the same value for the mixed polarization at the different angles of incidence. In addition, using the dependence $\phi_d(\phi_i)$, it is possible to define the dependence of the Brewster angle on the absorption and the polarization of an incident wave.

According to (16), Poynting's vector component, which is perpendicular to the incidence plane, can arise. In Fig. 4, the deviation angle of the refraction plane θ_d ($\cos\theta_d = (\mathbf{S}\cdot\boldsymbol{\kappa}_i)/(|[\mathbf{n}\times\mathbf{S}]||\boldsymbol{\kappa}_i|)$) vs the orientation of the incidence plane is depicted as a function of the polarization angle of the vector of electric field strength with reference to the incidence plane (a), and as a function of the angle of incidence (b). The dependence $\theta_d(\phi_i)$ (Fig. 4,b) loses its meaning in the case of normal incidence, so far as the projection of the wave vector on the interface $\boldsymbol{\kappa}_i$ and $[\mathbf{e}_z\times\boldsymbol{\kappa}_i]$ is absent in this case. So, it is possible only to define a rate with which θ_d tends to zero at $\phi_i \longrightarrow 0$.

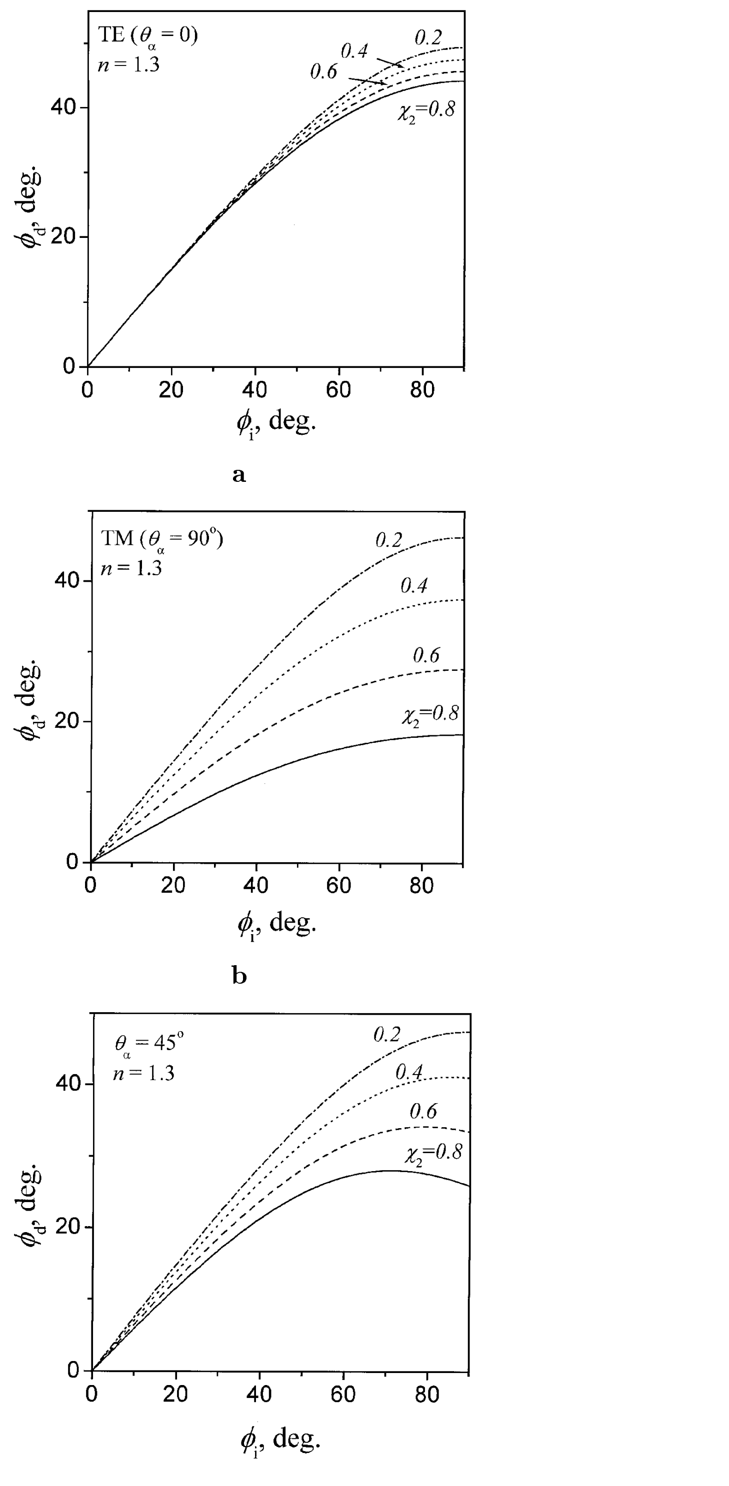


Fig. 3. Dependence of the refraction angle ϕ_d on the angle of incidence ϕ_i at $n_2 = 1.3$ and different values of the absorption coefficient χ_2 for TE (a), TM (b), and mixed (c) polarizations

Conclusions

Within the framework of the performed theoretical research, on the base of classical electrodynamics of continuous media and with an arbitrary polarization of the incident wave, the formulas for the ratios of the sines of the refraction angle and the angle of incidence of waves at the interface between the absorptive and transparent media have been obtained. A complicated dependence of the refraction angle on the incidence angle is established as well as the deviation of the refraction plane from the incidence plane. The expressions were obtained in the following approximations:

— the incident wave is planar;

— the interface is absolutely planar;

— the dielectric permeability varies stepwise on the interface;

— the dielectric permeability is homogeneous and isotropic;

— the spatial dispersion is absent (the dipole approximation).

