THE INVERSE PROBLEM OF DISTANT NEUTRINO DIAGNOSTICS OF INTRAREACTOR PROCESSES

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Using the known experimental data, we consider the inverse problem of neutrino diagnostics of a reactor core. On the one hand, its solution allows us to restore distantly (with known accuracy) the current values of the nuclear density of each component forming the isotope composition of a nuclear fuel and, on the other hand, gives a real possibility of creating a neutrino technology for diagnosing the temporal evolution of the isotope composition of nuclear fuel and the reactor power in the on-line mode.

It is known that a nuclear reactor represents an extremely powerful source of electron antineutrinos with a spectrum formed as a result of the beta decay of fission fragments of four nuclear fuel isotopes: ^{235}U, ^{239}Pu, ^{238}U, and ^{241}Pu. They are usually registered using the reaction of elastic capture of antineutrinos with the target protons or, in other words, the reverse beta decay reaction

$$\tilde{\nu}_e + \mathrm{p} \to \mathrm{e}^+ + \mathrm{n}. \tag{1}$$

The intensity of neutrino events n_ν registered with a detector in reaction (1) is related to the heat rating $\langle W_{\mathrm{NPP}}\rangle$ of a reactor as

$$n_\nu = \frac{\langle W_{\mathrm{NPP}}\rangle}{\langle E_f\rangle}\frac{\gamma\varepsilon_0}{4\pi\langle R\rangle^2}N_p\Sigma_\nu,\ \mathrm{s}^{-1}, \tag{2}$$

where

$$\Sigma_\nu = M_\nu\left\langle\sigma_{\nu p}\right\rangle, \quad M_\nu = \int\limits_{E_{\mathrm{thr}}}^{E_{\max}}\rho(E_\nu)dE_\nu,$$

$$\langle\sigma_{\nu p}\rangle = \int\limits_{E_{\mathrm{thr}}}^{E_{\max}}\sigma_{\nu p}(E_\nu)\rho(E_\nu)dE_\nu \Big/ \int\limits_{E_{\mathrm{thr}}}^{E_{\max}}\rho(E_\nu)dE_\nu. \tag{3}$$

Here, $\langle E_\nu\rangle = \sum(\alpha_i E_{\nu i})$ denotes the average energy absorbed in a reactor per one fission act at a given fuel composition, where α_i is the contribution of isotope i ($i = 5, 9, 8, 1$) to the total fission cross section [1—9]; $(4\pi\langle R\rangle^2)^{-1}$ is the effective space angle allowing for the actual distribution of energy release in the reactor core; N_p and $\gamma\varepsilon_0$ — characteristics of the detector (the number of hydrogen atoms in the target and the effectiveness of registration allowing for the portion γ of registered neutrons corresponding to reaction (1)); Σ_ν and $\langle\sigma_{\nu p}\rangle$ are the cross sections of the neutrino reaction measured in cm^2 per one fission act and cm^2 per ν-particle, respectively; for the fuel composition, $\Sigma_\nu = \sum(\alpha_i\sum_{\nu i})$; M_ν — the number of electron antineutrinos per one fission act; $\rho(E_\nu) = \sum(\alpha_i\rho_{\nu i})$ — the energy spectrum of antineutrinos (MeV^{-1}·fission event^{-1}) radiated by the mixture of products of fission of all actinoids forming the nuclear fuel; $\sigma_{\nu p}(E_\nu)$ — the cross section of interaction between monoenergetic antineutrinos with the energy E_ν with regard for recoil, weak magnetism, and radiation corrections [10—13].

It is easy to demonstrate [14] that (2) immediately gives the basic balance equation of the spectrometry of reactor antineutrinos which characterizes the contribution α_i of each actinoid to the energy spectrum $\eta(E_\nu)$ measured experimentally in the flux $\tilde{\nu}_e$ (at a given geometry and characteristics of the detector):

$$\eta(E_\nu) = \frac{\gamma\varepsilon_0 N_p}{4\pi\langle R\rangle^2}\sum_i\rho_i\sigma_{\nu p}(E_\nu)\lambda_i\Delta t, \quad \mathrm{MeV}^{-1}, \tag{4}$$

where

$$n_\nu = \frac{1}{\Delta t}\int\limits_{E_{\mathrm{thr}}}^{E}\eta\left(E_\nu\right)dE_\nu,\ \lambda_i = \alpha_i\lambda,\ \lambda = \langle W_{\mathrm{NPP}}\rangle/\langle E_f\rangle, \tag{5}$$

