INVESTIGATION OF CORRELATIONS OF NUCLEONS OF EVEN-EVEN NUCLEI IN THE FRAMEWORK OF THE ADIABATIC THREE-PARTICLE MODEL OF NUCLEI

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Stationary states of even-even atomic nuclei, whose mean self-consistent field is simulated by the Woods—Saxon potential, are described in the framework of the adiabatic three-particle model of nuclei. The description is carried out in the terms of collective variables, namely, the hyperradius R, hyperangle α, and conventional spherical angles (θ_i, φ_i), $i = 1, 2$. The efficiency of the adiabatic approach is illustrated by the example of the numerical calculation of the energy spectra of low-lying excited states of even-even atomic nuclei ^{40}Ca, ^{64}Zn, ^{74}Se, and ^{200}Hg which possess two valent nucleons in the external shell.

The impossibility to exactly solve the many-particle Schrodinger equation stimulates us to search for the approximate methods of its solution. One of the methods of the description of a many-particle system is the Hartree—Fock one which was developed for the first time in atomic physics in [1]. This method allows one to consider a many-particle problem as the problem of the movement of one particle in the self-consistent field created by all particles of the system under study. The condition for a successful application of the Hartree—Fock method in nuclear theory is the requirement that the nucleon-nucleon interaction potential be sufficiently smooth, i.e. it should have no singularity as $r \to 0$. The Hartree—Fock method in nuclear theory prevails in calculations for the magic nuclei. But the application of this method to non-magic nuclei meets essential mathematical difficulties if we take into account a repulsion in the nucleon-nucleon potential at small distances.

The angular and radial correlations of nucleons and the pairing effects for nucleons of the same sort play an important role in the formation of excited states of nuclei and appear, particularly, in the presence of gaps in the energy spectra of excited states of even-even nuclei and in their absence in the spectra of odd and odd-odd nuclei. Thus, it is necessary to develop another method for the calculation of the wave functions and the energy spectra of stationary states of even-even nuclei, which would go beyond the limits of the one-nucleon Hartree—Fock approach [1].

To this end, a hyperspherical adiabatic approach (HAA), which transcends the frames of the one-nucleon approach, was suggested in [3—6] to solve the mentioned problem.

As known, pairwise correlations of nucleons of the same sort, which result particularly in the existence of superfluid states of nuclei [7], are considered most logically and correctly in the superfluid model of nucleus [8, 9] within the secondary quantization formalism.

In the present paper, we suggest to consider the pairwise correlations between nucleons in the potential approach in the framework of the adiabatic three-particle model of nuclei [3—6], based on the assumption of separability of the motion of valence nucleons of a nucleus into the high-speed movement in angular variables, i.e. on the sphere $S^5(\Omega)$ and the adiabatic (low-speed) movement of nucleons along the hyperradius R and on the introduction of the notion of adiabatic potential term of nucleons of a nucleus $U_\mu(R)$ which is convenient for the description. We recall that the adiabatic three-particle model of nuclei is based on the assumption of the existence of an average self-consistent field in the model of independent particles with taking into account a short-range residual interaction of valence nucleons.

The further development and application of the adiabatic approach in nuclear theory to the investigation of the energy spectrum of both spherical and deformed even-even nuclei within the framework of the adiabatic three-particle model of nuclei and with consideration of the Coulomb interaction between valent protons besides the strong one are considered to be topical now.

1. Theoretical Description of the Energy Spectrum of a Spherical Nucleus

In [3—6] within the HAA method, a theoretical description of the energy spectrum of excited states of nuclei, which are modeled by a spherically symmetric

even-even "core" plus two nucleons on the outer unfilled shell, was carried out. For the ${}^{A}_{Z}\mathrm{X}$ nucleus with two valent nucleons, the nucleus description in the HAA method is carried out in terms of collective variables, whose role is played by the hyperradius R, hyperangle α,

$$R = (r_1^2 + r_2^2)^{1/2}, \quad \alpha = \mathrm{arctg}(r_2/r_1), \tag{1}$$

and usual spherical angles $\hat{r}_i = \{\varphi_i, \theta_i\}$, $i = 1, 2$.

In the adiabatic three-particle model of nuclei, the efficient self-consistent field is modeled by the static spherically symmetric Woods—Saxon potential [10]

$$U_i(r_i) = -V_0 \left(1 \pm 0.63 \frac{N-Z}{A}\right) \times$$

$$\times \left(1 + \exp\left(\frac{r_i - R_0}{a_0}\right)\right)^{-1} + V_k, \quad i = 1, 2, \tag{2}$$

where "+" should be taken for a proton and "−" for a neutron and $R_0 = r_0 A^{1/3}$.

If there are two valent protons on the external shell, then the Coulomb potential V_k can be modeled, for the sake of simplicity, as [10]

$$V_k = \sum_{i=1}^{2} V_k\left(r_i\right), \tag{3}$$

where

$$V_k\left(r_i\right) = \begin{cases} \left[\frac{3}{2} - \frac{1}{2}\left(\frac{r_i}{R_0}\right)^2\right] \frac{e^2(z-2)}{R_0}, & r_i \le R_0, \\ \frac{e^2(z-2)}{r_i}, & r_i > R_0. \end{cases} \tag{4}$$

Here, $V_k\left(r_i\right)$ is the potential energy of interaction between the i-th proton and the Coulomb field of the uniformly charged sphere.

For the simplification of further calculations, we can represent the residual strong interaction of valent nucleons between one another as the potential with zero interaction radius with regard for the repulsion of nucleons at short distances [11]

$$V_{\mathrm{res}}(\vec{r}_1, \vec{r}_2) = -V_{12}\left[1 - g\rho\left(\frac{\vec{r}_1 + \vec{r}_2}{2}\right)\right]\delta(\vec{r}_1 - \vec{r}_2). \tag{5}$$

The repulsion of nucleons is characterized by the term $\rho(\frac{\vec{r}_1+\vec{r}_2}{2})$ which denotes the total one-particle density of nucleons. The relative contribution of repulsion is defined by the constant g ($g > 0$). Such a choice of the residual interaction simplifies the algorithm of the energy spectrum computation, because it allows one to calculate, in the explicit analytic form, the matrix elements of this interaction and does not distort, possibly, the real situation. In the future, more realistic models of the interaction should be developed.

In the case of valent protons, their Coulomb interaction

$$V_{k12} = \frac{e^2}{|\vec{r}_1 - \vec{r}_2|} \tag{6}$$

must be added to (5).

The spin-orbit interaction of the i-th nucleon is given by

$$V_{l_i s_i}(r_i) = W_i(r_i)(\vec{l}_i \cdot \vec{s}_i),$$

$$W_i(r_i) = -\chi \frac{1}{r_i}\frac{\partial U_i(r_i)}{\partial r_i}, \quad i = 1, 2. \tag{7}$$

Thus, in the framework of the adiabatic three-particle model of nuclei in terms of collective variables (1), the potential energy $V\left(R, \Omega\right)$ of the system under study is given by

$$V\left(R, \Omega\right) = U_1\left(R\cos\alpha\right) + W_1\left(R\cos\alpha\right)\left(\vec{l}_1 \cdot \vec{s}_1\right) +$$

$$+U_2\left(R\sin\alpha\right) + W_2\left(R\sin\alpha\right)\left(\vec{l}_2 \cdot \vec{s}_2\right) + V_{\mathrm{res}} + V_{k12}. \tag{8}$$

We note that using the Hamiltonian with central two-particle and spin-orbit one-particle interactions for a spherical nucleus corresponds to the so-called intermediate coupling approximation.

As was shown in works [3—6], the problem of determination of the energy spectrum for spherical atomic nuclei in the framework of the adiabatic three-particle model of nuclei is reduced to solving the two following problems.

