HOPF BIFURCATION IN A SOLID-STATE SINGLE-MODE LASER WITH A CONTROLLED Q-FACTOR OF THE RESONATOR

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A criterion of stability for oscillations of the Hopf-bifurcation origin, an interval of stability for the pumping parameter A, a periodic solution in the quadratic approximation, an analytic form of the limit cycle in a first approximation, and the intervals of variations of the control parameters have been obtained for the classical model of dynamics of a solid-state single-mode laser with a Q-switch of the universal cusp-deformation type.

Introduction

An insertion of a non-linear element into a resonator of a solid-state laser, as was pointed out in [1], is one of the effective manners to affect its dynamics, which is proved to be true experimentally. Nevertheless, such an influence, to be studied in detail, should be described theoretically, which comprises a solution of differential equations, depending on a number of parameters, including those for a Q-switch control. But the absence of general methods for integrating nonlinear systems interferes in obtaining the coefficients defining how phase coordinates, and those of a photon irradiation field in the first place, are sensitive to parameters. The problem becomes more difficult if a dependence of the Q-switch on the photon field is not specified. It is practically impossible to obtain particular results providing such a formulation of the problem. An attempt [1] to realize such an approach has reduced to several comments.

In this context, a subtask arises to select an adequate local method of integration and a specific dependence of the nonlinear element on the photon intensity, which, nevertheless, is rather general in its class of dependences.

Formulation of the Problem

Let us consider the rate equations that describe a dynamics of a single-mode solid-state laser:

$$\dot{x} = Gx(y - 1 - \Psi(x)) \equiv f_1(x, y),$$

$$\dot{y} = A - y(x + 1) \equiv f_2(x, y). \qquad (1)$$

where x is an intensity of the irradiation photon field, y is the relative density of an inverse population of atomic energy levels, $G = T_1/T_2$ is the ratio of a relaxation time of a difference in level population to a photon lifetime in a resonator, A is a pumping parameter, $\tau = t/T_1, t$ is time, and the dot means a differentiation with respect to τ. Concerning system (1), the following tasks are set:
— to carry out a bifurcation analysis of the system in the case where the dependence of the resonator loss on the radiation power is described by an expression $1 + \Psi(x)$, and a universal cusp deformation $x^4 + ax^2 + bx$ with the control parameters a and b [2] is taken for $\Psi(x)$;
— to find a criterion for the stability of periodic oscillations, which arise in the system due to the Hopf bifurcation;
— to construct approximate periodic solutions;
— to demonstrate an example of the calculation of the limit cycle.

To achieve those aims, we use the Hopf bifurcation method [3]. All quantities in system (1), both phase coordinates and parameters, are dimensionless. Concerning the methods to vary the parameters themselves, those issues are studied in [4].

Elements of the Bifurcation Analysis of System (1)

A stationary solution x_c, y_c of system (1) is obtained from the equations

$$A - \alpha(1 + x_c^4 + ax_c^2 + bx_c) = 0,$$

$$y_c = A/\alpha, \quad \alpha = x_c + 1. \qquad (2)$$

But it has no reason to determine x_c from (2) not taking into account the criterion of stability for periodic oscillations, emerging due to the Hopf bifurcation. In practice, one can find x_c from Eq. (2) only numerically. But nobody knows in advance, what numerical values should be taken for the parameters, so that x_c and

y_c satisfy the criterion of stability that is constructed according to the Hopf bifurcation algorithm (HBA). Furthermore, it is important to construct a solution in a vicinity of a stationary one, provided that the latter obeys the Hopf theorem conditions [3]. This compensates, to some extent, a locality of the method. Therefore, it is more reasonable to determine, from Eq. (2), the dependence of any parameter on the others. For example,

