SINGLE-MODE SQUEEZING OF GLUONS IN QCD JET

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We study the evolution of colour gluon states in an isolated QCD jet at the non-perturbative stage. Fluctuations of gluons are less than those for coherent states under specific conditions. This fact suggests that there gluon squeezed states can arise. The angular and rapidity dependences of the normalized second-order correlation function for present gluon states are studied at this stage of jet evolution. It is shown that these new gluon states can have both sub-Poissonian and super-Poissonian statistics corresponding to, respectively, the antibunching and bunching of gluons by analogy with squeezed photon states.

Introduction

Many experiments at $e^+e^-, p\bar{p}, ep$ colliders are devoted to hadronic jet physics, since detailed studies of jets are important for a better understanding and testing of both perturbative and non-perturbative QCD and also for the finding of manifestations of new physics. Although the nature of jets is of a universal character, e^+e^-- annihilation stands out among hard processes, since jet events admit a straightforward and clear-cut separation in this process. In the reaction $e^+e^- \rightarrow$ hadron, four evolution phases are recognized by various time and space scales. These are (I) the production of a quark-antiquark pair: $e^+e^- \rightarrow q\bar{q}$; (II) the emissions of gluons and quarks from primary partons — perturbative evolution of the quark-gluon cascade; (III) the non-perturbative evolution and the hadronization of quarks and gluons; (IV) the decays of unstable particles.

The second phase of e^+e^-- annihilation has been well understood, and sufficiently accurate predictions for it have been obtained within the perturbative QCD (PQCD) [1, 2]. But predictions of the PQCD are limited by small effective coupling $\alpha(Q^2) < 1$ and the third phase is usually taken into account either through a constant factor which relates partonic features with hadronic ones (within local parton-hadron duality) or through the application of various phenomenological models of hadronization. As a consequence, theoretical predictions both for intrajet and for interjet characteristics remain unsatisfactory. For example, the width of the multiplicity distribution (MD) according to the predictions of PQCD is larger than the experimental one. The discrepancies between theoretical calculations and experimental data suggest that the quark-gluon cascade undergoes non-perturbative evolution after the perturbative stage after that hadronization effects come into play. New gluon states, generated at the non-perturbative stage, contribute to various features of jets. For example, such a contribution to the multiplicity distribution can be in the form of the sub-Poissonian distribution [3, 4]. Therefore, we must take into account both the perturbative and non-perturbative stages of the jet evolution.

Calculations performed within PQCD [5, 6] show that the multiplicity distribution at the end of the perturbative cascade is close to a negative binomial distribution. At the same time, gluon MD in the range of small transverse momenta (thin ring of jet) is Poissonian [7]. Thus, parton MD in the whole jet at the end of the perturbative cascade can be represented as a combination of Poissonian distributions each of which corresponds to a coherent state. Studying a further evolution of gluon states at the non-perturbative stage of jet evolution, we obtain new gluon states that are squeezed states (SS) [8—10]. These states are formed as a result of the non-perturbative self-interaction of gluons expressed by nonlinearities of the Hamiltonian. In this paper, we prove that the non-perturbative stage of jet evolution can be one of sources of a gluon SS by analogy with the nonlinear medium for photon SS [11—14]. Squeezed states possess uncommon properties: they display a specific behaviour of the factorial and cumulant moments [15] and can have both sub-Poissonian and super-Poissonian statistics corresponding to the antibunching and bunching of photons. Moreover, the oscillatory behaviour of MD of photon SS is differentiated from Poissonian and negative binomial distributions (NBD). Because of the analogy between photon and gluon, MD of gluon SS must have oscillations and, using Local parton hadron duality (LPHD), we can compare derived gluon MD with hadron MD. It is clear that, in this case, the behaviour of hadron MD in jet events is differentiated from NBD and this fact is confirmed by experiments for pp, p$\overline{\rm p}$-collisions [16—18].

In series of works [19—21], it was shown that the presence of a chaos amplifies the effect of squeezing. It was demonstrated that one of the causes of chaos is a local instability of a dynamical system [22] which can lead to the mixing of trajectories in the phase space and, as a result, to a non-regular behaviour of the considered system [23]. A keen interest to chaos in field theories [24] is connected with the facts that all four fundamental particle interactions have chaotic solutions [25]. Since the chaos phenomenon can be related with confinement [26] and with fractality for the factorial moments [27], the question about a condition of the appearance of chaos in a jet is important. In this connection, we investigate the SU(2)-jet model for the purpose of revealing the local instability which can lead to chaos.