In the first place, it is the problem of determination of adiabatic potential terms $U_\mu(R)$ of nucleons of the nucleus and the corresponding basis functions $\Phi_\mu(R, \Omega)$. This can be done by a numerical solution of the system of differential equations for the variable α,

$$\left[\frac{d^2}{d\alpha^2} - \frac{l_1(l_1+1)}{\cos^2\alpha} - \frac{l_2(l_2+1)}{\sin^2\alpha} + U_\mu(R)\right]\varphi^{(\mu)}_{j_1 j_2 l_1 l_2}(R, \alpha) +$$

$$+R^2 \sum_{j_1' j_2' l_1' l_2'} V^{j_1 j_2 l_1 l_2}_{j_1' j_2' l_1' l_2'}(R, \alpha)\varphi^{(\mu)}_{j_1' j_2' l_1' l_2'}(R, \alpha) = 0, \tag{9}$$

where the coefficients

$$\varphi^{(\mu)}_{j_1 j_2 l_1 l_2}(R,\alpha) = \sin\alpha\cos\alpha\Phi^{(\mu)}_{j_1 j_2 l_1 l_2}(R,\alpha). \quad (10)$$

System (9) is supplemented by the boundary conditions that ensure a boundedness of the function $\varphi_\mu(R,\alpha)$ at zero and the validity of the Pauli principle:

$$\varphi^{(\mu)}_{j_1 j_2 l_1 l_2}(R,\alpha=0) = 0,$$

$$\varphi^{(\mu)}_{j_1 j_2 l_1 l_2}(R,\alpha)|_{\alpha=\pi/4} =$$

$$= (-1)^{j-j_1-j_2+1}\varphi^{(\mu)}_{j_2 j_1 l_2 l_1}(R,\pi/2-\alpha)|_{\alpha=\pi/4},$$

$$\partial\varphi^{(\mu)}_{j_1 j_2 l_1 l_2}(R,\alpha)/\partial\alpha|_{\alpha=\pi/4} =$$

$$= (-1)^{j-j_1-j_2}\partial\varphi^{(\mu)}_{j_2 j_1 l_2 l_1}(R,\alpha)/\partial\alpha|_{\alpha=\pi/4}. \quad (11)$$

Thus, we can find the adiabatic terms $U_\mu(R)$ and basis functions $\Phi_\mu(R,\Omega)$ by solving system (9) with boundary conditions (11) [3]. The expansion of the full wave function of the system in the hypergeometric adiabatic basis $\{\Phi_\mu(R,\Omega)\}$ [3] looks

$$\Psi(R,\Omega) = R^{-5/2}\sum_\mu F_\mu(R)\Phi_\mu(R,\Omega). \quad (12)$$

Secondly, we must determine the radial functions $F_\mu(R)$ and the energy spectrum E of bound states of nucleons through a numerical solution of the system of differential equations for the variable R

$$\left\{-\frac{d^2}{dR^2} - \frac{1}{4R^2} + U_\mu(R) - 2E\right\}F_\mu(R) +$$

$$+\sum_{\mu'}\left\{H_{\mu\mu'}(R)F_{\mu'}(R) + Q_{\mu\mu'}(R)\frac{d}{dR}F_{\mu'}(R) +\right.$$

$$\left.+\frac{d}{dR}[Q_{\mu\mu'}(R)F_{\mu'}(R)]\right\} = 0. \quad (13)$$

The radial functions $F_\mu(R)$ satisfy the boundary conditions

$$F_\mu(0) = F_\mu(\infty) = 0. \quad (14)$$

In numerical calculations, we substitute the interval $(0,\infty]$ of R by a finite interval $[0,R_{\max}]$ and truncate system (13) to that with a finite number of equations. Using the adiabatic approximation, in which expansion (12) contains only diagonal matrix elements, reduces system (13) to a single equation.

The explicit form of potentials (2) — (7) is given in [4, 5].

The efficiency of the HAA method in the framework of the adiabatic three-particle model of nuclei has been illustrated by the example of numerical calculations [12—15] of the energy spectra of nuclei ^{6}He, ^{10}Be, ^{14}C, ^{16}C, ^{18}Ne, ^{18}O, ^{42}Ca, and ^{58}Ni under assumption of spherical symmetry of the nucleus field. The calculated energies of excited states of nucleons for the certain studied nuclei indicate the necessity to take into account the polarization effects for the even-even core, i.e. to consider a deformation of the nucleus core field by nucleons from the external unfilled shell.

2. Theoretical Description of the Energy Spectrum of a Deformed Nucleus

In the calculations of stationary states of deformed nuclei, the oscillatory Nilsson potential was used for a long time as an effective potential of the average nuclear field of a core [16]. With the help of the Nilsson potential, a rather simple scheme for the determination of the one-particle levels and corresponding wave functions of states of deformed nuclei was developed. However, the Nilsson potential has a number of essential limitations. For example, it has infinite depth, which yields the improper behaviour of wave functions on the nucleus boundary and outside it. Moreover, the spin-orbit interaction in the Nilsson's scheme is independent of the mass number A and the deformation parameters.

Therefore, a more realistic finite anisotropic Woods—Saxon potential becomes recently to be widely used in calculations of the energy spectrum of deformed nuclei [17, 18]. For the first time, the problem of determination of one-particle levels and wave functions of states in a deformed Woods—Saxon potential was investigated by Nemirovskii and Chepurnov in [17]. Later on, other methods of solving the Schrodinger equation with anisotropic Woods—Saxon potential were proposed in the one-particle approximation [18].

It is necessary to note that the integral of motion for deformed nuclei with the form of an ellipsoid of revolution is the projection K of the total angular momentum of a nucleon on the nucleus symmetry axis,

i.e. one-particle nucleon states are characterized by energy, parity, and projection K.

In the adiabatic three-particle model of nuclei, the stationary states of two valent nucleons in the deformed nucleus field, which is simulated by the anisotropic Woods—Saxon potential, are determined [19] from the Schrodinger's equation

$$\left(-\frac{\hbar^2}{2\mu_1}\Delta_1 - \frac{\hbar^2}{2\mu_2}\Delta_2 + \hat{V} - E\right)\Psi = 0, \tag{15}$$

where the potential energy operator of the system is given [19] by

$$\hat{V} = U_1(\vec{r}_1,\beta) + V_{\rm so}(\vec{r}_1,\vec{\sigma}_1,\beta) + U_2(\vec{r}_2,\beta) +$$

$$+V_{\rm so}(\vec{r}_2,\vec{\sigma}_2,\beta) + V_{\rm res}(\vec{r}_1,\vec{r}_2). \tag{16}$$

Here, $U_i\,(\vec{r}_i,\beta)$ is purely the nuclear potential energy of the i-th nucleon at the point $\vec{r}_i$ in the deformed axially symmetric Woods—Saxon field [10]:

$$U_i(\vec{r}_i,\beta) = -V_0\left(1 \pm 0.63\frac{N-Z}{A}\right)\times$$

$$\times\left(1+\exp\left[\frac{(r_i - R(\theta_i,\beta)}{a}\right]\right)^{-1} + V_k, \quad i = 1,2. \tag{17}$$

The radius $R(\theta_i,\beta)$ of the deformed axially symmetric field of a nucleus depends on the deformation parameter β and the angle θ_i relative to the symmetry axis of a nucleus and is chosen as

$$R(\theta_i,\beta) = R_0[1+\beta Y_{20}(\theta_i)]. \tag{18}$$

As is well known, spin-orbit interaction operators in the case of the nuclear potential $U_i\,(\vec{r}_i,\beta)$ have the form [17]

$$V_{\rm so}(\vec{r}_i,\vec{\sigma}_i,\beta) = -\chi[\vec{p}_i\times\vec{\sigma}_i]\cdot{\rm grad}U_i(\vec{r}_i,\beta). \tag{19}$$