$$a = (A - \alpha(1 + bx_c + x_c^4))(\alpha x_c^2)^{-1}.$$

The Jacobi matrix of the right-hand sides of system (1) is

$$M = \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix},$$

$$a_1 = -G\varphi(x_c),$$

$$\varphi(x_c) = \alpha^{-1}[2(x_c^4 - 1)\alpha + 2A - \alpha b x_c],$$

$$b_1 = Gx_c,\ c_1 = \frac{A}{\alpha};\ d_1 = -\alpha. \tag{3}$$

Its eigenvalues

$$2\lambda_k = \mathrm{Spur}M \pm [(\mathrm{Spur}M)^2 - 4\det M]^{1/2} \tag{4}$$

$(k = 1, 2;$

$\mathrm{Spur}M = (a_1 + d_1), \quad \det M = a_1 d_1 - c_1 b_1)$

belong to a stable focus if $\mathrm{Spur}M < 0$ and $(\mathrm{Spur}M)^2 - 4\det M < 0$. At a bifurcation value of one of the parameters, which is determined from the equation

$$G\varphi(x_c) + \alpha = 0, \tag{5}$$

the eigenvalues λ_k become purely imaginary. If one takes into account that the parameter G is of the order of $O(10^5)$ and the value of $\frac{\alpha}{G}$ can be neglected, then, according to (5), we obtain

$$b_0 = (\alpha x_c)^{-1}[2\alpha(x_c^4 - 1) + 2A], \tag{6}$$

where the subscript "0" indicates the bifurcation value of the quantity. A linear part of system (1) has a periodic solution, whose frequency ω_0 is determined from the bifurcation value of the matrix M:

$$M_0 = \begin{pmatrix} \alpha & Gx_c \\ -\frac{A}{\alpha} & -\alpha \end{pmatrix}. \tag{7}$$

Its eigenvalues are $\lambda_k = \pm i\omega_0$, where

$$\omega_0 = \pm\sqrt{\det M_0} = (\frac{Gx_c A}{\alpha} - \alpha^2)^{1/2} \approx \sqrt{\frac{Gx_c A}{\alpha}},$$

which is a zero approximation to a modulation frequency ω. Taking into account (6), the parameter a becomes as follows:

$$a_0 = -(\alpha x_c^2)^{-1}[A + \alpha(3x_c^4 - 1)]. \tag{8}$$

An application of the HBA makes it possible to construct a periodic solution of the nonlinear system (1) in the vicinity the bifurcation value of the parameter b_0, which describes a limit cycle around the stationary solution, and to determine a criterion of the cycle stability. For this purpose, the following steps should be made. We find an eigenvector of matrix (7), which corresponds to the eigenvalue $i\omega_0$: $\mathbf{R} = \mathrm{Re}\,\mathbf{R} + i\,\mathrm{Im}\,\mathbf{R}$. Using its real and imaginary parts, we construct a transformation matrix P, with the help of which we introduce new variables z_1 and z_2:

$$P = \begin{pmatrix} 1 & 0 \\ -\frac{\alpha}{Gx_c} & -\frac{\omega_0}{Gx_c} \end{pmatrix},$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_c \\ y_c \end{pmatrix} + P\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}.$$

Then the transformed system is

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = P^{-1}\begin{pmatrix} f_1(z_1, z_2) \\ f_2(z_1, z_2) \end{pmatrix} \equiv \begin{pmatrix} F_1 \\ F_2 \end{pmatrix}, \tag{9}$$

where P^{-1} is a matrix inverse to P. The nonlinear terms of the second and third orders of the joint power of the variables z_1 and z_2 on the right-hand sides of system (9) are needed only for the further consideration. The summands of higher orders are supposed as those which may be neglected. The resulting, truncated in such a way, right-hand sides of system (9) are marked hereafter by a bar over F_1 and F_2:

$$\bar{F}_1 = -(GBz_1^2 + \omega_0 x_c^{-1} z_1 z_2 + GDz_1^3),$$

$$\bar{F}_2 = \frac{\alpha G}{\omega_0}Bz_1^2 + x_c^{-1} z_1 z_2 + \frac{\alpha}{\omega_0}Gz_1^3,$$

$$B = (\alpha\gamma - A)(\alpha x_c)^{-1},$$

$$D = Bx_c^{-1} + 4x_c^2;\ \gamma = 3x_c^4 + 1. \tag{10}$$

Then, using the partial derivatives of the second and third orders of the functions $\bar{F}_k$, calculated at origin, we find the complexes

$$g_1 = \frac{1}{2}\left[-GB + \frac{\alpha i}{\omega_0}(GB - 1)\right],$$

$$g_{2,3} = \frac{1}{2}\left[-GB \mp x_c^{-1} + i\left(\frac{\alpha}{\omega_0}(GB-1) \mp \frac{\omega_0}{x_c}\right)\right],$$