1. Squeezed Gluon States in QCD Jet Ring

The Hamiltonian of the four-gluon self-interaction V in the jet ring of thickness $d\theta$ has the next form [9, 10]

$$V = 2\pi\, u_2 f_{abc} f_{ade} \left\{ \left(1 + u_1 - \frac{u_1}{2}\sin^2\theta\right) \times \right.$$

$$\left. \times \left[a^{bcde}_{1212} + a^{bcde}_{1313}\right] + (1 + u_1 \sin^2\theta) a^{bcde}_{2323} \right\} \sin\theta\, d\theta. \qquad (1)$$

Here, $u_1 = \left(1 - \dfrac{q_0^2}{k_0^2}\right)$, $u_2 = \dfrac{k_0^4}{4(2\pi)^3}\dfrac{g^2}{2}\sqrt{u_1^3}$, $a^{bcde}_{lmlm} = a^{b+}_l a^{c+}_m a^d_l a^e_m + a^{b+}_l a^c_m a^{d+}_l a^e_m + a^b_l a^{c+}_m a^{d+}_l a^e_m + \text{h.c.}$, $a^b_l (a^{b+}_l)$ is the operator annihilating (creating) a gluon of colour b and vector component l, q_0^2 and k_0 are correspondingly the virtuality and energy of the gluon at the end of the perturbative cascade, g is the coupling constant, f_{abc} stands for the structure constants of the $SU_c(3)$ group, θ is the angle between the jet axis and the momentum $\mathbf{k}$ $(0 \le \theta \le \theta_{\max}, \theta_{\max}$ is half of the opening angle of the jet cone).

It is obvious that Hamiltonian (1) includes both the squares of the creation and annihilation operators for gluons with fixed colour and vector indices and the product of the corresponding operators with different colour and vector indices. As is known from quantum mechanics and quantum optics, the presence of such a structure in the Hamiltonian and, consequently, in the evolution operator is a necessary condition for the emergence of single- and multi-mode squeezed states [13], since the unitary squeezing operator $S(z)$ can involve both quadratic combinations of creation and annihilation operators (single-mode squeezing) and the product of the considering operators (multimode squeezing)

$$S(z) = \exp\left\{\frac{z^*}{2} a_i^2 - \frac{z}{2}(a_i^+)^2\right\},$$

$$S(z) = \exp\left\{z^* a_i a_j - z a_i^+ a_j^+\right\}, \qquad (2)$$

where $z = re^{i\delta}$ is an arbitrary complex number, r is a squeeze factor, phase δ defines the direction of squeezing maximum [13].

In order to verify whether the final gluon state vector describes the single-mode SS, it is necessary to introduce the phase-sensitive Hermitian operators $(X^b_l)_1 = \left[a^b_l + (a^b_l)^+\right]/2$ and $(X^b_l)_2 = \left[a^b_l - (a^b_l)^+\right]/2i$ by analogy with quantum optics and to establish conditions under which the variance of one of them can be less than the variance of a coherent state.

Mathematically, the condition of squeezing is expressed as the inequality [14]

$$\left\langle N\left(\Delta (X^b_l)_{\frac{1}{2}}\right)^2\right\rangle < 0. \qquad (3)$$

Here, N is the normal-ordering operator such as

$$\left\langle N\left(\Delta (X^b_l)_{\frac{1}{2}}\right)^2\right\rangle = \frac{1}{4}\left\{ \pm \left[\left\langle \left(a^b_l\right)^2\right\rangle - \right.\right.$$

$$-\left\langle a^b_l\right\rangle^2\right] \pm \left[\left\langle\left(a^{b+}_l\right)^2\right\rangle - \left\langle a^{b+}_l\right\rangle^2\right] +$$

$$\left. +2\left[\left\langle a^{b+}_l a^b_l\right\rangle - \left\langle a^{b+}_l\right\rangle\left\langle a^b_l\right\rangle\right]\right\}. \qquad (4)$$

The expectation values of the creation and annihilation operators for gluons with specified colour and vector components are taken for the vector

$$|f\rangle = \prod_{c=1}^{8}\prod_{l=1}^{3}|\,\alpha^c_l(t)\rangle \simeq \prod_{c=1}^{8}\prod_{l=1}^{3}|\,\alpha^c_l(0)\rangle - itV\prod_{c=1}^{8}\prod_{l=1}^{3}|\,\alpha^c_l(0)\rangle. \qquad (5)$$