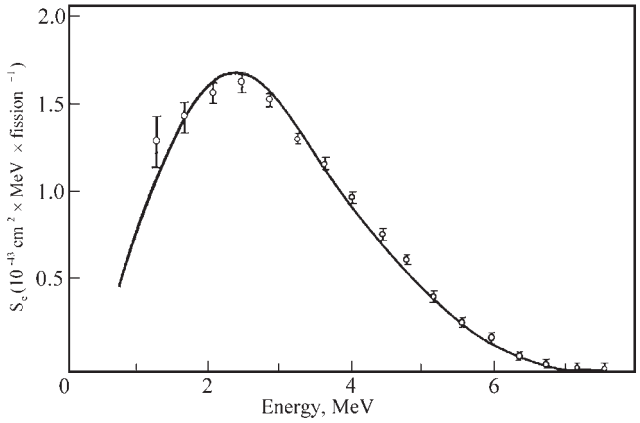

Fig. 1. Spectrum of positrons of reaction (1) at a distance of 18 m (positron energy is given in units of 10^{-43} cm^2·MeV^{-1}·fission^{-1}) [15, experiment 3c]. Full curve represents the calculated (expected) spectrum [15]. The histogram step — 300 keV

Δt is the duration of measurement; λ and λ_i are the average and partial ($i = 5, 9, 8, 1$) fission rates of nuclear fuel.

It is worth noting that theoretical calculations developed for today (together with the contemporary level of experimental ideology and technique of antineutrinos measurements) not only allow one to pose a number of fundamental problems of the physics of reactor antineutrinos but also give a real possibility to effectively solve the basic problem of the neutrino spectrometry of intrareactor processes: the determination of the nuclear density of each component forming the isotope composition of a nuclear fuel and their dynamics directly in the course of the reactor operation. The development of an effective technique allowing one to solve Eq. (4) with respect to λ_i evidently represents a crucial point for creating a neutrino technology for diagnosing the temporal evolution of the isotope composition of a nuclear fuel in the on-line mode. Along with searching for optimal numerical methods, such a technique should also include the physical substantiation of procedures determining the basic energy functions $\eta(E_\nu)$, ρ_ν, and $\sigma_{\nu p}(E_\nu)$ contained in this equation. That was the purpose of this paper.

1. Problem Statement

Though there exists a great number of publications dealing with the problems of registration of reactor antineutrinos, we managed to find only three experiments completely reflecting the essence of the basic equation (4) for the spectrometry of reactor

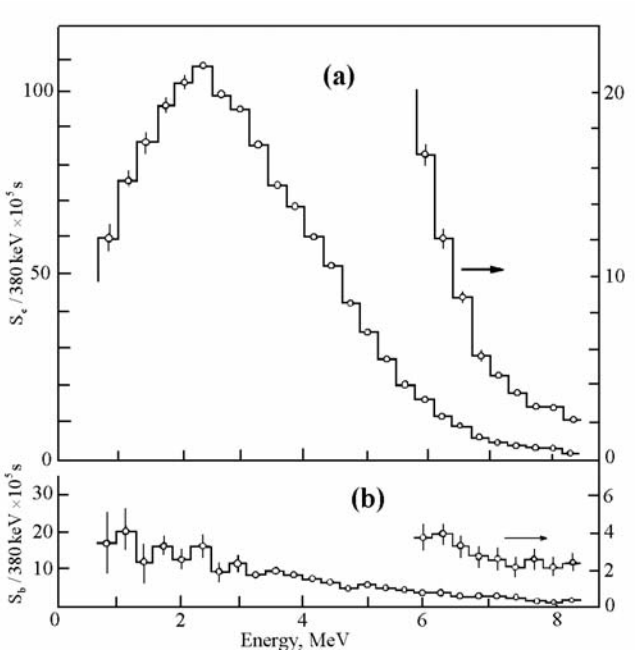

Fig. 2. Experimental spectra [4]: *a* — the sum of the effect and correlated noise, *b* — correlated noise

antineutrinos [4, 15, 16]. Figs. 1—3 represent the energy spectra of positrons $S_{e^+}(E_{e^+})$ of reaction (1) obtained in these papers. The main parameters characterizing the geometries of the experiments, detector, and reactor are given in Table 1.

Now we should pay attention to the procedure of obtaining the energy spectrum $\eta(E_\nu)$ or, in other words, the spectrum "with respect to the registration place" of $\tilde{\nu}_e$. The spectrum of antineutrinos can be directly determined using the kinetic energy of a positron T_{e^+}, as it differs from the energy of an antineutrino absorbed by a proton only by the threshold energy of 1.804 MeV. But as was demonstrated in [4, 15—16], the partial absorption of annihilation quanta in the volume of a spectrometer results in the fact that the observed energy of a positron is always larger than the actual kinetic energy by the value ΔT_{e^+} ($\sim$0.6 MeV) on the average. That's why the shift procedure for the spectrum of positrons $S_{e^+}(E_{e^+})$ must be refined, and this fact should be taken into account when determining the energy spectrum $\eta(E_\nu)$:

$$E_\nu = (T_{e^+} - \Delta T_{e^+}) + 1.804 + O(E_\nu/M_n), \text{ MeV}. \qquad (6)$$