$$g_4 = \frac{3}{4}GD\left(-1 + i\frac{\alpha}{\omega_0}\right). \quad (11)$$

They are employed to create a quantity

$$\Phi = \frac{i}{2\omega_0}\left(g_1 g_3 - 2|g_1|^2 - \frac{1}{3}|g_2|^2\right) + \frac{g_4}{2}, \quad (12)$$

where the notation $|\ldots|$ means the absolute value of a complex number. The real part, $\mathrm{Re}\Phi$, is a principal summand of the Floquet index and the imaginary one, $\mathrm{Im}\Phi$, is used to find a correction to the oscillation period $T = \frac{2\pi}{\omega_0}$. An isolation of the real part of (12) making use of complexes (11) results in a following value:

$$\mathrm{Re}\Phi = \frac{1}{8\omega_0}\left[2GB(GB-1)\frac{\alpha}{\omega_0} + \right.$$

$$\left. + GB\frac{\omega_0}{x_c} - \frac{\alpha}{x_c\omega_0}(GB-1)\right] - \frac{3}{8}GD. \quad (13)$$

It turns out that the sign of (13) is determined by the summands that involve the factor G, provided that x_c is far enough from zero, which means, in practice, that $x_c > 0,1$. Hence, the terms, which do not involve G or involve it, raised to a negative power, are discarded. Moreover, if one take into account that $\omega_0^2 = \frac{GAx_c}{\alpha}$, the criterion of the periodic oscillation stability is of the form:

$$\mathrm{Re}\Phi_0 = \frac{G\Omega}{8A\alpha x_c^3} < 0,$$

$$\Omega = A^2(2x_c+1) - \alpha A(8(x_c+2\alpha) + 6x_c^5) + \alpha^3\gamma^2 < 0. \quad (14)$$

According to (14), the inequality is valid if the pumping parameter A is in an interval (A_1, A_2):

$$A_k = \frac{1}{2}[\alpha H \mp \alpha\gamma\sqrt{x_c(17x_c+8)}](2x_c+1)^{-1},$$

$$H = \gamma(5x_c+2) - 2x_c. \quad (15)$$

Derivation of the Approximate Periodic Solution

The approximate periodic solution is obtained according to formulae presented in [3]:

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_c \\ y_c \end{pmatrix} + P\begin{pmatrix} z_1 \\ z_2 \end{pmatrix};$$

$$z_1 = \mathrm{Re}z; z_2 = \mathrm{Im}z,$$

$$z = \varepsilon e^{2\theta i} + \frac{i\varepsilon^2}{6\omega_0}[g_2 e^{-4\theta i} - 3g_2 e^{4\theta i} + 6g_1] + O(\varepsilon^3),$$

$$\theta = \frac{\pi t}{T}, T = \frac{2\pi}{\omega_0}(1 + \tau_1\varepsilon^2 + O(\varepsilon^4)),$$

$$\tau_1 = -\frac{1}{\omega_0}(\mathrm{Im}\Phi_0 - \mathrm{Re}\Phi_0\frac{\omega_0'}{\lambda_0'}),$$

$$\varepsilon^2 = -\frac{(b-b_0)\lambda_0'}{\mathrm{Re}\Phi_0} + O((b-b_0)^2).$$

To use them, there is nothing to do but to determine the values of ω_0', λ_0', and $\mathrm{Im}\Phi_0$:

$$\lambda_0' \equiv \frac{\partial \mathrm{Re}\lambda}{\partial b} = \frac{Gx_c}{2},$$

$$\omega_0 \equiv \frac{\partial \mathrm{Im}\lambda}{\partial b} = -\frac{\alpha}{2}\sqrt{\frac{Gx_c\alpha}{A}},$$

$$\mathrm{Im}\Phi_0 = \frac{1}{6}\sqrt{\frac{G\alpha}{Ax_c}}(B^2G + \frac{1}{4}S),$$