Let us consider the specific case where the colour index is $b = 1$ and the vector index l is arbitrary. Then we have

$$\left\langle N\left(\Delta(X_l^1)_{\frac{1}{2}}\right)^2\right\rangle = \pm\, 4\pi\, u_2\, t\, \sin\theta\, d\theta \Big\{(1+u_1)\times$$

$$\times\Big[\delta_{l1}(Z_{33}+Z_{22}) + (1-\delta_{l1})Z_{11}\Big] + \Big[\delta_{l2}Z_{33}+\delta_{l3}Z_{22}\Big]+$$

$$+u_1\sin^2\theta\,\Big[-\frac{1}{2}\delta_{l1}(Z_{22}+Z_{33}) + \delta_{l2}(Z_{33}-\frac{1}{2}Z_{11})+$$

$$+\delta_{l3}(Z_{22}-\frac{1}{2}Z_{11})\Big]\Big\}. \qquad (6)$$

Here, $Z_{mn} = \sum_{k=2}^{7}{}' \langle (X_m^k)_1\rangle\langle (X_n^k)_2\rangle \quad (m,n = 1,2,3)$. Since, at small values of the squeeze factor,

$$r\cos\delta = \mp\, 2\left\langle N\left(\Delta(X)_{\frac{1}{2}}\right)^2\right\rangle, \qquad (7)$$

expression (7) can be rewritten taking into account formula (6) in the form

$$r_l^1\cos\delta = -\,8\pi\, u_2\, t\, \sin\theta\, d\theta \Big\{(1+u_1)\Big[\delta_{l1}(Z_{33}+Z_{22})+$$

$$+(1-\delta_{l1})Z_{11}\Big] + \delta_{l2}Z_{33}+\delta_{l3}Z_{22}+$$

$$+u_1\sin^2\theta\,\Big[-\frac{1}{2}\delta_{l1}(Z_{22}+Z_{33}) + \delta_{l2}(Z_{33}-\frac{1}{2}Z_{11})+$$

$$+\delta_{l3}(Z_{22}-\frac{1}{2}Z_{11})\Big]\Big\}. \qquad (8)$$

Evidently that $\left\langle N\left(\Delta(X)_{\frac{1}{2}}\right)^2\right\rangle \neq 0$ if $r \neq 0$ and $\delta \neq \frac{\pi}{2}, \frac{3\pi}{2}$. In the final state under consideration, fluctuations of one of the squared components of the gluon field, $\Delta(X_l^1)_2$, are less than those in the initial coherent state under the following conditions: $\langle (X_m^k)_1\rangle < 0, \langle (X_m^k)_2\rangle < 0$ or $\langle (X_m^k)_1\rangle > 0$ and $\langle (X_m^k)_2\rangle > 0 \quad (k \neq 1, m \neq l)$. Then, as follows from (8), $\frac{\pi}{2} < \delta < \frac{3\pi}{2}$. In this case, we have phase-squeezed gluon states by analogy with quantum optics [12, 13]. If the conditions $\langle (X_m^k)_1\rangle > 0, \langle (X_m^k)_2\rangle < 0$ or $\langle (X_m^k)_1\rangle < 0$ and $\langle (X_m^k)_2\rangle > 0 \quad (k \neq 1, m \neq l, -\frac{\pi}{2} < \delta < \frac{\pi}{2})$ are satisfied, fluctuations in another squared component of the gluon field, $\Delta(X_l^1)_1$, will be less in the final state vector $\prod_{c=1}^{8}\prod_{l=1}^{3}|\,\alpha_l^c(t)\rangle$ than in the coherent state. In this case, we arrive at the amplitude-squeezed states (as in the case of photons [12, 13]).