Numerical values of the additional shift energy ΔT_{e^+} obtained in experiments [4, 15, 16] are given in Table 1. Numerical values of the energy spectra $\eta(E_\nu)$

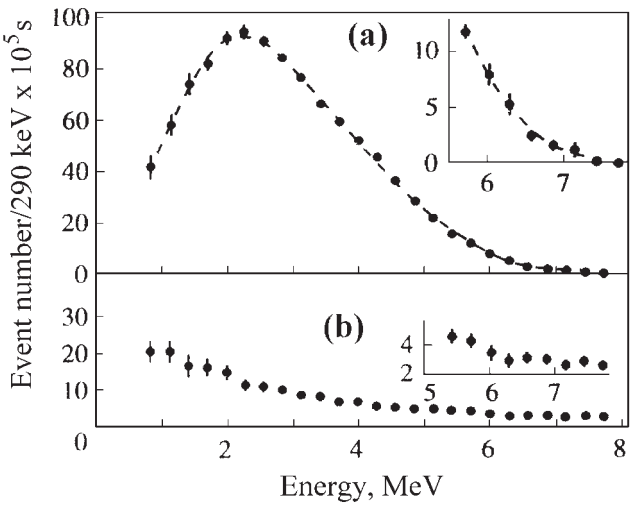

Fig. 3. Measured spectra of positrons of reaction (1) (*a*) and events of correlated noise (*b*) [16]. Dots — experiment, dotted curve — approximation [16]. The histogram step — 290 keV

for the energy interval 2—9 MeV obtained using the shift procedure (6) for the spectra of positrons taken from [4, 15, 16] (see Figs. 1, 2 and 3, respectively) are given in Table 2.

Besides defining the spectrum $\eta(E_\nu)$ "with respect to the registration place" of $\tilde{\nu}_e$, it's necessary to examine the technique of obtaining the spectra ρ_i "with respect to the origination place". They differ not only in the "place" but also in the mode of determination. The calculational technique for obtaining the spectrum ρ_i "with respect to the origination place" of $\tilde{\nu}_e$ for a given fissionable nucleus is based on the fact that the ratio of the ν-spectrum (ρ_i) to the β-spectrum (ρ_β) of the mixture of fission products of each actinoid under the condition of secular equilibrium is characterized by a high stability not depending on hypotheses about the character of unknown models of fission [5, 17]. Though the stability of this ratio has no clear physical substantiation [7], the experimental data obtained when measuring the β-spectra of fission fragments of ^{235}U, ^{239}Pu, and ^{241}Pu [9, 16, 18] confirm the results of numerical simulation [5, 17] with high accuracy. The spectra ρ_i necessary for further calculations and converted in such a way are given in Table 2. The β-spectrum for ^{238}U was not measured, that's why we use a calculated spectrum [5].

The cross section $\sigma_{\nu p}(E_\nu)$ of the interaction between monoenergetic antineutrinos with energy E_ν for reaction (1) is the most theoretically developed energy function among the mentioned ones: $\eta(E_\nu)$, ρ_i, and $\sigma_{\nu p}(E_\nu)$. The technique of its calculation is described in detail in a number of papers, for example in [10—12]. In this case, the cross section of the capture of an antineutrino by a proton with regard for the behavior of the cross section near the reaction threshold ($\delta_{\rm thr}$), recoil ($\delta_{\rm rec}$), weak magnetism ($\delta_{\rm WM}$), and the external radiation correction ($\delta_{\rm rad}$) has a form:

$$\sigma_{\nu p}(E_\nu) = \sigma_0(E_\nu)(1+\delta_{\rm thr})(1+\delta_{\rm WM}+\delta_{\rm rec})\times$$
$$\times(1+\delta_{\rm rad}). \qquad (7)$$

Analytical relations for all the corrections and their detailed discussion are given in [11, 12]. The "naive" cross section $\sigma_0(E_\nu)$ [11, 12] corresponding to the approximation of infinitely heavy nucleons or, in other words, $E_n \approx m_n$, $E_{e^+} \ll m_n$ ($h = c$=1), is given by

$$\sigma_0(E_\nu) = \frac{2\pi^2\hbar^3 \ln 2}{m_e^5 c^7 (f\tau_{1/2})}\frac{1}{c}\left[(E_\nu-\Delta)^2 - m_e^2c^4\right]^{1/2}\times$$
$$\times(E_\nu-\Delta), \qquad (8)$$

where $E_\nu - (m_n - m_p)c^2 = E_\nu - \Delta = E_{e^+}$ represents the total energy of a positron in reaction (1) because the recoil energy of a neutron in this reaction can be neglected at the energies of antineutrinos appearing in the reactor; Δ=1.293 MeV is the physical threshold of reaction (1), $f\tau_{1/2}$ is the so-called reduced half-life of a neutron [7] with the phase space factor for a neutron f =1.7146 [13] determined to within 0.01% and the half-life period $\tau_{1/2} = \tau\ln 2$, where τ= 887.4 s ± 0.2% [19].