$$S = 2B\alpha(5B\alpha^2 - 2A)(Ax_c)^{-1} - 27\alpha x_c^2. \quad (16)$$

The solution is written down as a truncated power series of a functional parameter ε that must satisfy the inequality $\varepsilon < 1$. For the variables z_1 and z_2, we obtain

$$\begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \varepsilon\begin{pmatrix} \cos 2\theta \\ \sin 2\theta \end{pmatrix} + \frac{\varepsilon^2 G}{6\omega_0}\left[\begin{pmatrix} -2\sin 4\theta \\ \cos 4\theta \end{pmatrix} B + \right.$$

$$+ \begin{pmatrix} \cos 4\theta \\ 2\sin 4\theta \end{pmatrix}\frac{\alpha B}{\omega_0} + \begin{pmatrix} 2\cos 4\theta \\ \sin 4\theta \end{pmatrix}\frac{A}{\omega_0\alpha} -$$

$$\left. -3\begin{pmatrix} \alpha\omega_0^{-1} \\ 1 \end{pmatrix} B\right] + O(\varepsilon^3), \quad (17)$$

where $\varepsilon^2 = -4(b-b_0)A\alpha x_c^4/\Omega$. Since $-\Omega > 0$ in the interval of stability, for the condition $\varepsilon^2 > 0$ to be fulfilled, it has to be $b > b_0$, i.e. the bifurcation is supercritical.

Calculation of the Limit Cycle Elements

The results obtained make it possible to find an interval of the parameter b variation, an analytic expression of the limit cycle, at least in a first approximation, a shift between the photon field and the inversion, an oscillation period with an accuracy of ε^2, an average output intensity on a Q-switch over a period, and the stability limits for the parameter A at x_c selected. It follows from $\varepsilon < 1$ that the parameter b varies in the interval

$$b_0 < b < b_0 - \frac{\Omega}{4Ax_c^4\alpha},$$

where A is selected from interval (15) at x_c selected. An interval of the b_0-variation, in its turn, is determined from (6), where the interval limits (15) should be substituted into. With the help of (17) and the transformation matrix P, we find the intensity of the photon field in a square approximation:

$$x = x_c + \varepsilon\cos 2\theta + \frac{\varepsilon^2 G}{6\omega_0^2}\times$$

$$\times[(-2\omega_0\sin 4\theta + \alpha\cos 4\theta - 3\alpha)B+$$

$$+2A\alpha^{-1}\cos 4\theta] + O(\varepsilon^3).$$

An expression for the limit cycle in a first approximation is obtained after the exclusion of the parameter θ from the solution

$$x = x_c + \varepsilon\cos 2\theta,$$

$$y = y_c - \frac{\varepsilon}{Gx_c}(\alpha\cos 2\theta + \omega_0\sin 2\theta) =$$

$$= y_c - \frac{\varepsilon N}{Gx_c}\cos(2\theta - \theta_1),$$

$$\theta_1 = \operatorname{arctg}\frac{\omega_0}{\alpha}, \quad N = (\alpha^2 + \omega_0)^{\frac{1}{2}}.$$

Since $\omega_0 = (GAx_c\alpha^{-1})^{\frac{1}{2}} >> \alpha$, the initial phase θ_1 is close to $\frac{\pi}{2}$ for solid-state lasers but always less than $\frac{\pi}{2}$. In a first approximation, the curve of the limit cycle, which surrounds the stationary solution, has the form

$$(x - x_0)^2(\omega_0^2 - \alpha^2) + (y - y_c)^2(Gx_c)^2 =$$

$$= \varepsilon^2\omega_0^2 + 2\alpha\omega_0(x - x_c)\sqrt{\varepsilon^2 - (x - x_c)^2}.$$

The expression for the period can be simplified taking into account the large value of the parameter G. First, we find

$$\mathrm{Re}\Phi_0\omega_0'(\lambda_0')^{-1} \approx \frac{\Omega}{8Ax_c^3}\sqrt{\frac{G\alpha}{Ax_c}},$$

then the first-order correction is

$$\tau_1 = -\frac{1}{6}\sqrt{\frac{G\alpha}{Ax_c}}\left[B^2G + \frac{1}{8}\left(2S - \frac{\Omega}{Ax_c^3}\right)\right],$$

which results in a following approximate value for the period T:

$$T = \frac{2\pi}{\omega_0}\left[1 + \frac{\varepsilon^2}{6}\sqrt{\frac{G\alpha}{Ax_c}}\left[B^2G + \frac{1}{8}\left(2S - \frac{\Omega}{Ax_c^3}\right)\right]\right].$$