In (15), we separate the spherically symmetric part of interaction and the additional term which sets a deviation of the interaction symmetry from the spherical one. As a result, we obtain the equation

$$\left(-\frac{\hbar^2}{2\mu_1}\Delta_1 - \frac{\hbar^2}{2\mu_2}\Delta_2 + V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta) -\right.$$

$$-V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta=0) + V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta=0) +$$

$$\left.+V_{\rm res}(\vec{r}_1,\vec{r}_2) - E\right)\Psi = 0. \tag{20}$$

It is convenient to seek for solutions of Eq. (20) in hyperspherical coordinates (1) in the form of a superposition of solutions Ψ_{nJK},

$$\Psi_K(R,\Omega) = \sum_n\sum_J C_{nJK}\Psi_{nJK}(R,\Omega), \tag{21}$$

of the stationary Schrodinger's equation

$$\left(-\frac{\hbar^2}{2\mu_1}\Delta_1 - \frac{\hbar^2}{2\mu_2}\Delta_2 + V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta=0) +\right.$$

$$\left.+V_{\rm res}(\vec{r}_1,\vec{r}_2) - \varepsilon_{nJK}\right)\Psi_{nJK} = 0 \tag{22}$$

with the spherically symmetric potential $V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,$ $\beta=0)$.

The stationary states

$$\Psi_{nJK} \equiv \Psi_{nJK}(R,\Omega) = F_{nJK}(R)\Phi_{nJK}(R,\Omega) \tag{23}$$

of the corresponding spherical nucleus can be obtained from (22) according to the scheme introduced in [3—6] and briefly given in Section 1.

After the substitution of (21) to (20), multiplication of all terms of the equation by $\Psi^*_{n'J'K'}(R,\Omega)$, and integration over the whole region of hyperspherical coordinates, we obtain

$$\sum_n\sum_J(\varepsilon_{nJ}-E)C_{nJK}\delta_{nn'}\delta_{JJ'} + \sum_n\sum_J C_{nJK}\times$$

$$\times\langle\Psi_{n'J'K'}|\tilde{V}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)|\Psi_{nJK}\rangle = 0, \tag{24}$$

where

$$\tilde{V}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta) = V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta) -$$

$$-V(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta=0) =$$

$$=\sum_{i=1}^{2}[\tilde{U}_i(\vec{r}_i,\beta)+\tilde{V}_{\rm iso}(\vec{r}_i,\vec{\sigma}_i,\beta)], \tag{25}$$

$$\tilde{U}_i(\vec{r}_i,\beta) = U_i(\vec{r}_i,\beta) - U_i(\vec{r}_i,\beta=0), \tag{26}$$

$$\tilde{V}_{\rm iso}(\vec{r}_i,\vec{\sigma}_i,\beta)=V_{\rm iso}(\vec{r}_i,\vec{\sigma}_i,\beta)-V_{\rm iso}(\vec{r}_i,\vec{\sigma}_i,\beta=0). \quad (27)$$

We can represent the spin-orbit addition $\tilde{V}_{\rm so}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)$ (27) in potential (25) as [17,18]

$$\tilde{V}_{\rm so}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)=W_1+W_2+W_3, \quad (28)$$

where

$$W_1=-\chi\sum_{i=1}^{2}\frac{1}{r_i}\frac{\partial\tilde{U}_i(r_i,\beta)}{\partial r_i}\left(p_{\theta_i}\sigma_{\varphi_i}-\frac{1}{\sin\theta_i}p_{\varphi_i}\sigma_{\theta_i}\right), \quad (29)$$

$$W_2=-\chi\sum_{i=1}^{2}\frac{1}{r_i^2\sin\theta_i}\frac{\partial\tilde{U}_i(r_i,\beta)}{\partial\theta_i}p_{\varphi_i}\sigma_{r_i}, \quad (30)$$

$$W_3=\chi\sum_{i=1}^{2}\frac{1}{r_i}\frac{\partial\tilde{U}_i(r_i,\beta)}{\partial\theta_i}p_{r_i}\sigma_{\varphi_i}, \quad (31)$$

and

$$p_{r_i}=-i\hbar\frac{\partial}{\partial r_i},\quad p_{\theta_i}=-i\hbar\frac{\partial}{\partial\theta_i},\quad p_{\varphi_i}=-i\hbar\frac{\partial}{\partial\varphi_i}. \quad (32)$$

In (29)—(31), $\sigma_{r_i},\sigma_{\theta_i},\sigma_{\varphi_i}$ is the Pauli matrices which are given explicitly in [17].

To numerically solve the system of equations (24), we need to know the matrix elements of the potentials of both purely nuclear and spin-orbit interactions.

For the determination of matrix elements in (24), it is convenient to expand $\tilde{V}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)$ in a series in terms of spherical functions. For the nuclear terms of potential (25), we obtain

$$\tilde{U}_i(\vec{r}_i,\beta)=\sum_{\lambda_i m_i}A_{\lambda_i m_i}(r_i,\alpha,\beta)Y_{\lambda_i m_i}(\theta_i,\varphi_i). \quad (33)$$

Respectively

$$\frac{\partial\tilde{U}_i(r_i,\beta)}{\partial r_i}=\sum_{\lambda_i m_i}B_{\lambda_i m_i}(r_i,\alpha,\beta)Y_{\lambda_i m_i}(\theta_i,\varphi_i), \quad (34)$$

where the expansion coefficients $A_{\lambda_i m_i}(r_i,\alpha,\beta)$ and $B_{\lambda_i m_i}(r_i,\alpha,\beta)$ should be obtained numerically.

In the examined case of an axisymmetric nucleus, $m_1=m_2=0$.

Non-diagonal matrix elements of the operator $\tilde{V}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)$ in Eq. (24) are given in Appendix.

The system of homogeneous equations (24) has nonzero solutions if the determinant composed from the coefficients of the unknowns C_{nJK} equals zero. Expanding this determinant, we obtain an algebraic equation for the determination of E.

By solving system (24) in the standard way, we can find the energy spectrum E of the deformed nucleus, coefficients C_{nJK}, and, hence, the corresponding wave functions of stationary states of the deformed nucleus. We can obtain the unknown energy E of the deformed nucleus for $\beta<1$ by the method of perturbation theory with respect to the deformation parameter $\beta<1$.

For the case of minor deformations $\beta<1$, we can consider the operator $\tilde{V}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)$ in Eq. (24) as an operator of perturbation which represents the difference between a weakly deformed Woods—Saxon potential with deformation parameter $\beta<1$ and a spherical Woods—Saxon potential. The energy of an arbitrary level E_{nJK} of a deformed nucleus in the first approximation of perturbation theory is given by

$$E=E^{(1)}_{nJK}=\varepsilon_{nJ}+V_{nJK,nJK}, \quad (35)$$

where ε_{nJ} is the energy of the j-th level of a spherically symmetric nucleus and $V_{nJK,nJK}$ is the unknown matrix element of the operator $\tilde{V}(\vec{r}_1,\vec{\sigma}_1,\vec{r}_2,\vec{\sigma}_2,\beta)$.

As seen from formula (35), due to the axial symmetry of the Woods—Saxon potential, the energy levels ε_{nJ} that were found for a spherically symmetric Woods—Saxon field decay in the axially deformed Woods—Saxon field into the energy levels that correspond to different values of the quantum number K of the angular momentum projection J on the nucleus symmetry axis. That is, the degeneration $2J+1$ in $|K|$ is removed, but the twofold degeneration of levels in the sign of K remains.

Thus, to determine the energy spectrum of a deformed nucleus ${}^{A}_{Z}\mathrm{X}$ in the framework of the considered adiabatic three-particle model of nuclei, it is necessary, following works [3—6], to obtain the spectra of levels ε_{nJ} and the corresponding wave functions of stationary states in the assumption of the spherical symmetry of the field of a nucleus ${}^{A}_{Z}\mathrm{X}$, and then to take into account the deformation of the nucleus field while numericaly solving system (24).