Now we have all necessary preconditions for solving Eq. (4) which turns into a system of linear equations after discretization. The results of discretization of Eq. (4) are seen from the analysis of data given in Table 2. It allows us to draw the evident conclusion that the general determinant of the system has a lot of "zeros" and the system can be quasiconfluent. This implies that we deal

T a b l e 1. Basic detector and reactor characteristics and the geometry of experiments

	Experiment		
	[15]	[4]	[16]
Target	Liquid scintillator (C_nH_{2n})		
$N_p(10^{28})$	1.506±1.5 %	5.820±1.5 %	5.820±1.5 %
ε_0	0.322±5.5 %	0.321±5.5 %	0.321±5.5 %
γ	1.00	0.75	0.75
$(\langle R^2\rangle)^{1/2}$, m	18.18±0.3 %	18.00±0.3 %	18.00±0.3 %
$\langle W\rangle$, MW	1452±2.0 %	1375±2.0 %	1375±2.0 %
ΔT_{e^+}, MeV	0.50	0.55	0.60
n_ν per 10^5 s	309.5±0.7 %	—	835.0±0.3 %
Number of measurements	66	70	174

with an ill-conditioned system of linear equations and its solutions can be unstable with respect to small variations of input data. In other words, such a problem belongs to a class of incorrect problems which can be solved with the help of the regularization method of Tikhonov [20, 21].

2. Dynamic Renewal of the Isotope Composition of a Nuclear Fuel

We'll demonstrate the effectiveness of solving the inversed problem (4) of the neutrino spectrometry of intrareactor processes discussing the experiments described in [4, 15, 16].

For this purpose, let's put down the original equation (4) as

$$y = \sum_{i=1}^{n} \lambda_i x^{(i)},$$

where

$$y = \eta(E_\nu), \quad x^{(i)} = \frac{\gamma \varepsilon_0 N_p}{4\pi R^2} \Delta t \sigma_{\nu p}(E_\nu) \rho_i,$$

or, after discretization,

$$y_k = \sum_{i=1}^{n} \lambda_i x_k^{(i)}, \quad k = 1, 2, ..., N,$$

where N— the number of energy values taken for measuring the spectrum.

At this stage, the values of λ_i are usually evaluated by the least-squares procedure using the condition for a discrepancy minimum:

$$\chi = \sum_{k=1}^{N} \left(\sum_{i=1}^{n} \lambda_i x_k^{(i)} - y_k \right)^2, \tag{9}$$

which states that partial derivatives $\partial\chi/\partial\lambda_i$ are equal to zero. It results in such a system of linear algebraic equations with respect to λ_i:

$$\sum_{i=1}^{n} \sum_{k=1}^{N} \lambda_i x_k^{(i)} x_k^{(j)} = \sum_{k=1}^{N} x_k^{(j)} y_k, \quad j = 1, 2, ..., n,$$

It can be written shortly as

$$A\lambda = u, \tag{10}$$

where

$$A_{ij} = \sum_{k=1}^{N} x_k^{(i)} x_k^{(j)}, \quad u_j = \sum_{k=1}^{N} x_k^{(j)} y_k.$$

Let the quantities A_{ij} and u_j be given with errors h and δ, respectively. Then, according to the regularization method of Tikhonov [20], the determination of the normal solution of Eq. (4) or, which is the same, the system of equations (10) is reduced to searching for the minimum norm vector over the set of vectors $\lambda = (\lambda_1, ..., \lambda_n)$ satisfying the condition $\|A\lambda - u\| = 2(h\|\lambda\| + \delta)$. Abiding by [20], we'll solve this problem using the Lagrange method of undetermined multipliers, that is, we'll search for the vector λ^α minimizing the smoothing functional

