REDUCED PROBABILITIES OF $E2$ TRANSITIONS AND QUADRUPOLAR MOMENTS OF THE EXCITED STATES OF DEFORMABLE NONAXIAL EVEN-EVEN NUCLEI

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The rotation-vibration excited states of deformable nonaxial even-even nuclei are investigated. The ratios of reduced probabilities of $E2$ transitions and quadrupolar moments of the excited states of deformable nonaxial even-even nuclei ^{104}Ru, ^{166}Er, and ^{238}U are calculated including the high-spin states.

Introduction

The wave function of the longitudinal quadrupolar β-type vibrations of deformable nonaxial even-even nuclei, satisfying the Schrödinger equation [1]

$$\left\{-\frac{\hbar^2}{2B}\frac{1}{\beta^3}\frac{d}{d\beta}\left(\beta^3\frac{d}{d\beta}\right)-V_0\exp\left\{-\frac{(\beta-\beta_0)^2}{\beta_0^2}\right\}+\right.$$

$$\left.+\frac{\hbar^2}{4B\beta^2}\varepsilon_{I\tau}-E_{I\tau}\right\}\varphi_{I\tau}(\beta)=0, \tag{1}$$

where B is a mass parameter, $\varepsilon_{I\tau}$ — eigenvalues of the equation for a rigid asymmetric top, V_0 — energy-measured theory parameter, β_0 — deformation parameter of an even-even nucleus in the ground state, is written as

$$\varphi(\xi)=N_\nu H_\nu(\xi)e^{-\xi^2/2},\quad \varphi_{I\tau}(\xi)=0$$

$$\text{under}\quad \xi\to 0\quad\text{and}\quad \xi\to\infty \tag{2}$$

with N_ν being a normalization factor, $H_\nu(\xi)$ — Hermite function of the 1st kind, and ν is a solution of a transcendental equation [1]. The variable $\xi=\frac{p_{I\tau}(\beta-\beta_{I\tau})}{\mu_{I\tau}\cdot\beta_{I\tau}}$ varies within the interval $-\frac{p_{I\tau}}{\mu_{I\tau}}\le\xi\le\infty$, and $p_{I\tau}=\frac{\beta_{I\tau}}{\beta_0}\ge 1$ determines an augmentation of the equilibrium deformation β_0 of the ground state when the nucleus transits to the $I\tau$ states and meets the condition

$$p_{I\tau}^3(p_{I\tau}-1)\exp\left\{-\left(p_{I\tau}-1\right)^2\right\}=\frac{\varepsilon_{I\tau}+1.5}{4g}, \tag{3}$$

where $g=\frac{BV_0\beta_0^2}{\hbar^2}$ is the dimensionless parameter. The $\mu_{I\tau}$ value determines the deformability of a nucleus:

$$\mu_{I\tau}=\frac{1}{\beta_0}\left[\frac{\hbar^2}{BC_{I\tau}}\right]^{\frac{1}{4}}=\left\{4g\left(2p_{I\tau}+1-p_{I\tau}^2-\frac{3}{2p_{I\tau}}\right)\times\right.$$

$$\left.\times\exp\left[-(p_{I\tau}-1)^2\right]\right\}^{-1/4}. \tag{4}$$

The transversal γ-vibrations are taken into account by the introduction of the effective parameter $\gamma=\gamma_{\text{eff}}$.

In this work, we consider, within the frame of the nonadiabatic collective theory [2—5], a possibility of description of reduced probabilities of the electrical quadrupolar transitions, and average values of the electrical quadrupolar moments of the main, anomalously, and rotational γ- and β-vibration bands, including the high-spin states, for the case of the deformable nonaxial even-even nuclei.