Rewriting expression (8) in terms of the amplitude and phase of the gluon coherent states ($\alpha_l^b = |\,\alpha_l^b|\, e^{i\gamma_l^b}$)

$$r_l^1\cos\delta = -\,8\pi\, u_2\, t\, \sin\theta\, d\theta \sum_{k=2}^{7}{}' \Big\{\Big(1+u_1-\frac{u_1}{2}\sin^2\theta\Big)\times$$

$$\times\Big[\delta_{l1}\sum_{n=2}^{3}|\,\alpha_n^k|^2\sin(2\gamma_n^k) + (1-\delta_{l1})|\,\alpha_1^k|^2\sin(2\gamma_1^k)\Big]+$$

$$+\,(1+u_1\sin^2\theta)\Big[\delta_{l2}|\,\alpha_3^k|^2\sin(2\gamma_3^k)+$$

$$+\delta_{l3}|\,\alpha_2^k|^2\sin(2\gamma_2^k)\Big]\Big\}, \qquad (9)$$

we see that the effect of single-mode squeezing is absent ($r_l^1\cos\delta = 0$). Then the initial gluon coherent fields are either real ($\gamma_n^k = 0, n \neq l, k \neq 1$) or imaginary ($\gamma_n^k = \pi/2, n \neq l, k \neq 1$). Similar conclusions will also be valid for a gluon field featuring other colour indices.

Thus, the vector $\prod_{c=1}^{8}\prod_{l=1}^{3}|\,\alpha_l^c(t)\rangle$ describes the squeezed state of gluons that are produced at the non-perturbative stage of the jet evolution within a small interval of time t. Here, the corresponding fluctuations of the squared components of the gluon field will be less than those in the case of the initial coherent state.

It should be noted that the Hamiltonian of the three-gluon self-interaction which is proportional to f_{abc} in the momentum representation does not lead to the single-mode squeezing effect. In fact, rewriting the expression for $\left\langle N\left(\Delta(X_l^h)_{\frac{1}{2}}\right)^2\right\rangle$ (4) for small times as

$$\left\langle N\left(\triangle(X_l^h)_{\frac{1}{2}}\right)^2\right\rangle = \mp\,\frac{i\,t}{4}\Big\{\langle\alpha\,|\,[a_l^h\,(k)\,,[a_l^h\,(k)\,,V\,]\,]|\alpha\rangle-$$

$$-\langle\alpha\,|[\,[V, a_l^{+h}\,(k)], a_l^{+h}\,(k)]\alpha\rangle\Big\}, \qquad (10)$$

it can be shown that $[a_l^h(k),[a_l^h(k),V']] = 0$, $[[V',a_l^{+h}(k)],a_l^{+h}(k)] = 0$ since $f_{hhb} = 0$. That is, the squeezing condition (3) does not hold.

There is also a direct relation between squeezing and chaos in some quantum-mechanical systems [19 — 21].

The problem of the existence of a chaos in the SU(2)-jet model merits an especial attention. Here, we take the case of SU(2)-group only for a simplification of calculations. The respective Hamiltonian of the interaction V is obtained from the Hamiltonian for SU(3)-jet (1) by replacing the structure constants of the SU(3)-group by the corresponding SU(2)-group constant, that is $f_{abc} \to \varepsilon_{abc}$ $(a,b,c=\overline{1,3})$.

The local instability for the given Hamiltonian has been verified by the Toda criterion [28]. Analysis was made numerically accordingly to the next algorithm:

1) we come to the classical Hamiltonian by keeping the order of operators a^+, a and consider them as c-numbers (α^*, α);

2) we have 18 variables and calculate the instability matrix 18×18 for this case;

3) next step is the calculation of its eigenvalues to find out whether they are real or imaginary.

As a result, the following conclusions have been obtained:

1. If all variables α and α^* are real or imaginary, then the system of SU(2)-gluons is strictly ordered and the effect of the squeezing is absent.

2. If at least one of α or α^* is imaginary and other is real or at least one of α and α^* is real and other is imaginary — we have the local instability, which can lead to a chaotic system.

2. Correlation Functions for Squeezed Gluon States

The behaviour of a correlation function can serve as one of the criteria of the existence of squeezed gluon states. It is common to define a normalized second-order correlation function as [27]

$$K_{(2)}(\theta_1,\theta_2) = \frac{C_{(2)}(\theta_1,\theta_2)}{\rho_1(\theta_1)\rho_1(\theta_2)}, \tag{11}$$

$C_{(2)}(\theta_1,\theta_2) = \rho_2(\theta_1,\theta_2) - \rho_1(\theta_1)\rho_1(\theta_2)$, with $\rho_2(\theta_1,\theta_2)$ $(\rho_1(\theta))$ being the two-particle (single-particle) inclusive distribution. Then, for gluons with a colour b and a vector component l, we can write

$$K^b_{l(2)}(\theta_1,\theta_2) = \frac{\rho^b_{l(2)}(\theta_1,\theta_2)}{\rho^b_{l(1)}(\theta_1)\rho^b_{l(1)}(\theta_2)} - 1. \tag{12}$$