$$M^\alpha(\lambda, u, A) = \|A\lambda - u\|^2 + \alpha\|\lambda\|^2. \tag{11}$$

T a b l e 2. Experimental $\eta(E)$ and converted spectra $\rho_i(E_\nu)$

E_ν,	$\eta(E)$, $(\text{MeV})^{-1}$			ρ_i, (MeV·fission act)$^{-1}$			
MeV	$\eta(E)$ [15]	$\eta(E)/0{,}30$ [4]	$\eta(E)/0{,}29$ [16]	^{235}U [9]	^{239}Pu [9]	^{238}U [5]	^{241}Pu [9]
2.0	23.032±3.6	42.857±2.85	21.285±2.21	0.130(+1)±4.2	0.107(+1)±4.5	0.153(+1)	0.124(+1)±4.3
2.5	57.928±3.6	73.571±2.76	53.285±2.23	0.900(0)±4.2	0.710(0)±4.3	0.111(+1)	0.870(0)±4.0
3.0	79.091±2.4	80.357±3.83	74.143±2.40	0.673(0)±4.2	0.491(0)±4.3	0.835(0)	0.623(0)±4.0
3.5	87.267±2.4	91.428±2.91	81.357±3.30	0.473(0)±4.2	0.317(0)±4.3	0.586(0)	0.420(0)±3.9
4.0	84.077±3.4	82.857±5.95	73.042±3.11	0.283(0)±4.2	0.190(0)±4.4	0.386(0)	0.270(0)±3.9
4.5	71.503±2.3	78.428±3.11	60.714±2.28	0.172(0)±4.2	0.107(0)±4.8	0.245(0)	0.157(0)±4.2
5.0	62.129±3.2	60.000±4.13	53.357 ± 4.73	0.105(0)±4.2	0.576(−1)±5.2	0.152(0)	0.920(−1)±4.4
5.5	49.378±2.2	52.142±3.17	35.757±2.18	0.617(−1)±4.2	0.350(−1)±5.9	0.908(−1)	0.525(−1)±4.9
6.0	35.377±3.2	37.857±3.75	25.500±1.91	0.370(−1)±4.3	0.177(−1)±6.8	0.549(−1)	0.267(−1)±5.6
6.5	26.007±2.7	22.499±2.63	15.714±1.93	0.203(−1)±4.4	0.940(−2)±7.4	0.328(−1)	0.139(−1)±6.1
7.0	16.744±2.7	14.999±2.67	7.428±1.73	0.105(−1)±4.7	0.468(−2)±11	0.176(−1)	0.683(−2)±7.0
7.5	8.693±0.48	9.285±1.85	2.642±1.83	0.429(−2)±5.0	0.180(−2)±19	0.787(−2)	0.254(−2)±8.0
8.0	3.562±0.42	6.071±2.31	0.134±0.56	0.136(−2)±6.0	0.500(−3)±35	0.313(−2)	0.890(−3)±11
8.5	1.033±0.45	2.142±1.82	—	0.237(−3)±10	0.220(−3)±80	0.526(−3)	0.235(−3)±24
9.0	0.139±0.36	1.428±1.82	—	0.560(−4)±27	0.398(−4)	0.124(−3)	0.470(−4)±90

N o t e s: 1) errors are given in percents; 2) a decimal exponent for converted spectra ρ_i is given in parentheses, errors are given in percents with probability 90 %.

The parameter α is determined from the condition

$$\|A\lambda^\alpha - u\| = 2\left(h\|\lambda^\alpha\| + \delta\right). \quad (12)$$

Trial values λ_i represent the solution of the system of linear algebraic equations

$$\alpha\lambda_i + \sum_{j=1}^{n}\sum_{k=1}^{n} A_{ki}A_{kj}\lambda_j = \sum_{j=1}^{n} A_{ji}u_j, \quad i = 1, ..., n, \quad (13)$$

which result from the condition for a minimum of functional (11): $(\partial M^\alpha/\partial\lambda_j^\alpha) = 0, j = 1, ..., n$.

The substitution of the trial sequence $\{\lambda_i\}$ obtained from (13) (at a given initial estimate of the regularization parameter (α)) to condition (12) results in an equation for the parameter α. Solving it by means of numerical methods, we can obtain its actual value. Then the solution of Eq. (13) at the determined value of the parameter α will be a final required solution of Eq. (4) (or (10) with respect to λ_i^α).

The regularized solutions $\{\lambda_i^\alpha\}$ of system (10) that characterize the values of fission rates for the nuclear fuel actinoids are given in Table 3, while Table 4 demonstrates the normed values of these quantities $\{a_i^\alpha\}$:

$$a_i^\alpha = \lambda_i^\alpha \Big/ \sum_i \lambda_i^\alpha, \quad (14)$$

This corresponds to the contribution of fissile isotopes averaged over the measurement time ($\Delta t = 10^5$ s) to the total average number of fission acts a_i(5).

The comparison of contributions of fissile isotopes to the total number of fission acts (14) and (5) characterizing the same experiment but obtained in different ways (see Table 4), indicates their good agreement.