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1. Reduced Probabilities of $E2$ Transitions

The reduced probability of quadrupolar transitions between excited states $i \equiv \{\nu I\tau\}$ and $f \equiv \{\nu' I'\tau'\}$ of deformable nonaxial even-even nuclei is [2]

$$B(E2, \nu I\tau \to \nu' I'\tau') = B_a(E2, I\tau \to I'\tau')S_{if}^2, \quad (5)$$

where $B_a(E2, I\tau \to I'\tau')$ are the probabilities of the $E2$ transitions between excited states of deformable nonaxial even-even nuclei in the adiabatic approximation [2]. The factor S_{if} accounts for the nucleus deformability:

$$S_{\nu I\tau\nu' I'\tau'} \equiv \langle \nu I\tau|\frac{\beta}{\beta_0}|\nu' I'\tau'\rangle =$$

$$= \int_0^\infty \varphi_{\nu' I'\tau'}(\beta)\frac{\beta}{\beta_0}\varphi_{\nu I\tau}(\beta)d\beta. \quad (6)$$

The general expression for the matrix elements (6) is cumbersome. Because of this, let us consider particular cases. Let states $\nu I\tau$ and $\nu' I'\tau'$ be related to the collective excited states of the main ($\nu = 0$), anomalous ($\nu = 0$), and first ($\nu = 1$) rotation-β-vibration bands. Then we have to take $\nu = 0, 1$ in Eq. (6).

So, to calculate the reduced probability of $E2$ transitions in nonaxial even-even nuclei, we suppose $\nu = 0, 1$ in the collective excited states related to the main and first rotation-β-vibration bands with the wave function (2), and obtain the following formulae:

$$B(E2, \nu I\tau \to \nu' I'\tau') = \Bigg| N_0(I\tau)N_0(I'\tau')c^{-2}(if)e^{-\frac{a(if)}{2}}\times$$

$$\times\left\{1 + \frac{\sqrt{\pi}b(if)}{\sqrt{2}c(if)}\left[1 + \Phi\left(\frac{b(if)}{\sqrt{2}c(if)}\right)\right]e^{\frac{b^2(if)}{2c^2(if)}}\right\}\Bigg|^2\times$$

$$\times B_a(E2, I\tau \to I'\tau') \quad (7)$$

— inside the main and anomalous rotation-β-vibration bands in which

$$N_0(I\tau) = \left\{\mu_{I\tau}\frac{\sqrt{\pi}}{2}[1 + \Phi(\kappa_{I\tau})]\right\}^{-\frac{1}{2}} \quad (8)$$

is a normalization coefficient for the case of $\nu = 0$,

$$\Phi(\kappa) = \frac{2}{\sqrt{\pi}}\int_0^{\kappa_{I\tau}} e^{-z^2}dz,$$

is the probability integral or the Kramp function;

$$\kappa_{I\tau} = \frac{p_{I\tau}}{\mu_{I\tau}}; \quad a(if) = \kappa_{I\tau}^2 + \kappa_{I'\tau'}^2;$$

$$b(if) = \frac{\kappa_{I\tau}}{\mu_{I\tau}} + \frac{\kappa_{I'\tau'}}{\mu_{I'\tau'}}; \quad c(if) = \sqrt{\mu_{I\tau}^{-2} + \mu_{I'\tau'}^{-2}}; \quad (9)$$

$$B(E2, \nu I\tau \to \nu' I'\tau') =$$

$$= \Bigg| N_1(I\tau)N_0(I'\tau')c^{-2}(if)\frac{e^{-\frac{a(if)}{2}}}{\mu_{I\tau}c^2(if)}\left\{2\left[\frac{b(if)}{c^2(if)} - p_{I\tau}\right] + \right.$$

$$+\frac{\sqrt{2\pi}}{c(if)}\left[\frac{b^2(if)}{c^2(if)} - b(if)p_{I\tau} + 1\right]\times$$

$$\left.\times\left[1 + \Phi\left(\frac{b(if)}{\sqrt{2}c(if)}\right)\right]e^{\frac{b^2(if)}{2c(if)}}\right\}\Bigg|^2 B_a(E2, I\tau \to I'\tau') \quad (10)$$