At the same time, we have

$$\int_\Omega \rho_1(\theta)\,d\theta = \langle n\rangle = \langle a^+a\rangle = \int_\Omega \langle f(\theta,t)|a^+a|f(\theta,t)\rangle d\theta, \tag{13}$$

where $|f(\theta,t)\rangle$ is the final state vector. We find from (13) that the single- and two-particle inclusive distributions can be represented as

$$\left.\begin{array}{l}\rho_1(\theta) = \langle f(\theta,t)|a^+a|f(\theta,t)\rangle,\\ \rho_2(\theta_1,\theta_2) =\\ = \langle f(\theta_2,t),f(\theta_1,t)|a^+a^+a\,a|f(\theta_1,t),f(\theta_2,t)\rangle.\end{array}\right\} \tag{14}$$

By taking the expectation values over the vector[1] $\prod_{c=1}^{8}\prod_{l=1}^{3} |\,\alpha_l^c(\theta_1,t),\alpha_l^c(\theta_2,t)\rangle$, we obtain the explicit form of the normalized second-order correlation function for squeezed gluon states:

$$K^b_{l(2)}(\theta_1,\theta_2) = -\,M_1(\theta_1,\theta_2)/\{|\,\alpha_l^b\,|^4 -$$
$$-2\,|\,\alpha_l^b\,|^2\,M_1(\theta_1,\theta_2) + M_2(\theta_1,\theta_2)\}. \tag{15}$$

For the colour index $b = 1$ and an arbitrary vector component l, we have

$$M_1(\theta_1,\theta_2) = 24\,t\,u_2\,\pi\,|\,\alpha\,|^2|\,\beta\,|^2 \sin\left(\delta + \frac{\pi}{2}\right) \times$$
$$\times\Big\{(1+\delta_{l1})(2+u_1-\delta_{l1})(\sin\theta_1 + \sin\theta_2) -$$
$$-\frac{1}{2}u_1\,(3\delta_{l1}-1)(\sin^3\theta_1 + \sin^3\theta_2)\Big\}, \tag{16}$$

$$M_2(\theta_1,\theta_2) = 80\,t\,u_2\,\pi\,|\,\alpha\,|^3|\,\beta\,|^3 \sin\left(\frac{\delta}{2} + \frac{\pi}{4}\right) \times$$
$$\times\Big\{(1+\delta_{l1})(2+u_1-\delta_{l1})(\sin\theta_1 + \sin\theta_2) -$$
$$-\frac{1}{2}u_1\,(3\delta_{l1}-1)(\sin^3\theta_1 + \sin^3\theta_2)\Big\}. \tag{17}$$

[1] That this vector also describes squeezed gluon states can be proven by verifying the squeezing condition (3).

In deriving these formulae, we assumed for simplicity that $\alpha_l^1 = |\alpha|\, e^{i\gamma_1}$, $l =$ any, and $\alpha_l^b = |\beta|\, e^{i\gamma_2}$, when $b \neq 1$ and an arbitrary l, $\gamma_1 - \gamma_2 = \delta/2 + \pi/4$.

Let us perform a comparative analysis of the correlation function (15) for gluon squeezed states and the corresponding function for photon squeezed states, which was thoroughly studied in quantum optics.

In quantum optics, a normalized second-order correlation function is defined as [14]

$$K_{l(2)} = g_l^{(2)} - 1 \quad = \quad \frac{\langle \hat{a}_l^+ \, \hat{a}_l^+ \, \hat{a}_l \, \hat{a}_l \rangle}{\langle \hat{a}_l^{1+} \, \hat{a}_l \rangle^2} - 1, \tag{18}$$

where the expectation values are taken over the evolved state vector at the instant t. If the correlation function is positive, there occurs photon bunching (super-Poissonian distribution); otherwise ($K_{l(2)} < 0$), we have photon antibunching (sub-Poissonian distribution) [12, 14]. For a coherent field obeying the Poissonian statistics, the normalized second-order correlation function vanishes ($K_{l(2)} = 0$).