3. Numerical Calculations of the Energy Spectra of Nuclei ^{40}Ca, ^{64}Zn, ^{74}Se, and ^{200}Hg

Below we illustrate the main points of the numerical calculation of the energy spectrum of the nuclei in the framework of the adiabatic three-particle model of nuclei. It will be done by the example of low-lying excited

states of even-even nuclei ^{40}Ca, ^{64}Zn, ^{74}Se, and ^{200}Hg which possess two valent nucleons in the external shell, namely, ^{40}Ca and ^{64}Zn have two protons, and ^{74}Se and ^{200}Hg have two neutrons. For the simplification of calculations, we simulate the strong interaction of valent nucleons by the spherically symmetric Woods—Saxon potential. For the valent protons, we also consider the Coulomb interaction in addition to the strong one.

Accordingly to the asymptotic behavior of the terms investigated l in [5] in detail, the calculations of the energy spectra of nuclei ^{40}Ca, ^{64}Zn, ^{74}Se, and ^{200}Hg under assumption of the spherically symmetric field of a nucleus core were carried out as follows. Parameters of the Woods—Saxon potential were selected in such a way that the potential terms of nuclei ^{40}Ca, ^{64}Zn, ^{74}Se, and ^{200}Hg tend to the corresponding levels of isotopes with mass numbers less by unit as $R \to \infty$, i.e. to the corresponding levels of isotopes ^{39}K, ^{63}Cu, ^{73}Se, and ^{199}Hg, respectively. The values of the Woods—Saxon potential parameters for nuclei ^{40}Ca, ^{64}Zn, ^{74}Se, and ^{200}Hg defined in such a way are shown in Table 1. Then, following works [3—6], the spectra of levels and the corresponding wave functions of stationary states without regard for the matrix elements $H_{\mu\mu'}$ and $Q_{\mu\mu'}$ were determined by using the determined parameters of potentials. As the reference zero point, we took the energy when both valent nucleons were in the ground state, i.e. zero reference point, we took the energy of the state where both valent nucleons were in the ground state: two protons on the levels $2s_{1/2}$ and $1f_{5/2}$ for nuclei ^{40}Ca and ^{64}Zn, and two neutrons on the levels $2p_{1/2}$ and $2f_{5/2}$ for the nuclei ^{74}Se and ^{200}Hg, respectively.

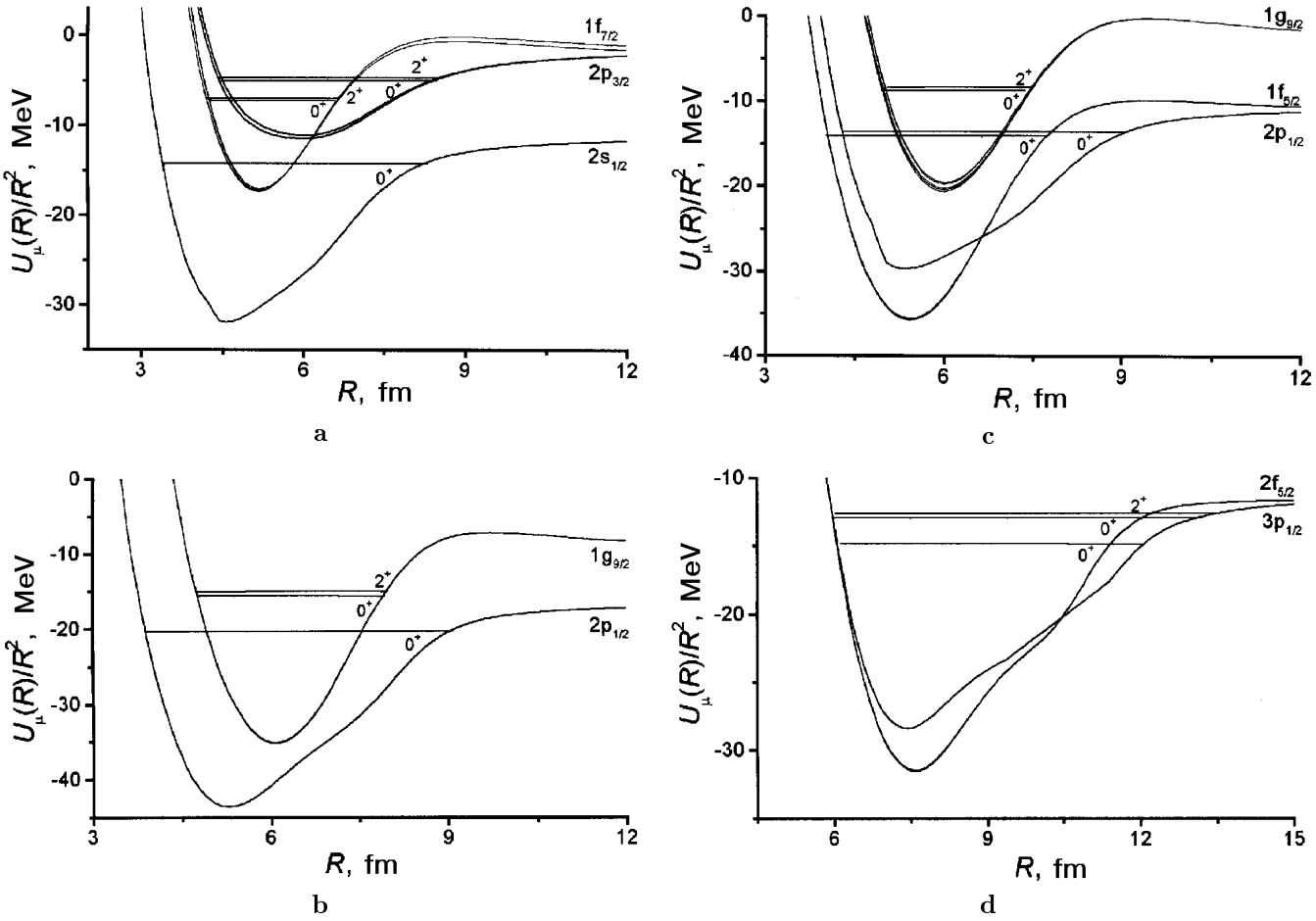
The results of calculations of the energy spectrum ε_{nJ} of low-lying excited states of nuclei ^{40}Ca, ^{64}Zn, ^{74}Se, and ^{200}Hg under assumption of a spherically symmetric field are given in Table 2, and their positions on the adiabatic potential terms $U_\mu(R)/R^2$ of the nuclei are presented, respectively, by straight lines in Figure. As a null, we took the energy of separation of two nucleons from

T a b l e 1. Parameters of the Woods—Saxon potential for nuclei ^{40}Ca, ^{64}Zn, ^{74}Se, and ^{200}Hg

Nucleus	V_0, MeV	V_{12}, MeV	r_0, fm	a_0, fm	χ, fm^2
^{40}Ca	45.5	33.0	1.24	0.63	0.263
^{64}Zn	49.0	33.0	1.24	0.63	0.296
^{74}Se	59.0	33.0	1.24	0.63	0.310
^{200}Hg	57.3	33.0	1.24	0.63	0.368

T a b l e 2. Results of calculations of the energy of nuclei ^{40}Ca, ^{64}Zn, ^{74}Se, and ^{200}Hg states under assumption of the spherically symmetric Woods—Saxon potential