— between the levels of the main and first anomalous rotation-β-vibration bands;

$$B(E2, \nu I\tau \to \nu' I'\tau') =$$

$$= \Bigg| N_1(I\tau)N_1(I'\tau')c^{-4}(if)\frac{e^{-\frac{a(if)}{2}}}{\mu_{I\tau}\mu_{I'\tau'}}\times$$

$$\times\left\{1 + \frac{b^2(if)}{2c^2(if)} - \frac{\sqrt{\pi}b(if)}{\sqrt{2}c(if)}\left[\frac{3}{2} + \frac{b^2(if)}{c^2(if)}\right]\times\right.$$

$$\left.\times\left[1 + \Phi\left(\frac{b(if)}{\sqrt{2}c(if)}\right)\right]e^{\frac{b^2(if)}{2c(if)}}\right\} - (p_{I\tau} + p_{I'\tau'})\times$$

$$\times e^{-\frac{a(if)}{2}}c^{-3}(if)\left\{\frac{b(if)}{c(if)} - \frac{\sqrt{\pi}}{\sqrt{2}}\left[1 + \frac{b^2(if)}{c^2(if)}\right]\times\right.$$

$$\left.\times\left[1 + \Phi\left(\frac{b(if)}{\sqrt{2}c(if)}\right)\right]e^{\frac{b^2(if)}{2c(if)}}\right\} +$$

$$+p_{I\tau}p_{I'\tau'}e^{-\frac{a(if)}{2}}c^{-2})(if)\times$$

$$\times\left\{1 + \sqrt{\frac{\pi}{2}}\frac{b(if)}{c(if)}\left[1 + \Phi\left(\frac{b(if)}{\sqrt{2}c(if)}\right)\right]e^{\frac{b^2(if)}{2c^2(if)}}\right\}\Bigg|^2\times$$

$$\times B_a(E2, I\tau \to I'\tau') \quad (11)$$

— inside the first rotation-β-vibration band. With $\nu = 1$, the normalization coefficient $N_1(I\tau)$ equals

$$N_1(I\tau) = \Bigg| 4\left\{\frac{\mu_{I\tau}}{2}\left[\kappa_{I\tau}e^{-\kappa_{I\tau}^2} + \right.\right.$$

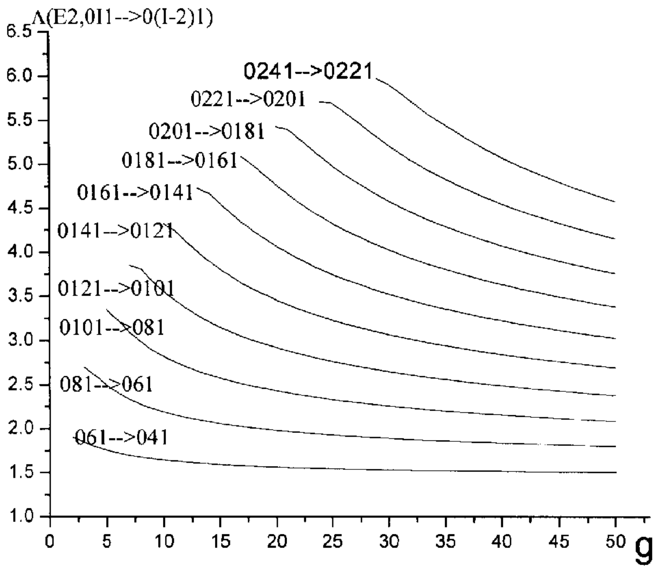

Fig.1. Ratio of (reduced probabilities of $E2$ transitions)/(reduced probability of the $E2$ transition from the first 2^+-spin excited state to the fundamental one) versus the parameter g under the fixed parameter $\gamma = 10°$

$$+\sqrt{\pi}\left(\frac{1}{2}+\kappa_{I\tau}^2\right)(1+\Phi(\kappa_{I\tau}))\Bigg]-$$

$$-p_{I\tau}\left[e^{-\kappa_{I\tau}^2}+\kappa_{I\tau}\sqrt{\pi}(1+\Phi(\kappa_{I\tau}))\right]+$$

$$+\sqrt{\pi}\mu_{I\tau}[1+\Phi(\kappa_{I\tau})]\frac{\kappa_{I\tau}^2}{2}\Bigg\}\Bigg|^{-\frac{1}{2}}. \qquad (12)$$