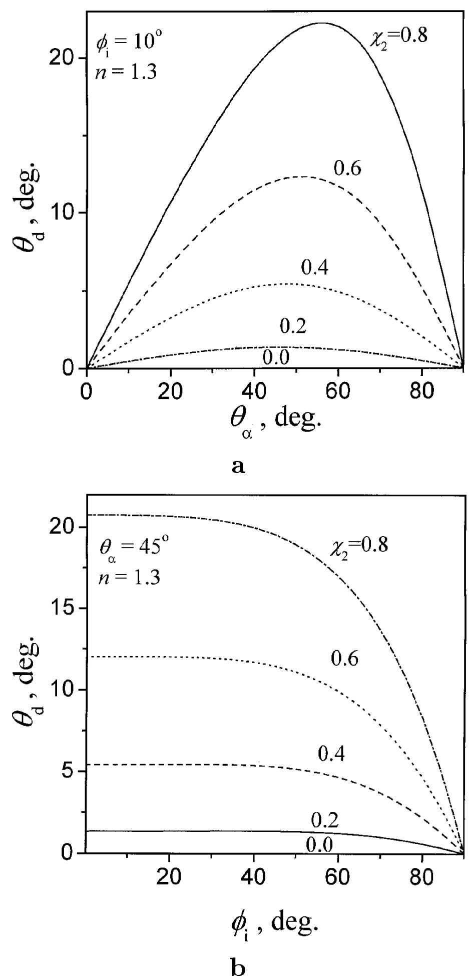


Fig. 4. Dependences of the deviation angle of the refraction plane θ_d on the polarization angle θ_α at the angle of incidence of 10° (a) and on the angle of incidence at fixed polarization of 45° (b) when $n_2 = 1.3$ and χ_2 has various values

Such approximations are fully justified in the case where the mean-square deviation of the interface from the plane is far less than 4 nm (for 4 nm, the experimental and theoretical values of a change of the absorpsion coefficient are less than 10% [9]) and where the characteristic length of variations in dielectric permeability is far less than the wavelength (this takes place for sharp transitions). In the case where the distance to the radiation source is far more than the size of the irradiated area, it is possible to consider the incident wave to be plane. The dipole approximation widely used in theoretical physics is fully justified for wavelengths corresponding to the solar spectrum (approximately 200 — 1000 nm), as far as they are much more than the sizes of the interaction region which are determined by the absorption mechanisms such as fundamental, exciton, and phonon ones.

The mentioned approximations are widely used, so the reliability of the obtained theoretical results is grounded. However, practical observations of the dependence of the refraction angle on the polarization of an incident wave and the phenomenon of a refraction plane turn in relation to the plane of incidence are complex tasks. Indeed, the deviation angle is small in the case of weak absorption, while it is necessary to use very thin films in the case of strong absorption. The similar situation exists in the presence of the spatial dependence of the refraction index, when the effect of the polarization plane twisting arises (see, for example, [10] and references therein). This effect is experimentally observed in long waveguides.

The deviation of the refraction plane from the plane of incidence can be explained by the appearance of longitudinal and transverse surface electromagnetic waves at the expense of the complex interaction of TM and TE waves with the environment. According to (16), a tangent component of Poynting's vector (in relation to the interface) in an absorptive medium changes its direction in relation to Poynting's vector of the incident TM wave that corresponds to the excitation of surface waves [11]. In this case, the real part of dielectric permeability can be zero ($n = \chi$). This situation is intermediate between the occurrences of volume and surface waves in the absorptive medium under the incidence of the TM wave on the interface. Moreover, the appearance of surface waves was considered in the literature (see, for example, [11, 12]) under the condition of weak absorption ($|\mathrm{Re}\tilde{\epsilon}| \gg \mathrm{Im}\tilde{\epsilon}$), that is conditioned by the convenience of experimental observation, when the appearance of surface waves is possible only under resonance conditions and only the tangent component of Poynting's vector exists, while the normal one is absent. We have considered the case of an arbitrary relation between $|\mathrm{Re}\tilde{\epsilon}|$ and $\mathrm{Im}\tilde{\epsilon}$, where, in addition to the tangent component of Poynting's vector, its normal component is present. Their coexistence can be explained by the simultaneous existence of volume and surface waves and by the non-resonance excitation of surface waves.

Moreover, in the case of strong absorption where $n \ll \chi$, the field fades away on distances comparable with the wavelength, and it is already difficult to tell about the presence of waves in the medium. But for many practical applications, a part of the electromagnetic wave energy that has penetrated into the medium is substantial. It is determined by the z-component of Poynting's vector. It is shown that, in the case of an absorptive medium, the approximate formula for Snell's law with totally real refraction indices can be used only for TE polarization.

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ОСОБЛИВОСТІ ЗАЛОМЛЕННЯ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ НА ПОВЕРХНІ ПОГЛИНАЛЬНОГО СЕРЕДОВИЩА

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Р е з ю м е

Отримано вирази для кута заломлення електромагнітних хвиль в поглинальному середовищі з класичних векторних рівнянь Максвелла для електродинаміки суцільного середовища. Встановлено, що напрямок поширення енергії електромагнітного поля збігається з напрямком нормалі до площини рівних фаз лише у випадку ТЕ-поляризації падаючої хвилі та при майже нормальному падінні; тоді кут заломлення можна визначати з використанням лише дійсної частини показника заломлення. У всіх інших випадках поляризації падаючої хвилі виявлено складну залежність напрямку поширення електромагнітного поля в поглинальному середовищі від поляризації падаючої хвилі; для змішаної поляризації падаючої хвилі виявлено відхилення площини поширення енергії електромагнітного поля від площини падіння.