3. Error of the Regularized Solution

Here, abiding by [22], we'll briefly discuss a somewhat unexpected but very fundamental question: is it possible in principle to estimate the error of an approximate solution of an incorrect problem? Unfortunately, in the general case, it is not. This main and important result was obtained by Bakushinskii (see [23] or [24]). Despite his pessimism, this result is very logical from the physical point of view: if nothing is known about the characteristics of the accurate solution, that is, there are no model ideas describing the investigated phenomenon, we can't expect any additional information (for example, information concerning the accuracy of an approximate solution). In other words, in the absence of any *a priori* information, the fact of obtaining an approximate solution of an incorrect problem represents a certain achievement. But it results in the basic uncertainty of the error of this solution. It is a really nontrivial outbreak in our understanding of the problem stimulated by the efforts of Tikhonov's school [20—24], which is proved by the fact that, till 1960s, a lot of investigators influenced by Hadamard [25] considered the attempts of solving incorrect problems to be a demonstration of mathematical tastelessness. On the other hand, a persistent mistake (which has been very popular recently in the scientific literature) concerning the expectations that there exists a hypothetical method of estimating an error of the approximate solution of a problem incorrect by Tikhonov in the absence of any *a priori* information is obviously explained by unwillingness of an investigator (who has overcome though the traditional but serious difficulties related to the statement of the problem by means of sound estimates of the error of the operator $\|A_h - A\| \le h$ and experimental data $\|u_\delta - u\| \le \delta$) to understand that it is not enough. However, knowing the error of the operator and the right-hand side of Eq. (10) is absolutely insufficient for obtaining an estimate of the error of an approximate solution, but (and it is very important!) it is essentially necessary for obtaining an approximate solution in general [26]. In particular, as was demonstrated in [27] and [28], using the error h is necessary for constructing the stable methods of solving ill-conditioned systems of linear algebraic equations such as Eq. (4) or (10).

T a b l e 3. Regularized solutions $\{\lambda_i^\alpha\}$ of the system of equations (10)

$\lambda \cdot 10^{19}$, fission·s^{-1}	[15]	[4]	[16]
λ_5	2.688	2.486	2.457
λ_9	1.223	1.203	1.232
λ_8	0.320	0.296	0.298
λ_1	0.196	0.196	0.195
$\lambda_0 = \sum\lambda_i$	4.427±5.42 %	4.181±6.36 %	4.182±5.18 %

T a b l e 4. Relative contributions of fissile isotopes a_i to the total number of fission acts λ_0

a_i	Our results	[15]	Our results	[4]	Our results	[16]
a_5	0.607	0.606	0.595	0.593	0.588	0.586
a_9	0.276	0.274	0.288	0.286	0.295	0.292
a_8	0.072	0.074	0.071	0.075	0.071	0.075
a_1	0.044	0.046	0.047	0.047	0.047	0.047
W_{NPP}, MW	1451.7	1452	1374.6	1375	1375.6	1375

Thus, a stable method of solving incorrect problems doesn't guarantee the possibility of obtaining an estimate of the error or the rate of convergence of an approximate solution. It can only guarantee that an approximate solution, if it exists, asymptotically approaches the accurate one [20] but says nothing about how close or how far it is situated from the accurate solution. But the situation changes substantially if we know some additional information concerning the unknown accurate solution. In particular, if this additional information concerns the structure of the set including the accurate solution, then there exist sufficiently effective methods of finding the error of an approximate solution.

Here, we observe two kinds of mostly developed methods. The first one occurs in the case where it is known *a priori* that the accurate solution belongs to a compact set. It implies that the problem is reduced to a rather common case where it is known for sure that the unknown accurate solution represents, for example, a monotonic or convex function. Then, if the operator A_h of the problem is injective (embedded) and continuous, the problem can be easily reformulated giving a well-conditioned (correct by Hadamard) problem which can be dealt with applying traditional numerical methods [24]. Examples of effective calculations also including the estimation of the error of approximate solutions for such a kind of compact sets can be found in [29].

The other kind of the mentioned approaches occurs in cases where the accurate solution admits a source-like image with a compact operator [30]. In particular, it is shown for inverse problems of heat conduction (see references in [30]). In this case, there exists a possibility of calculating the so-called *a posteriori* error of an approximate solution. A method of estimating the *a posteriori* error and its domain of applicability for such a kind of problems is given and discussed in [22, 30].

Now let's discuss the question concerning the accuracy of a stable approximation of the solution or, speaking more accurately, the normal solution (the normal pseudo-solution) of our system (4) or (10). Applying the regularization method to solving ill-conditioned systems of linear equations is characterized with one particularity. Its essence is determined by such an important theorem [31]: while searching for a normal solution λ of a consistent system like (10) with the help of the regularization method using the smoothing functional (11), it is possible to choose the regularization parameter $\alpha = \alpha(h)$ in such a way that, for $0 \le h \le h_0 = \text{const}$ and for any $\sigma \ge 0$, the error of an approximate solution satisfies the estimate

$$\left\|\lambda^{\alpha(h)} - \lambda\right\| \le \frac{\left\|\bar{A}^{+}\right\|\left[\sigma + h(1+\sqrt{2})\|\lambda\|\right]}{1 - h\left\|\bar{A}^{+}\right\|} = O(h+\sigma), \tag{15}$$

where $\|\cdot\|$ is the Euclidean norm.