It is convenient to consider the ratios of "reduced probabilities of $E2$ transitions" to the "reduced probability of the $E2$ transition" from the first 2^+-spin excited level to the ground state:

$$\Lambda(E2,\nu I\tau\to\nu' I'\tau') = \frac{B(E2,\nu I\tau\to\nu' I'\tau')}{B(E2,021\to 001)}. \qquad (13)$$

It is seen from formulae (7), (10), and (11) that the ratios of the reduced probabilities of $E2$ transitions of deformable nonaxial even-even nuclei are described by two parameters g and γ. The results of calculations for $\Lambda(E2,\nu I\tau\to\nu' I'\tau')$ as a function of the parameter g of deformable nonaxial even-even nuclei with a fixed γ value are presented in Fig. 1.

It is seen that the ratios of reduced probabilities of many-stage $E2$ transitions inside the main rotation band with a fixed γ value, decrease with different rates. Under fixed values of g and γ, the ratios of reduced probabilities of many-stage $E2$ transitions inside the main rotation band increase with the spin of nuclear levels. Only when $g\to\infty$, we obtain results for a rigid asymmetric rotator [2].

The theoretical and experimental ratios of the reduced probabilities of $E2$ transitions inside the main, anomalous rotation, and first rotation-β-vibration bands in certain deformable nonaxial even-even nuclei to the reduced probability of the $E2$ transition from the first excited 2^+-spin level to the ground state for mass numbers $104<A<238$ are presented in Table 1.

It is seen from Table 1 that a theory that employs two parameters g and γ, describes satisfactorily the relations between reduced probabilities $E2$ transitions inside the main, anomalous rotation, and first rotation-β-vibration bands.

2. Electric Quadrupolar Moments

If one takes into account the deformability of nucleus, this should introduce, in addition to the reorganization

Table 1. Ratios of reduced probabilities of $E2$ transitions

Nucleus and parameters	$\frac{B(E2,\nu I\tau\to\nu' I'\tau')}{B(E2,021\to 001)}$	Theory	Experiment
^{104}Ru	$021\to 001/021\to 001$	1	1
$\gamma=29°$	$041\to 021/021\to 001$	1.40	1.3(6)
$g=400$	$061\to 041/021\to 001$	1.77	2.2(6)
[6]	$081\to 061/021\to 001$	1.99	2.2(2)
	$0101\to 081/021\to 001$	2.15	2.1(6)
	$042\to 022/021\to 001$	0.57	0.6(6)
	$051\to 031/021\to 001$	1.29	0.79(8)
	$062\to 042/021\to 001$	0.76	1.2(6)
	$082\to 062/021\to 001$	1.13	2.1(7)
	$042\to 021/021\to 001$	0.0053	0.003(9)
	$062\to 041/021\to 001$	0.0002	0.004(6)
^{166}Er	$021\to 001/021\to 001$	1	1
$\gamma=28°$	$041\to 021/021\to 001$	0.41	1.4(1)
$g=350$	$061\to 041/041\to 021$	1.26	0.9(3)
[6]	$0101\to 081/081\to 061$	1.08	1.2(1)
	$0101\to 081/081\to 061$	1.08	1.1(1)
	$0121\to 0101/081\to 061$	1.06	1.0(1)
	$0121\to 0101/081\to 061$	1.06	1.9(1)
	$042\to 022/042\to 021$	35.59	53.5
^{238}U	$001\to 021/001\to 021$	1	1
$\gamma=8°$	$081\to 061/001\to 021$	0.45	0.40(6)
$g=400$	$0101\to 0121/001\to 021$	0.45	0.44(6)
[6]	$0121\to 0141/001\to 021$	0.46	0.44(5)
	$0141\to 0161/001\to 021$	0.48	0.43(4)
	$0161\to 0181/001\to 021$	0.49	0.34(6)
	$0181\to 0201/001\to 021$	0.51	0.38(6)
	$0201\to 0221/001\to 021$	0.53	0.33(6)
	$0221\to 0241/001\to 021$	0.55	0.44(4)
	$0241\to 0261/001\to 021$	0.57	0.48(2)
	$0261\to 0281/001\to 021$	0.59	0.44(2)