For photon squeezed states whose state vector is defined as $|\alpha, z\rangle = S(z)|\alpha\rangle$, the corresponding correlation function has the form (at small values of the squeezing parameter r_l)

$$K_{l(2)} = - \frac{r_l[\alpha_l^2 e^{-i\delta} + (\alpha_l^*)^2 e^{i\delta}]}{|\, \alpha_l \,|^4 - 2r_l \,|\, \alpha_l \,|^2 \, [\alpha_l^2 e^{-i\delta} + (\alpha_l^*)^2 e^{i\delta}]}. \tag{19}$$

In contrast to the correlation function for squeezed photon states, $K_{l(2)}$, the corresponding function for the squeezed gluon states, $K_{l(2)}^b$, includes, as follows from (15), $M_2(\theta_1, \theta_2)$ which appears because Hamiltonian (1) of the gluon self-interaction involves a nonlinear combination of the creation and annihilation operators of gluons with different colours and vector components.

The angular dependence of the correlation function for squeezed gluon states (with colour $b = 1$) that are formed at the non-perturbative stage after a lapse of $t = 0.001$ is investigated graphically at the following parameter values: $\theta_2 = 0$; $g^2 = 4\pi$ because $\alpha_s = \dfrac{g^2}{4\pi} \sim 1$; $q_o^2 = 1$ GeV^2 that corresponds to the gluon virtuality at the beginning of the non-perturbative stage; $k_o = \dfrac{\sqrt{s}}{2\langle n_{\text{gluon}}\rangle}$ corresponds to a gluon energy in the case of 2-jet events; $\sqrt{s} = 91$ GeV and $\langle n_{\text{gluon}}\rangle = |\,\alpha|^2 + 7|\,\beta|^2$.

If the amplitude $|\alpha|$ of the gluon field being considered is equal to the amplitudes $|\beta|$ of the cophased gluon fields having different colours and vector components, then the values of the correlation function lie in the negative region (Fig.1,*a*), and there occurs the antibunching of gluons with the corresponding sub-Poissonian distribution. In this case, the correlation function tends to a constant ($K_{1(2)}^1(\theta_1, \theta_2 = 0) = -2.80094$) as the angle θ_1 increases. The behaviour of the angular correlations of the cophased squeezed gluon states ($\delta = 0$) is similar to the correlations of analogous photon states at small values of the squeezing parameter [11]. If the amplitude of a selected gluon field with the colour ($b = 1$) α begins to dominate in relation to the amplitudes of other colour fields ($b \neq 1$), that is, $\alpha > \beta$, then the correlation function involves a singularity (Fig.1,*b*) at $\theta_1 \approx 1.208725 \times 10^{-11}$ ($|\alpha|^2 = 3$, $|\beta|^2 = 1$).

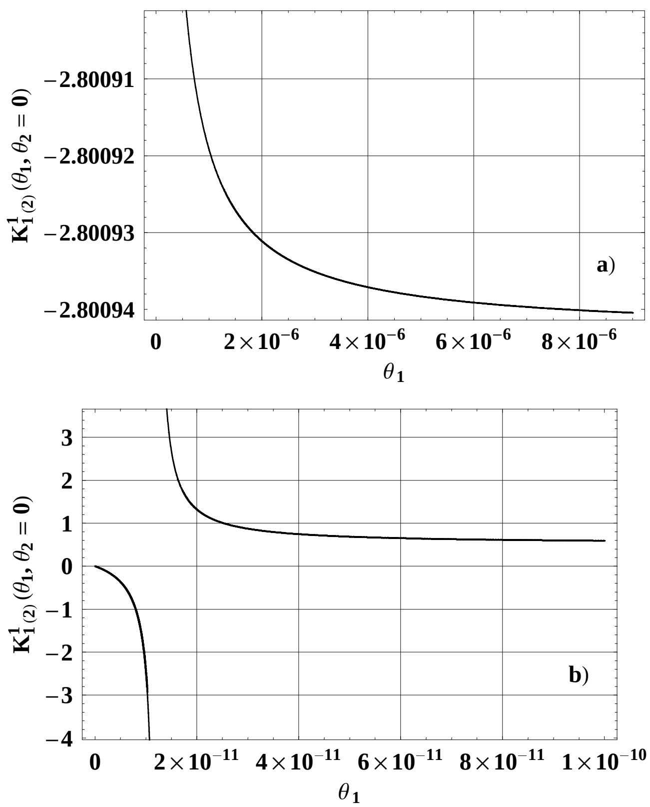

Fig. 1. The angular dependence of the cophased ($\delta = 0$) squeezed gluon correlation function at: *a* — $|\alpha|^2 = 1$, $|\beta|^2 = 1$; *b* — $|\alpha|^2 = 3$, $|\beta|^2 = 1$

For the antiphased squeezed states of gluons ($\delta = \pi$), the correlation function lies in the positive region and there occurs the gluon bunching with the corresponding super-Poissonian distribution. In this case, the correlation function grows fast at small angles θ_1 and tends to a constant irrespective of values of the amplitudes α and β.