Nucleus ^{A}X	Configuration of nucleons	J^π	ε_{nJ}, MeV	$\varepsilon_{\exp}$, MeV	$U_\mu(R)/R^2$ for R = 12 fm, MeV	$\varepsilon_{\exp}$ for $^{A-1}$X, MeV
^{40}Ca	$2s_{1/2}2s_{1/2}$	0^+	0	0	–11.8375	–3.8560
	$1f_{7/2}1f_{7/2}$	0^+	7.4532	5.2116	–1.1285	–3.5642
	$1f_{7/2}1f_{7/2}$	2^+	7.4856	5.2488	–1.1285	–3.5642
	$1f_{7/2}1f_{7/2}$	4^+	7.5267	5.2788	–1.6424	–3.5642
	$1f_{7/2}1f_{7/2}$	6^+	7.6029	—	–1.6423	–3.5642
	$2p_{3/2}2p_{3/2}$	0^+	9.6124	7.3007	–2.2736	–2.4397
	$2p_{3/2}2p_{3/2}$	2^+	9.8108	7.4664	–2.2648	–2.4397
^{64}Zn	$1f_{5/2}1f_{5/2}$	0^+	0	0	–10.5294	–5.1589
	$1f_{5/2}1f_{5/2}$	2^+	0.0586	0.9915	–10.5294	–5.1589
	$1f_{5/2}1f_{5/2}$	4^+	0.1093	2.3067	–10.5294	–5.1589
	$2p_{1/2}2p_{1/2}$	0^+	0.1727	1.9103	–11.1537	–5.4513
	$1g_{9/2}1g_{9/2}$	0^+	4.9490	3.2400	–1.5556	–3.9131
	$1g_{9/2}1g_{9/2}$	2^+	5.0637	3.2972	–1.5555	–3.9131
	$1g_{9/2}1g_{9/2}$	4^+	5.1504	3.3069	–1.5553	–3.9131
	$1g_{9/2}1g_{9/2}$	6^+	5.3822	3.4650	–1.5550	–3.9131
	$1g_{9/2}1g_{9/2}$	8^+	5.4606	—	–1.5549	–3.9131
^{74}Se	$2p_{1/2}2p_{1/2}$	0^+	0	0	–17.3143	–8.3024
	$1g_{9/2}1g_{9/2}$	0^+	4.4014	1.6575	–8.3838	–8.3930
	$1g_{9/2}1g_{9/2}$	2^+	4.4239	1.8387	–8.3838	–8.3930
	$1g_{9/2}1g_{9/2}$	4^+	4.4272	2.1080	–8.3838	–8.3930
	$1g_{9/2}1g_{9/2}$	6^+	4.4498	2.2315	–8.3838	–8.3930
	$1g_{9/2}1g_{9/2}$	8^+	4.4594	3.1984	–8.3838	–8.3930
^{200}Hg	$2f_{5/2}2f_{5/2}$	0^+	0	0	–13.0605	–6.4906
	$2f_{5/2}2f_{5/2}$	2^+	0.0282	0.3679	–13.0552	–6.4906
	$2f_{5/2}2f_{5/2}$	4^+	0.0428	0.9472	–13.0525	–6.4906
	$3p_{1/2}3p_{1/2}$	0^+	1.4823	1.0293	–15.1259	–6.6490



Behavior of the potential curves (terms) $U_\mu(R)/R^2$ and the energy levels of nucleus ^{40}Ca (*a*), ^{64}Zn (*b*), ^{74}Se (*c*), ^{200}Hg (*d*) under assumption of the spherically symmetric Woods—Saxon potential

nuclei ^{40}Ca, ^{64}Zn, ^{74}Se, and ^{200}Hg, respectively: $E_{2p}(^{40}\mathrm{Ca}) = 14.708$ MeV, $E_{2p}(^{64}\mathrm{Zn}) = 13.83$ MeV, $E_{2n}(^{74}\mathrm{Se}) = 20.463$ MeV, and $E_{2n}(^{200}\mathrm{Hg}) = 14.678$ MeV [18].

As for deformed nuclei, we have shown in Section 2 that the problem of determination of the energy spectrum of stationary states of an axisymmetric deformed nucleus is divided into three stages: 1) numerical determination of the coefficients $A_{\lambda_i m_i}(r_i, \alpha, \beta)$ and $B_{\lambda_i m_i}(r_i, \alpha, \beta)$ of expansions (33) and (34) in series in spherical functions $Y_{\lambda_i 0}(\theta_i)$ for the purely nuclear potential $\tilde{U}_i(\vec{r}_i, \beta)$ and its derivatives $\frac{\partial \tilde{U}_i(r_i,\beta)}{\partial r_i}$, 2) following formulas (A.2), (A.5), (A.9), and (A.13), the determination of matrix elements of the interaction operators $\tilde{U}(\vec{r}_1, \vec{\sigma}_1, \vec{r}_2, \vec{\sigma}_2, \beta)$ and $\tilde{V}_{\mathrm{so}}(\vec{r}_1, \vec{\sigma}_1, \vec{r}_2, \vec{\sigma}_2, \beta)$ in analytic form with the basis functions $\Phi_{nJK}(R, \Omega)$, 3) numerical calculations of the corresponding matrix elements according to (A.1) and (A.3) and determination of the energy spectrum of a deformed nucleus according to (35).

In future, for the numerical calculation of the energy spectrum of the stationary states of deformed nuclei, it is necessary to develop a package of applied computer programs, which would give us a possibility to use more realistic interaction potentials. With regard for the deformation of the nucleus core field and the spin-orbit interaction (19), we hope to improve the accuracy of calculations of the energy spectra of deformed nuclei.

Numerical calculations of the energy spectra of deformed nuclei in the framework of the adiabatic three-particle model of nuclei are actual for further investigations. Thus, the adiabatic three-particle model of nuclei developed by us allows one to carry out, in the potential approach, the adequate theoretical description

of pairing effects of nucleons and their angular and radial correlations which result, in particular, in the creation of superfluid nuclear states.

APPENDIX

Non-diagonal matrix elements of the purely nuclear interaction $\tilde{U} = \sum\limits_{i=1}^{2} \tilde{U}_i(\vec{r}_i, \beta)$ which appear in (24) are defined in terms of hyperspherical coordinates as

$$\left\langle \Psi_{n'J'K'} \left| \tilde{U} \right| \Psi_{nJK} \right\rangle = \int\limits_0^\infty dR \int\limits_0^{\pi/2} d\alpha \times$$

$$\times F^*_{n'J'K'}(R) \left\langle \Phi_{n'J'K'} \left| \tilde{U} \right| \Phi_{nJK} \right\rangle F_{nJK}(R), \qquad (A.1)$$

where the matrix elements of the operator $\tilde{U}$ in terms of basis functions $\Phi_{nJK}(R,\Omega)$ in analytic form are

$$\left\langle \Phi_{n'J'K'} \left| \tilde{U} \right| \Phi_{nJK} \right\rangle = \sum_{j'_1 l'_1 m'_1} \sum_{j_1 l_1 m_1} \sum_{j'_2 l'_2 m'_2} \sum_{j_2 l_2 m_2} C^{*J'K'}_{j'_1 m'_1 j'_2 m'_2} \times$$

$$\times C^{JK}_{j_1 m_1 j_2 m_2} \Phi^{*n'J'K'}_{j'_1 l'_1 j'_2 l'_2}(R,\alpha) \Phi^{nJK}_{j_1 l_1 j_2 l_2}(R,\alpha) \times$$

$$\times \left(\sum_{\lambda_1} (-1)^{(l'_1 - j'_1 - m'_1 - m_1 - \lambda_1 - 1/2)} \left\{ \begin{array}{ccc} l'_1 & l_1 & \lambda_1 \\ j_1 & 1/2 & j'_1 \end{array} \right\} \times \right.$$

$$\times C^{l'_1 0}_{l_1 0 \lambda_1 0} C^{j_1(-m_1)}_{\lambda_1 (m'_1 - m_1) j'_1 (-m'_1)} A_{\lambda_1 0}(r_1, \alpha, \beta) \times$$

$$\times \sqrt{\frac{(2\lambda_1+1)(2l_1+1)(2l'_1+1)}{4\pi}} \delta_{j'_2 j_2} \delta_{l'_2 l_2} \delta_{m'_2 m_2} +$$

$$+ \sum_{\lambda_2} (-1)^{(l'_2 - j'_2 - m'_2 - m_2 - \lambda_2 - 1/2)} \left\{ \begin{array}{ccc} l'_2 & l_2 & \lambda_2 \\ j_2 & 1/2 & j'_2 \end{array} \right\} \times$$

$$\times C^{l'_2 0}_{l_2 0 \lambda_2 0} \cdot C^{j_2(-m_2)}_{\lambda_2 (m'_2 - m_2) j'_2 (-m'_2)} A_{\lambda_2 0}(r_2, \alpha, \beta) \times$$

$$\left. \times \sqrt{\frac{(2\lambda_2+1)(2l_2+1)(2l'_2+1)}{4\pi}} \delta_{j'_1 j_1} \delta_{l'_1 l_1} \delta_{m'_1 m_1} \right). \qquad (A.2)$$