of energy levels and a change of probabilities of electric quadrupolar transitions between excited levels, a variation of the quadrupolar moments' values of excited levels. Let us consider an impact of the parameter g, defining the deformability of a nucleus, on average values of the electric quadrupolar moments of excited states of deformable nonaxial even-even nuclei. Average values of the electric quadrupolar moments of nonaxial even-even nuclei in the state $i = \{I\tau\nu\}$ are determined by the expression [2]

$$\langle Q_2\rangle_i = \langle Q_2\rangle^a_{I\tau} \cdot S_{ii}, \tag{14}$$

where $\langle Q_2\rangle^a_{I\tau}$ are average values of the electric quadrupolar moment of a nonaxial even-even nucleus in the states $I\tau$ calculated in the adiabatic approximation; the factor S_{ii} is a diagonal matrix element. In the states of collective excitation of the main ($\nu = 0$), anomalous ($\nu = 0$), and first ($\nu = 1$) rotation-β-vibration bands ($\nu = 1$), we obtain the following formulae for the calculation of the quadrupolar moments of excited states of deformable nonaxial even-even nuclei:

$$\langle Q_2\rangle_i = p_{I\tau}\left\{1 + \frac{\exp(-\kappa_{I\tau})}{\sqrt{\pi}\kappa_{I\tau}a}\right\}\langle Q_2\rangle^a_{I\tau} \tag{15}$$

— for the levels of the main and anomalous rotation bands;

$$\langle Q_2\rangle_i = N_1^2(I\tau)\Big|e^{-\kappa^2_{I\tau}}\mu^2_{I\tau}\Big\{1 + \kappa^2_{I\tau} +$$

$$+\kappa_{I\tau}\sqrt{\pi}\left[1 + \Phi\left(\kappa_{I\tau}\right)\right]\Big\} -$$

$$-4p_{I\tau}\mu_{I\tau}\left\{\kappa_{I\tau}e^{-\kappa^2_{I\tau}} + \sqrt{\pi}\left(\frac{1}{2} + \kappa^2_{I\tau}\right)\left[1 + \Phi\left(\kappa_{I\tau}\right)\right]\right\} +$$

$$+2p^2_{I\tau}\left\{e^{-\kappa^2_{I\tau}} + \kappa_{I\tau}\sqrt{\pi}[1 + \Phi\left(\kappa_{I\tau}\right)]\right\}\Big|\langle Q_2\rangle^a_{I\tau} \tag{16}$$

— for the levels of the first rotation-β-vibration band.

It is seen from Eqs. (15) and (16) that the quadrupolar moments of excited levels depend only on

T a b l e 2. (Quadrupolar moment of an excited level)/(quadrupolar moment of the 2^+-spin first excited level) ratios

Nucleus and parameters	$Q_{0I\tau}/Q_{021)}$	Theory	Experiment
^{104}Ru	Q_{021}/Q_{021}	1	1
$\gamma = 29°$	Q_{041}/Q_{021}	1.29	0.43(3)
$g = 400$	Q_{061}/Q_{021}	1.47	0.5(3)
[6]	Q_{081}/Q_{021}	1.57	0.6(3)
^{166}Er	Q_{021}/Q_{021}	1	1
$\gamma = 29°$	Q_{041}/Q_{021}	1.41	1.4(2)
$g = 20$[6]	Q_{022}/Q_{021}	–0.6(2)	–0.054