An average radiation intensity over the period T, which is generated at the modulator, is calculated below in a first approximation according to the formula

$$\bar{x} = \frac{G}{T}\int_0^T x(t)\psi(x(t))dt =$$

$$= \frac{G}{T}\int_0^T (x_c^5 + a_0x_c^3 + b_0x_c^2)dt.$$

The substitution $x = x_c + \varepsilon\cos 2\theta$ results in

$$\bar{x} = G\left[x_c^5 + a_0x_c^3 + b_0x_c^2+\right.$$

$$\left.+\varepsilon^2\left(5x_c^3 + \frac{3}{2}a_0x_c + \frac{b_0}{2}\right) + +\frac{15}{8}x_c\varepsilon^4\right].$$

As is seen, the stationary solution x_c makes the main contribution to the integral.

To define the safe and dangerous limits of stability for the limit cycle, it is necessary to find the first Lyapunov's value. To do this, we have to know the coefficients of expansions of the right-hand sides of system (1) in a power series of $x - x_c$ and $y - y_c$ up to the third order inclusive. The coefficients of the linear terms were obtained earlier. Let a_{qj} and b_{qj} stand for the coefficients of the expansions of the functions f_1 and f_2, respectively, with $q + j = 2$ for the second-order terms and $q + j = 3$ for the third-order ones. Then,

$$a_{20} = -\frac{G}{\alpha x_c}\Gamma_1;\ a_{11} = G;\ a_{02} = 0;\ a_{30} = -\frac{3G\Gamma_2}{\alpha x_c};$$

$$a_{21} = a_{12} = a_{03} = 0;\ b_{11} = -1;\ b_{20} = b_{02} = 0;$$

$$b_{qj} = 0;\ q + j = 3;$$

$$\Gamma_1 = 4x_c^6 + 11x_c^5 + 9x_c^4 - 4x_c^2 - x_c - A + 1;$$

$$\Gamma_2 = 11x_c^5 + 17x_c^4 + 8x_c^3 - x_c - A + 1.$$

The first and second subscripts indicate here the order of the derivative with respect to the first or second variable, respectively. The Lyapunov value for a system of the second order is quoted in [5]. In the present case, it equals

$$L_1 = -\frac{\pi}{4b_1\omega_0^3}[a_1c_1a_{11}^2 + a_1b_1(b_{11}^2 + b_{11}a_{20}) -$$

$$-2a_1b_1a_{20}^2 + (b_1c_1 - 2a_1^2)a_{11}a_{20} + 3(a_1^2 + b_1c_1)b_1a_{30}].$$

A substitution of the coefficient values gives

$$L_1 = -\frac{\pi G^2}{4b_1\omega_0^3\alpha^2}Q;$$

$$Q = 4G^2x_c^3\Gamma_1\rho(\Gamma_1x_c^{-2} - 2\rho)\alpha^{-1} +$$

$$+G\left[2\rho x_c^2(A - \Gamma_1) - A\Gamma_1 - \frac{36}{\alpha}x_c^3\rho^2\Gamma_2\right] + 9\Gamma_2,$$

$$\rho = x_c^4 - 1.$$

Below, we present a numerical example of the calculation of the solution elements, the criterion of stability, and the intervals of the parameter variations.

Let $x_c = 1,5$. Then, $A \in (11.26 \quad 82.97)$. One may select $A = 30$. The bifurcation value of b_0 and the relevant value of a_0 are calculated according to formulae (6) and (8):

$$b_0 = (\alpha x_c)^{-1}[2\alpha(x_c^4 - 1) + 2A] = 18.166;$$

$$a_0 = -(\alpha x_c^2)^{-1}[A + \alpha(3x_c^4 - 1)] = -11.638.$$

Then, for Ψ, we obtain

$$\Psi = x_c^4 + a_0x_c^2 + b_0x_c = 6.126 > 0.$$

The principal factor of the criterion of stability is

$$\Omega(1.5, 30) = -3614.2 < 0.$$

Then, $-\frac{\Omega}{4Ax_c^4\alpha} = 2.379$. Therefore, $b \in (18.166; 20.545)$. The phase shift is

$$\frac{\omega_0}{\alpha} = \frac{1}{\alpha}\sqrt{\frac{GAx_c}{\alpha}} = 536; \ \mathrm{arctg}536 = 89.893°.$$