Here, $A_{\lambda_1 0}(r_1,\alpha,\beta)$, $A_{\lambda_2 0}(r_2,\alpha,\beta)$ are the coefficients of expansion of potential (33) for the first and second valent nucleons, respectively.

Non-diagonal matrix elements of the spin-orbit interaction $\tilde{V}_{\rm so} = \hat{W}_1 + \hat{W}_2 + \hat{W}_3 = \sum\limits_{q=1}^{3} \hat{W}_q$ can be represented as

$$\left\langle \Psi_{n'J'K'} \left| \tilde{V}_{\rm so} \right| \Psi_{nJK} \right\rangle = \int\limits_0^\infty dR \int\limits_0^{\pi/2} d\alpha F^*_{n'J'K'}(R) \times$$

$$\times \left\langle \Phi_{n'J'K'} \left| \tilde{V}_{\rm so} \right| \Phi_{nJK} \right\rangle F_{nJK}(R) = \int\limits_0^\infty dR \int\limits_0^{\pi/2} d\alpha F^*_{n'J'K'}(R) \times$$

$$\times \left\langle \Phi_{n'J'K'} \left| \sum_{q=1}^{3} \hat{W}_q \right| \Phi_{nJK} \right\rangle F_{nJK}(R). \qquad (A.3)$$

Matrix elements of the operator of spin-orbit interaction $\tilde{V}_{so}$ in terms of basis functions $\Phi_{nJK}(R,\alpha)$ are given by

$$\left\langle \Phi_{n'J'K'} \left| \sum_{q=1}^{3} \hat{W}_q \right| \Phi_{nJK} \right\rangle =$$

$$\left\langle \Phi_{n'J'K'} \left| \sum_{q=1}^{3} \left(\hat{W}^{(1)}_q + \hat{W}^{(2)}_q \right) \right| \Phi_{nJK} \right\rangle =$$

$$= \sum_{j'_1 l'_1 m'_1} \sum_{j_1 l_1 m_1} \sum_{j'_2 l'_2 m'_2} \sum_{j_2 l_2 m_2} C^{*J'K'}_{j'_1 m'_1 j'_2 m'_2} C^{JK}_{j_1 m_1 j_2 m_2} \times$$

$$\times \Phi^{*n'J'K'}_{j'_1 l'_1 j'_2 l'_2}(R,\alpha) \Phi^{nJK}_{j_1 l_1 j_2 l_2}(R,\alpha) \times$$

$$\times \left[\left(\Phi^{m'_1}_{j'_1 l'_1}, \hat{W}^{(1)}_q \Phi^{m_1}_{j_1 l_1} \right) \delta_{j'_2 j_2} \delta_{l'_2 l_2} \delta_{m'_2 m_2} + \right.$$

$$\left. + \left(\Phi^{m'_2}_{j'_2 l'_2}, \hat{W}^{(2)}_q \Phi^{m_2}_{j_2 l_2} \right) \delta_{j'_1 j_1} \delta_{l'_1 l_1} \delta_{m'_1 m_1} \right], \qquad (A.4)$$

where matrix elements of the operator $\hat{W}_1$ in terms of spin-angular functions $\Phi^{m_i}_{j_i l_i}(\theta_i, \varphi_i)$ in (A.4) have the following analytic form:

$$\left(\Phi^{m'_i}_{j'_i l'_i}, W^{(i)}_1 \Phi^{m_i}_{j_i l_i} \right) = -\frac{\chi}{\sqrt{4\pi}} \frac{1}{r_i} \left[\sum_j \sum_{j'} \sum_{\lambda_i} \sum_l \left(-1\right)^{(2j'+j'_i+2l_i+l'_i-m'_i-m_i+\lambda_i+1/2)} \left\{ \begin{array}{ccc} j_i & j' & 1 \\ 1/2 & 1/2 & l_i \end{array} \right\} \left\{ \begin{array}{ccc} j' & j & 1 \\ l & l_i & 1/2 \end{array} \right\} \times \right.$$

$$\left. \times B_{\lambda_i 0}(r_i,\alpha,\beta) D_l(l_i) \frac{1}{\sqrt{(2\lambda_i+1)(2j'_i+1)}} \sqrt{\frac{3}{2}(j_i+1)(2j'+1)} \right] \times$$

$$\times \left\{ \left(P_1 \frac{\sqrt{(j+m_i+1)(j-m_i+1)}}{j+1} + P_2 \frac{\sqrt{(j-m_i)(j-m_i+1)}}{\sqrt{2}(j+1)} - P_3 \frac{\sqrt{(j+m_i)(j+m_i+1)}}{\sqrt{2}(j+1)} \right) (-1)^{2l} \times \right.$$

$$\times \sqrt{2l+3} C^{l'_i 0}_{(l+1)0\lambda_i 0} C^{(j+1)(-m_i)}_{\lambda_i (m'_i - m_i) j'_i (-m'_i)} \left\{ \begin{array}{ccc} l+1 & \lambda_i & l'_i \\ j'_i & 1/2 & j+1 \end{array} \right\} - \left(P_1 \frac{m_i}{j(j+1)} + P_2 \frac{\sqrt{(j+m_i+1)(j-m_i)}}{\sqrt{2}j(j+1)} + \right.$$

$$+P_3\frac{\sqrt{(j+m_i)(j-m_i+1)}}{\sqrt{2}j(j+1)}\Big)(-1)^{2l'}\times\sqrt{2l'+1}C^{l'_i0}_{l'0\lambda_i0}C^{j(-m_i)}_{\lambda_i(m'_i-m_i)j'_i(-m'_i)}\left\{\begin{array}{ccc} l' & \lambda_i & l'_i \\ 1/2 & 1/2 & j \end{array}\right\}+\Big(P_1\frac{\sqrt{(j-m_i)(j+m_i)}}{j}-$$

$$-P_2\frac{\sqrt{(j+m_i)(j+m_i+1)}}{\sqrt{2}j}+P_3\frac{\sqrt{(j-m_i)(j-m_i+1)}}{\sqrt{2}j}\Big)(-1)^{2l}\sqrt{2l-1}C^{l'_i0}_{(l-1)0\lambda_i0}C^{(j-1)(-m_i)}_{\lambda_i(m'_i-m_i)j'_i(-m'_i)}\left\{\begin{array}{ccc} l-1 & \lambda_i & l'_i \\ j'_i & 1/2 & j-1 \end{array}\right\}\Bigg\}, \tag{A.5}$$

where

$$P_1=C^{j'(m_i-1)}_{j_im_i1(-1)}C^{jm_i}_{j'(m_i-1)1(1)}-C^{j'(m_i+1)}_{j'_im_i1(1)}C^{jm_i}_{j'(m_i+1)1(-1)}, \tag{A.6}$$

$$P_2=C^{j'(m_i+1)}_{j_im_i1(1)}\cdot C^{j(m_i+1)}_{j'(m_i+1)1(0)}-C^{j'm_i}_{j_im_i1(0)}\cdot C^{j(m_i+1)}_{j'm_i1(1)}, \tag{A.7}$$

$$P_3=C^{j'(m_i-1)}_{j_im_i1(-1)}\cdot C^{j(m_i-1)}_{j'(m_i-1)1(0)}-C^{j'm_i}_{j_im_i1(0)}\cdot C^{j(m_i-1)}_{j'm_i1(-1)}, \tag{A.8}$$