This estimate is accurate by the order of magnitude of the errors σ, h and the pseudoinverse matrix $\|A^{+}\|$ contained in it. So, particular is the fact that the approximate solution obtained with the help of this method using approximate data of problem (10) has *the optimal order of accuracy* (15) only in the case where the system is solvable (see [32] in more details). If the system is not consistent and we search for its normal pseudosolution, then the regularization method may not give the optimal order of accuracy in the general case. It's worth noting that it isn't possible in the general case to judge about the solvability of an accurate system from its approximate data [32].

It isn't our case. The system is obviously solvable as the existence of the accurate solution is known *a priori*. The latter represented in the generalized form is equivalent to the nominal heat rating of a reactor $\langle W_{\text{NPP}}\rangle$ (see Table 4) which can be easily estimated from (5):

$$W_{\text{NPP}} = \sum_i \lambda_i{}^i E_f = \lambda \sum_i \alpha_i{}^i E_f.$$

In practice, the quantity $\langle W_{\text{NPP}}\rangle$ is usually determined and controlled in independent way, for example, using the calorific method characterized with the approximately 2% error extent [15]. The obtained approximate solutions $\{\lambda_i^{\alpha}\}$ of system (10) coincide with the values of the nominal power of a Rivne reactor VVER-440 determined in the mentioned experiments [4, 15, 16] within the limits of 0.07 % (see Table 4). A comparative estimate denotes that the total error of the approximate solution with allowance for the 2% error of the calorific method of defining the reactor power exceeds 2—3%. Let's show it by means of estimate (15) which can be used for estimating the error of approximate solutions of the experiments [4, 15, 16] due to the solvability of system (10).

While solving system (4) in the energy spectrum "with respect to the registration place" $\eta(E_\nu)$, we've chosen the values of this function at energies E_ν =2.5; 3.0; 3.5; 4.5 MeV. That's why the relative error of the experimental data (see Table 2) amounted to: $\sigma \approx$2.71% in [15], $\sigma \approx$3.18 % in [4], $\sigma \approx$2.59 % in [16]. On the other hand, the error of the operator $\|A_h - A\| \le h$

in (4) is determined mainly by the average error of the energy spectrum of electron antineutrinos $\rho(E_\nu) = \sum \alpha_i \rho_i$) radiated by the mixture of products of fission of all actinoids forming the nuclear fuel. In this case, the relative error is equal to $h \approx 5.7$ % (see Table 2). But we should take into account the fact that the error of experimental data σ is much larger in absolute magnitude than the average error h of the energy spectrum of electron antineutrinos, that is, $\sigma >> h$ (see Table 2). So, in our case, the optimal order of accuracy (15) is defined mainly by the order of magnitude of the error of the experimental data σ in problem (10):

$$\left\| \lambda^{\alpha(h)} - \lambda \right\| = \|\Delta\lambda\| \le O(\sigma). \tag{16}$$

Then, with allowance for the obtained estimates of the average errors σ of experimental data [15, 4, 16] and the presumption about the uniform distribution of errors over the components of the regularized solution $\lambda = (\lambda_1, \ldots \lambda_4)$, we can obtain the estimated accuracy of each component of the approximate regularized solution as

$$\Delta\lambda = \Delta\lambda_i = \frac{1}{2}\|\Delta\lambda\| = \begin{cases} O(1,355\%) & [15], \\ O(1,590\%) & [4], \\ O(1,295\%) & [16]. \end{cases} \tag{17}$$

On the other hand, the values of the total fission rates λ_0 being the sum of the components of the regularized solution $\{\lambda_i^\alpha\}$ of system (10) for each case [15, 4, and 16], respectively (see Table 3), allow us to estimate the summarized approximate solution and its accuracy in such a form:

$$\lambda_0 = \sum_i \lambda_i^\alpha \pm \|\Delta\lambda\| =$$

$$= \begin{cases} 4,427 \cdot 10^{19} \pm O(2.71\%) & [15], \\ 4,181 \cdot 10^{19} \pm O(3.18\%) & [4], \\ 4,182 \cdot 10^{19} \pm O(2.59\%) & [16]. \end{cases} \tag{18}$$

Now there arises a natural question: how should we pass from the estimate of the optimal order of accuracy of the approximate solution (18) directly to the estimate of the accuracy of the solution itself? On the ground of the above-mentioned facts, the general plan of this procedure is clear enough: basing on some model ideas about the solution, we should make use of the additional information which will allow us to restrict the set of solutions including the accurate one and, by that, will make possible obtaining a sound estimate of the error of the chosen set of solutions. Below, we'll explain how it was done in this case.