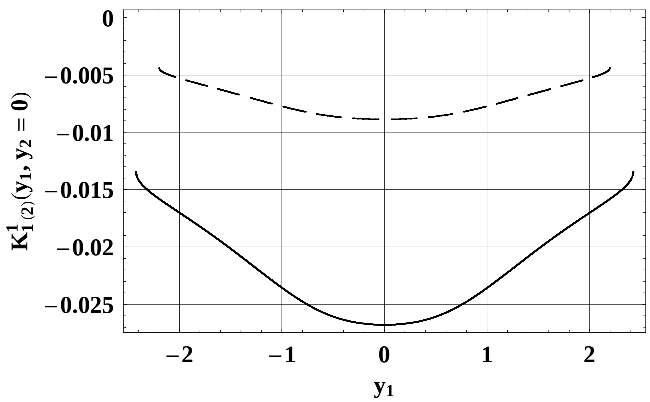

Fig. 2. The rapidity dependence of the cophased ($\delta = 0$) gluon squeezed correlation function at $y_2 = 0$: $|\alpha|^2 = 1, |\beta|^2 = 1$ (solid line), $|\alpha|^2 = 3, |\beta|^2 = 1$ (dotted line)

By using the transformation

$$\sin\theta = \sqrt{1 - \frac{\tanh^2 y}{u_1}}, \tag{20}$$

we can rewrite the correlation function for squeezed gluon states in terms of rapidities

$$K^b_{l(2)}(\theta_1, \theta_2) = -M_1(y_1, y_2)/\{|\ \alpha^b_l\ |^4 -$$

$$-2\ |\ \alpha^b_l\ |^2\ M_1(y_1, y_2) + M_2(y_1, y_2)\}. \tag{21}$$

Rapidity correlations of the cophased gluon squeezed states (Fig.2) fall within the region of negative values and have a minimum at the center ($K^b_{l(2)}(y_1 = y_2 = 0) = -0.0267894$ at $|\alpha|^2 = 1$). For $|\alpha| > |\beta|$, the correlation function has a less pronounced minimum at the center $K^b_{l(2)}(y_1 = y_2 = 0) = -0.00887147$ at $|\alpha|^2 = 3$.

Rapidity correlations of the antiphased gluon squeezed states fall within the region of positive values and have a maximum at the center ($K^b_{l(2)}(y_1 = y_2 = 0) = 0.0241966$ at $|\alpha|^2 = 1$, $K^b_{l(2)}(y_1 = y_2 = 0) = 0.0080179$ at $|\alpha|^2 = 3$).

It should be noted that the behaviour of the cophased gluon squeezed correlation function $K^b_{l(2)}(y_1, y_2)$ at $\sqrt{s} = 35$ GeV (Fig.3) is similar to that of hadron correlations with a distinctive minimum at $y_1 - y_2 = 0.45$ [27].

Thus, the behavior of rapidity correlations for the cophased gluon squeezed states under investigation suggests that, at the non-perturbative stage of evolution of a QCD jet, there exists the effect of the gluon antibunching with the corresponding sub-Poissonian statistics.