Similarly, non-diagonal matrix elements of the operator $\hat{W}_3$ in terms of spin-angular functions are as follows:

$$\left(\Phi^{m'_i}_{j'_il'_i},W^{(i)}_2\Phi^{m_i}_{j_il_i}\right)=-\tfrac{\chi}{\sqrt{4\pi}}\tfrac{1}{r_i^2}\left[\begin{array}{l}\sum\limits_j\sum\limits_{j'}\sum\limits_{\lambda_i}\sum\limits_l(-1)^{(2j'+j'_i+l_i+l'_i-m'_i-m_i+\lambda_i+1)}\left\{\begin{array}{ccc} j_i & j' & 1 \\ 1/2 & 1/2 & l_i \end{array}\right\}A_{\lambda_i0}(r_i,\alpha,\beta)\times \\ \times\sqrt{\frac{3}{2}(2j_i+1)}\sqrt{\lambda_i(\lambda_i+1)(2\lambda_i+1)(2j'_i+1)}\end{array}\right]\times$$

$$\times\Bigg((-1)^{l_i}\,M_1D_l(l_i+1)C^{j(m_i+1)}_{(j'+1)m_i1(1)}\left\{\begin{array}{ccc} j' & j & 1 \\ l & l_i+1 & 1/2 \end{array}\right\}+(-1)^{l'}\,M_2D_l(l')C^{j(m_i+1)}_{j'm_i1(1)}\left\{\begin{array}{ccc} j' & j & 1 \\ l & l' & 1/2 \end{array}\right\}+$$

$$+\,(-1)^{l_i}\,M_3D_l(l_i-1)C^{j(m_i+1)}_{(j'-1)m_i1(1)}\left\{\begin{array}{ccc} j'-1 & j & 1 \\ l & l_i-1 & 1/2 \end{array}\right\}\Bigg)(-1)^{2l-1/2}\times\sqrt{2l+1}C^{j(-m_i-1)}_{\lambda_i(m'_i-m_i-1)j'_i(-m'_i)}C^{l'_i0}_{l0\lambda_i0}\times$$

$$\times\left\{\begin{array}{ccc} l & \lambda_i & l'_i \\ j'_i & 1/2 & j \end{array}\right\}+\Bigg((-1)^{l_i+1}\,M_1D_l(l_i+1)C^{j(m_i-1)}_{(j'+1)m_i1(-1)}\left\{\begin{array}{ccc} j'+1 & j & 1 \\ l & l_i+1 & 1/2 \end{array}\right\}+(-1)^{l'+1}\,M_2D_l(l')\times$$

$$\times C^{j(m_i-1)}_{j'm_i1(-1)}\left\{\begin{array}{ccc} j' & j & 1 \\ l & l' & 1/2 \end{array}\right\}+(-1)^{l_i-1}\,M_3D_l(l_i-1)C^{j(m_i-1)}_{(j'-1)m_i1(-1)}\left\{\begin{array}{ccc} j'-1 & j & 1 \\ l & l_i-1 & 1/2 \end{array}\right\}\Bigg)\times$$

$$\times(-1)^{2l+3/2}\sqrt{2l+1}C^{l'_i0}_{l0\lambda_i0}C^{j(-m_i+1)}_{\lambda_i(m'_i-m_i+1)j'_i(-m'_i)}\left\{\begin{array}{ccc} l & \lambda_i & l'_i \\ j'_i & 1/2 & j \end{array}\right\}\Bigg\}, \tag{A.9}$$

where

$$M_1=\frac{\sqrt{(j'-m_i)(j'+m_i+1)}}{2(j'+1)}C^{j'(m_i+1)}_{j_im_i1(1)}-$$

$$-\frac{\sqrt{(j'-m_i+1)(j'+m_i+1)}}{\sqrt{2}(j'+1)}C^{j'm_i}_{j_im_i1(0)}-$$

$$-\frac{\sqrt{(j'+m_i)(j'+m_i+1)}}{2(j'+1)}C^{j'(m_i-1)}_{j_im_i1(-1)}, \tag{A.10}$$

$$M_2=-\frac{\sqrt{(j'+m_i)(j'+m_i+1)}}{2j'(j'+1)}C^{j'(m_i+1)}_{j_im_i1(1)}+$$

$$+\frac{m_i}{2\sqrt{2}j'(j'+1)}C^{j'm_i}_{j_im_i1(0)}+$$

$$+\frac{\sqrt{(j'+m_i)(j'-m_i+1)}}{2j'(j'+1)}C^{j'(m_i-1)}_{j_im_i1(-1)}, \tag{A.11}$$

$$M_3=\frac{\sqrt{(j'+m_i)(j'+m_i+1)}}{2j'}C^{j'(m_i+1)}_{j_im_i1(1)}+$$

$$+\frac{\sqrt{(j'-m_i)(j'+m_i)}}{\sqrt{2}j'}C^{j'm_i}_{j_im_i1(0)}+$$

$$+\frac{\sqrt{(j'-m_i)(j'-m_i+1)}}{2j'}C^{j'(m_i-1)}_{j_im_i1(-1)}. \tag{A.12}$$

At last, non-diagonal matrix elements of the operator $\hat{W}_3$ in terms of spin-angular functions are given by

$$\left(\Phi^{m_i'}_{j_i'l_i'}, W_3^{(i)}\Phi^{m_i}_{j_il_i}\right) = \frac{\chi}{\sqrt{4\pi}}\frac{1}{r_i^2}\left[\begin{array}{l}\sum\limits_j\sum\limits_{j'}\sum\limits_{\lambda_i}\sum\limits_l (-1)^{(j'+j_i'+j_i+l_i'+l_i-m_i'-m_i+\lambda_i)}\left\{\begin{array}{ccc} j_i & j & 1\\ l & l_i & 1/2\end{array}\right\}A_{\lambda_i 0}(r_i,\alpha,\beta)D_l(l_i)\times\\ \times\sqrt{(2j_i'+1)(2j_i+1)}\sqrt{\lambda_i(\lambda_i+1)(2\lambda_i+1)(2j_i'+1)}\end{array}\right]\times$$

$$\times\left\{\left(C^{j'(m_i+1)}_{(j+1)m_i1(1)}C^{j'(-m_i-1)}_{\lambda_i(m_i'-m_i-1)j_i'(-m_i')} + C^{j'(m_i-1)}_{(j+1)m_i1(-1)}C^{j'(-m_i+1)}_{\lambda_i(m_i'-m_i+1)j_i'(-m_i')}\right)(-1)^{3l+1/2}\sqrt{2l+3}\sqrt{\tfrac{3}{2}(2j+3)}I_1\times\right.$$

$$\times C^{l_i'0}_{(l+1)0\lambda_i0}\left\{\begin{array}{ccc} j+1 & j' & 1\\ 1/2 & 1/2 & l+1\end{array}\right\}\left\{\begin{array}{ccc} l+1 & \lambda_i & l_i'\\ j_i' & 1/2 & j'\end{array}\right\} + \left((-1)^{2l'-1/2}C^{j'(m_i+1)}_{jm_i1(1)}C^{j'(-m_i-1)}_{\lambda_i(m_i'-m_i-1)j_i'(-m_i')} + \right. \tag{A.13}$$