It is known that the burnout of ^{235}U and the accumulation of plutonium result in the temporal variation of both the spectrum of antineutrinos of a nuclear reactor and hence the cross section of reaction (1). As noted in [4], due to the burnout, the contributions of isotopes α_i change significantly during the operating period. Moreover, variations in the spectrum can reach 10%, while the total cross section of reaction (1) changes by nearly 6% during one operating period (as was demonstrated experimentally in [33]). It's comparable to the errors of the results given in expression (18) or even exceeds them. In practice, the contributions of isotopes α_i are usually calculated with the help of programs with the relative error of calculations reaching for example 5% for a reactor VVER-440 [33]. That is, if the values of the relative contributions of isotopes α_i used in the direct problem (2) and those obtained in the inverse problem (4) are close with all other things being equal (see Table 4), it is equivalent to the closeness of estimates of the optimal order of accuracy of solution (16) — (18) and the error of relative contributions of isotopes α_i obtained with the help of programs calculating the worked-out fuel [33]. In other words, it implies that the accuracy estimate of the error of each component of the approximate regularized solution (4) can be now written in a form acceptable for calculations:

$$\left\| \lambda^{\alpha(h)} - \lambda \right\| = \|\Delta\lambda\| \le O(\sigma) \approx 2\sigma, \tag{19}$$

Its essence supplements quite an abstract content of expressions (16)—(18). For example, expressions (17), (18) with condition (19) being satisfied allow one to estimate the error of relative contributions of isotopes α_i (Table 4) of each component forming the approximate regularized solution (18) of the balance equation (4) of the spectrometry of antineutrinos in experiments [15, 4, and 16]:

$$\alpha_i = \frac{\lambda_i}{\lambda_0} \Rightarrow \frac{\Delta\alpha_i}{\alpha_i} \approx \sqrt{\left(\frac{\Delta\lambda_i}{\lambda_i}\right)^2 + \left(\frac{\Delta\lambda_0}{\lambda_0}\right)^2} =$$

$$= \begin{cases} 5.6\% & [15], \\ 6.6\% & [4], \\ 5.3\% & [16]. \end{cases} \tag{20}$$

This estimate correlates well with data obtained from the calculations of a worked-out fuel [33]. The most important thing is that such a procedure of passing from estimating the optimal order of accuracy of approximate solution (18) directly to estimating the accuracy of this solution implies at the same time that condition (19) remains valid in all similar experiments and improves the

statistics of measurements (for example, at the expense of an increase of the sample size) or degrades.

And, at last, it's worth noting that there exists a method allowing to avoid the difficulties concerning the problem of consistency or inconsistency of systems of equations of kind (10). It turns out that the theory of a generalized discrepancy principle developed in [32] for solving nonlinear incorrect problems allows one to construct a method of solving ill-conditioned systems of linear equations with approximate data, and its accuracy doesn't depend on the solvability of an accurate system. This method (the so-called method of pseudoinverse matrix [32]) will be used in the following paper for the further analysis and the choice of a stable and reliable procedure allowing one to determine the accuracy of an approximate solution for the inverse problem of the neutrino diagnostics of intrareactor processes with allowance for the statistics of origination and registration of reactor antineutrinos [34—36].

Conclusions

We have proposed a basically new method for determining the nuclear density of each component forming the isotope composition of a nuclear fuel and, respectively, the dynamics of their variation in the course of the reactor operation. The neutrino-based method obviously has all the characteristics necessary for an independent and absolute technique, which is very actual for the remote diagnostics of the most important characteristics of the reactor core in on-line mode. It can be applied to both "short-range" problems connected with the determination of the current heat rating and the dynamics of variation of the nuclear density of each actinoid contained in the composition of a nuclear fuel and "long-range" ones like those of identification of a mixture of fission products and defining the integral density of a neutron flux.

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ОБЕРНЕНА ЗАДАЧА ДИСТАНЦІЙНОЇ НЕЙТРИННОЇ ДІАГНОСТИКИ ВНУТРІШНЬОРЕАКТОРНИХ ПРОЦЕСІВ

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Р е з ю м е

На прикладі відомих експериментальних даних розглянуто обернену задачу дистанційної нейтринної діагностики активної зони реактора, розв'язок якої, з одного боку, дозволяє з відомою точністю дистанційно визначати поточні значення ядерної щільності кожної з компонент ядерного палива, а з іншого — відкриває реальну можливість створення нейтринної технології діагностики часової еволюції ізотопного складу ядерного палива і потужності реактора в режимі on-line.