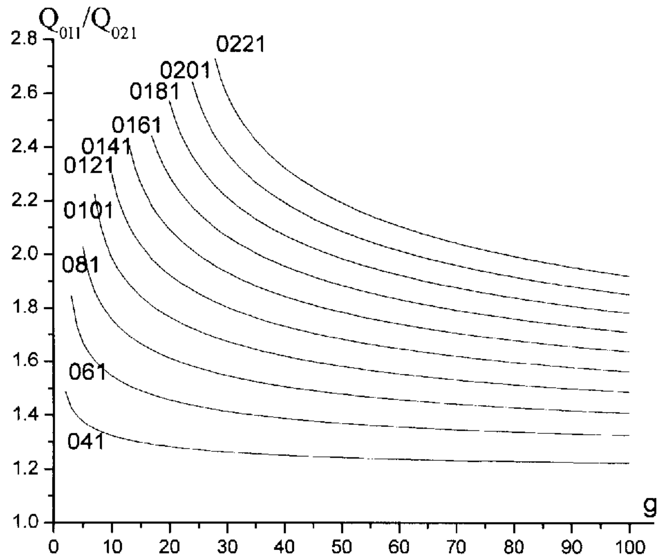

Fig. 2. Ratio of (quadrupolar moment of an excited level)/(quadrupolar moment of the 2^+-spin first excited level) versus the parameter g under the fixed parameter $\gamma = 10°$

the parameters g and γ, as this is the case for the $E2$ transitions ratio. Therefore, the ratios of the reduced probabilities of $E2$ transitions and the quadrupolar moments of deformable nonaxial even-even nuclei are expressed in terms of two parameters g and γ.

In Fig. 2, we presented the results of calculations of the ratio "quadrupolar moment of an excited level" to "quadrupolar moment of the 2^+-spin first excited level" as a function of the parameter g under a fixed γ value.

It is seen that the ratios of quadrupolar moments of the excited levels of the main rotation band under a fixed γ value have different rates of diminution. If one fixes the parameters g and γ then the ratios of quadrupolar moments of excited levels of the main band increase with the spin of levels of a nucleus. Only as $g \to \infty$, we obtain the results for a rigid asymmetric rotator [2].

The theoretical and experimental ratios of the excited levels' quadrupolar moments for some deformable nonaxial even-even atomic nuclei are presented in Table 2.

It is seen that the two-parameter (g and γ) theory describes satisfactory the ratios of excited levels' quadrupolar moments, as it takes place in the case of the reduced probabilities of $E2$ transitions of deformable nonaxial even-even atomic nuclei.

Conclusion

Thus, we have suggested a simplified consideration of the probabilities of $E2$ transitions along with the quadrupolar moments of excited states of deformable nonaxial even-even atomic nuclei taking into account the deformation variation if the spin of a nucleus is increased. We have derived also simple expressions describing the $E2$ transitions' probabilities and quadrupolar moments of excited states depending on two parameters g and γ which do describe satisfactorily experimental data.

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ЗВЕДЕНІ ІМОВІРНОСТІ $E2$-ПЕРЕХОДІВ
ТА КВАДРУПОЛЬНІ МОМЕНТИ ЗБУДЖЕНИХ
СТАНІВ ДЕФОРМОВАНИХ НЕАКСІАЛЬНИХ
ПАРНО-ПАРНИХ ЯДЕР

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Р е з ю м е

Досліджено обертально-коливальні збуджені стани деформованих неаксіальних парно-парних ядер. Відношення зведених імовірностей $E2$-переходів і квадрупольних моментів збуджених станів деформованих неаксіальних парно-парних ядер ^{104}Ru, ^{166}Er і ^{238}U було розраховано, в тому числі для станів з високими спінами.