The principal factor of the first Lyapunov value Q changes its sign depending on the sign of the first term. For example, if $x_c = 1$, then, in accordance with (15), we may take $A = 10$. Then, $Q = 600.69G + 664.37 < 0$, i.e. $L_1 > 0$. For $x_c = 1.5$, we may take $A = 30$. Then, $Q = 60(1004.133G^2 - 371.195G + 11.072) > 0$, i.e. $L_1 < 0$.

Although the selected values of the parameters satisfy the criterion of stability (14), the stability limit is "dangerous" in the former case and "safe" in the latter one [6].

Conclusions

A model of the Q-switch of the universal cusp-deformation type provides a stable periodic mode of photon irradiation in a wide interval of variation of the parameter A for each selected x_c. Those intervals are partially overlapped for close x_c's, which allows one not only to select A at the specific x_c, but to vary also x_c at the already selected A.

As the parameter x_c grows, the intervals for A as well as their overlap intervals widen. If a distance Δx_c between neighbor x_c's enlarges, the stretches, not overlapped by stability intervals (the so-called "lacunae"), emerge. However, as the value of x_c increases, the lacunae narrow and, above a certain x_c, disappear. For example, the intervals, corresponding to $x_c = 0.5, 1.5$, and 2.5, do not overlap. At the same time, the intervals, corresponding to $x_c = 2.5$ and 3.5, partially overlap.

Contrary to the criterion of stability, the sign of the first Lyapunov value is defined not only by a large value of the parameter G but also by a selected value of the parameter x_c. The Lyapunov value gives therefore a larger body of information concerning the stability limit of the dynamics of a solid-state laser operating in the mode of periodic irradiation, than the criterion of stability.

The theoretical results obtained can be tested experimentally. The use of a Q-switch as a nonlinear crystal is widely applied in practice and scientific researches. In particular, there are enough experimental evidences for the emergence of the Hopf bifurcation [1]. The problem is to ensure a task-oriented influence on one of the Q-switch parameters for the stable oscillations of the intensity, which modify the stationary mode of irradiation taking place at the beginning of the experiment, to start. Various methods can be used for the technical implementation of an appropriate influence, including an optoelectronic feedback. The latter is known to control laser parameters according to the intensity. It is the second approach that is advantageous, because it can change the phase-plane portrait of the system without a displacement of the stationary point, which means, in this case, a transition from the stable focus to a non-stable one. And just providing the latter, the Hopf bifurcation emerges. That is, the process of irradiation, having started from the stationary mode, then converts to the oscillation one around the starting stationary point under the action of an optoelectronic feedback on a derivative of the intensity.

One of the schemes of the optoelectronic feedback on a derivative of the photon field intensity is given in [1].

Perspectives

An application of the Hopf bifurcation theory [3] to laser dynamics, concerning both classical and semiclassical models, only gets under way. Wide perspectives of the HBA application open in those research directions, which practically have not been started yet for elaboration in theoretical aspects, e.g., the inverse problems of laser dynamics, the studies of the delay effect on the dynamical characteristics of the periodic mode of irradiation, the problems of secondary bifurcations, and so on.

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БІФУРКАЦІЯ ХОПФА
В ТВЕРДОТІЛЬНОМУ ОДНОМОДОВОМУ
ЛАЗЕРІ З КЕРОВАНОЮ ДОБРОТНІСТЮ РЕЗОНАТОРА

І.О.Шуда

Р е з ю м е

Для класичної моделі динаміки твердотільного одномодового лазера з модулятором добротності типу універсальної деформації згортки одержано критерій стійкості періодичних коливань, викликаних біфуркацією Хопфа, інтервал стійкості для параметра накачки A, періодичний розв'язок у квадратичному наближенні, аналітичний вигляд граничного циклу в першому наближенні, інтервал зміни параметрів керування.