$$\left.+(-1)^{3l'+1/2}C^{j'(m_i-1)}_{jm_i1(-1)}C^{j'(-m_i+1)}_{\lambda_i(m_i'-m_i+1)j_i'(-m_i')}\right)I_2\sqrt{2l'+1}\sqrt{\tfrac{3}{2}(2j+1)}C^{l_i'0}_{l'0\lambda_i0}\left\{\begin{array}{ccc} j & j' & 1\\ 1/2 & 1/2 & l'\end{array}\right\}\left\{\begin{array}{ccc} l' & \lambda_i & l_i'\\ j_i' & 1/2 & j'\end{array}\right\}+$$

$$+\left(C^{j'(m_i+1)}_{(j-1)m_i1(1)}C^{j'(-m_i-1)}_{\lambda_i(m_i'-m_i-1)j_i'(-m_i')} + C^{j'(m_i-1)}_{(j-1)m_i1(-1)}C^{j'(-m_i+1)}_{\lambda_i(m_i'-m_i+1)j_i'(-m_i')}\right)(-1)^{3l-1/2}\sqrt{2l-1}\sqrt{\tfrac{3}{2}(2j-1)}I_3\times$$

$$\left.\times C^{l_i'0}_{(l-1)0\lambda_i0}\left\{\begin{array}{ccc} j-1 & j' & 1\\ 1/2 & 1/2 & l-1\end{array}\right\}\left\{\begin{array}{ccc} l-1 & \lambda_i & l_i'\\ j_i' & 1/2 & j'\end{array}\right\}\right\},$$

where

$$I_1 = -\frac{\sqrt{(j-m_i)(j-m_i+1)}}{2(j+1)}C^{j(m_i+1)}_{j_im_i1(1)}+$$

$$+\frac{\sqrt{(j-m_i+1)(j+m_i+1)}}{\sqrt{2}(j+1)}C^{lm_i}_{j_im_i1(0)}-$$

$$-\frac{\sqrt{(j+m_i)(j+m_i+1)}}{2(j+1)}C^{l(m_i-1)}_{j_im_i1(-1)}, \tag{A.14}$$

$$I_2 = -\frac{\sqrt{(j-m_i)(j+m_i+1)}}{2j(j+1)}C^{j(m_i+1)}_{j_im_i1(1)}-$$

$$-\frac{m_i}{\sqrt{2}j(j+1)}C^{lm_i}_{j_im_i1(0)}+$$

$$+\frac{\sqrt{(j-m_i)(j-m_i+1)}}{2j(j+1)}C^{l(m_i-1)}_{j_im_i1(-1)}, \tag{A.15}$$

$$I_3 = \frac{\sqrt{(j+m_i)(j+m_i+1)}}{2j}C^{j(m_i+1)}_{j_im_i1(1)}+$$

$$+\frac{\sqrt{(j-m_i)(j+m_i)}}{\sqrt{2}j}C^{lm_i}_{j_im_i1(0)}+$$

$$+\frac{\sqrt{(j-m_i)(j-m_i+1)}}{2j}C^{l(m_i-1)}_{j_im_i1(-1)}. \tag{A.16}$$

In formulas (A.5), (A.9), and (A.13), the function

$$D_l(l_i) = \left\{\begin{array}{ll} l_i\sqrt{l_i+1} & \text{if } l = l_i+1,\\ 0 & \text{if } l \neq l_i+1,\\ (l_i+1)\sqrt{l_i} & \text{if } l = l_i-1,\end{array}\right. \tag{A.17}$$

In (A.5)—(A.17), $i = 1, 2$ and

$$l' = 2j - l = \left\{\begin{array}{l} j-1/2 \text{ if } l = j+1/2,\\ j+1/2 \text{ if } l = j-1/2.\end{array}\right.$$

1. *Hartree D.R.* The Calculation of Atomic Structures. — New York: Wiley, 1957.
2. *Barts B.I., Bolotin Yu. L., Inopin E.V., Gonchar V.Yu.* Hartree — Fock Method in Nuclear Theory. — Kyiv: Naukova dumka, 1982 (in Russian).
3. *Kapustey M.M., Pojda V.Yu., Khimich I.V.* // Ukr.Fiz.Zh.— 1995.— **40**, N 11.— P.1166—1170.
4. *Kapustey M.M., Pojda V.Yu., Khimich I.V.* // Dopovidi NANU. Ser. Mat.— 1995.— N 10.— P.71—74.
5. *Kapustey M.M., Pojda V.Yu., Khimich I.V.* // Ukr.Fiz.Zh.— 1999.— **44**, N 11.— P.1330—1336; *Kapustey M.M., Plekan R.M., Pojda V.Yu., Khimich I.V.* // Ibid.— 2001.— **46**, N 5—6.— P.524—528.
6. *Bogolyubov N.N.* // Dokl. AN SSSR.— 1958.— **119**, N 1.— P.52—55; *Solov'ev V.G.* // Zh. Eksp. Teor. Fiz.— 1959.— **36**, Iss. 6.— P.1869—1874.
7. *Belyaev S.T.* // Dan. Math. Fys. Medd.— 1959.— **31**, N11.— P.1—55.
8. *Solov'ev V.G.* The Theory of Atomic Nuclei: Nuclear Models.—Moscow: Energoizdat, 1981 (in Russian).
9. *Mikhailov V.M., Kraft O.E.* Nuclear Physics. — Leningrad: Leningrad University, 1988 (in Russian).
10. *Plekan R.M, Kapustey M.M., Khimich I.V., .*// Nauk. Visn. Uzhgorod Univ. Ser. Fiz.— 1999.— N 4.— P.50—54.
11. *Kapustey M.M., Plekan R.M., Pojda V.Yu., Khimich I.V.* // Ibid.— 2000.— N 7.— P.155—160.
12. *Kapustey M.M., Khimich I.V., Plekan R.M., Pojda V.Yu.* // Ibid.— 2000.— N 8.— P.98—103.

13. *Kapustey M.M., Plekan R.M., Pojda V.Yu., Khimich I.V.* // Ibid. – 2001, N 9.— P.155—161.

14. *Nilsson S.G.* // Mat. Fys. Medd. Dan. Vid. Selsk.— 1955.— **29**, N 16.— P.1—68.

15. *Nemirovskii P.E., Chepurnov V.A.* // Yad. Fiz.— 1966.— **3**.— P.998—1003.

16. *Gareev A.F., Ivanova S.P., Solov'ev V.G., Fedotov S.I.* // Fiz. Elem. Chast. At. Yadra.— 1973.— **4**.— P.357—455.

17. *Khimich I.V.*// Nauk. Visn. Uzhgorod Univ. Ser. Fiz.— 1998.— N 3.— P.53—56.

18. *Nemets O.F, Gofman Yu.V.* Reference Book on Nuclear Physics.— Kyiv: Naukova Dumka, 1971 (in Russian).

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ДОСЛІДЖЕННЯ КОРЕЛЯЦІЙ НУКЛОНІВ ПАРНО-ПАРНИХ ЯДЕР В РАМКАХ АДІАБАТИЧНОЇ ТРИЧАСТИНКОВОЇ МОДЕЛІ ЯДРА

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Р е з ю м е

У рамках адіабатичної тричастинкової моделі ядра проведено теоретичний опис стаціонарних станів парно-парних атомних ядер, середнє самоузгоджене поле яких моделюється потенціалом Вудса—Саксона. Опис проведено в термінах колективних змінних, а саме: гіперрадіуса R, гіперкута α і звичайних сферичних кутів (θ_i,φ_i), $i = 1,2$. Проілюстровано ефективність адіабатичного підходу на прикладі чисельного розрахунку енергетичного спектра низьколежачих збуджених станів парно-парних атомних ядер ^{40}Ca, ^{64}Zn, ^{74}Se, ^{200}Hg, в яких на зовнішній оболонці міститься два валентні нуклони.