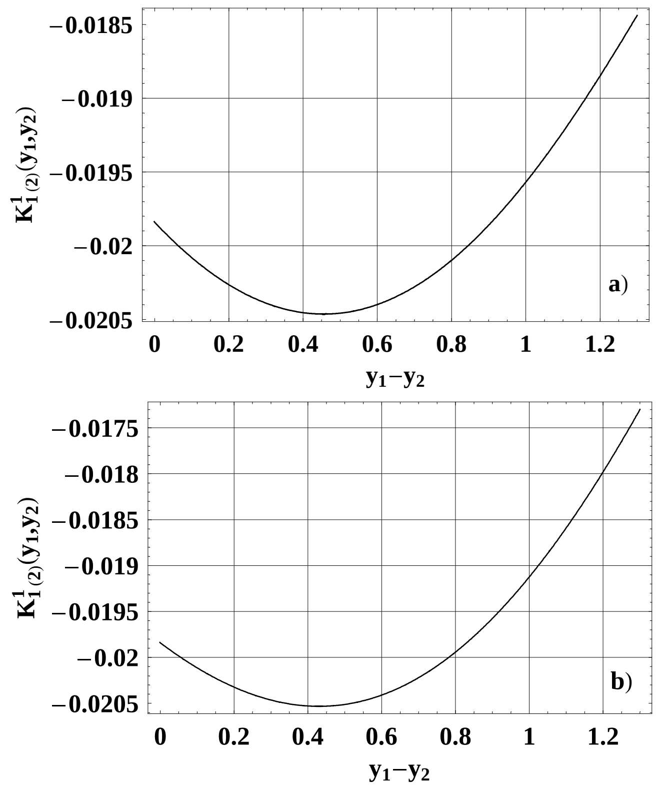

Fig. 3. The rapidity dependence of the cophased gluon squeezed correlation function at $\sqrt{s} = 35$ Gev, $|\alpha|^2 = 1, |\beta|^2 = 1$: a — $-1 \le y_1 + y_2 \le 0$, b — $0 \le y_1 + y_2 \le 1$

Conclusion

Investigating the gluon fluctuations, we have theoretically proved the possibility of the existence of the gluon single-mode SS at the non-perturbative stage of the QCD jet evolution. The emergence of such remarkable states becomes possible owing to the self-interaction of gluons with different colour indices.

As one of the identification criteria of the existence of gluon SS, a correlation function can serve. Therefore, we have analyzed the behaviour of angular and rapidity correlations and have compared our results with the corresponding correlation function for photon squeezed states, which was comprehensively investigated in quantum optics. The form of the normalized correlation function $K^b_{l(2)}$ for cophased squeezed states specifies the gluon antibunching effect if the amplitudes of all gluon fields (with various colours and vector components) are equal to one another. Such a behaviour of angular correlations is analogous to the behaviour of the corresponding correlations of the photon squeezed states at small values of the squeezing parameter. At the same time, there is a distinction between them: in contrast to the normalized correlation function known in quantum optics, the correlation function of the gluon SS has a singularity if the amplitude for the fixed-colour gluon field being studied is greater than the amplitudes for gluon fields with other colour indices. The correlations of the cophased squeezed states specifies the presence of the gluon antibunching effect, whereas the gluon bunching occurs for antiphased squeezed states. Hence, the non-perturbative gluon evolution makes a contribution to the parton distribution prepared by the perturbative stage of jet evolution in the form of a sub-Poissonian (cophased squeezed states) or a super-Poissonian (antiphased squeezed states) distributions.

Thus, the behaviour of the two-particle angular and rapidity correlations can serve as one of the criteria of the existence of squeezed gluon states. At the same time for a comparison of our results with experimental data, we must take into account the contribution of the perturbative stage of jet evolution and hadronization effects. This can be done by using Monte Carlo methods and will be the subject of our further investigations.

In this paper, we have investigated the possibility of coexistence both the condition of squeezing and chaos for some physical system: a mechanical model of Yang—Mills field for SU(2) gauge. Using the Toda criterion, we check the local instability in the corresponding classical system and determine conditions of the coexistence of this effect and squeezing at small times. Under investigation of the local instability within SU(2)-jet model, it was numerically shown that this effect exists under condition if at least one amplitude of the coherent fields α and α^* is imaginary and other is real and vice versa. Thus, for the SU(2)-jet model, the effects of squeezing and chaos can coexist under some conditions.

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ОДНОМОДОВЕ СТИСНЕННЯ ГЛЮОНІВ
У КХД-СТРУМЕНІ

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Р е з ю м е

Вивчається еволюція кольорових глюонних станів в окремому струмені КХД на непертурбативній стадії. За певних умов флуктуація глюонів менша, ніж у когерентних станах. Звідси випливає, що тут можуть виникнути стиснуті глюонні утворення. Нами вивчена кутова та швидкісна залежності нормованих кореляційних функцій для таких глюонних утворень на даній стадії еволюції струменя. Показано також, що ці нові глюонні утворення, за аналогією зі стиснутими фотонними утвореннями, можуть мати як суб-, так і суперпуассонівські статистики, які відповідають антигрупуванню або групуванню глюонів.